Econ 172A, Fall 2004: Problem Set 1

Possible Answers

1. (a) The answer comes from Excel. The spreadsheet is attached.
$x_1 = .5, x_2 = 0, x_3 = 10, x_4 = 0$ with the maximum value $x_0 = 100.5$.

   (b) The answer comes from Excel. The spreadsheet is attached. I modified the first part by adding the restriction that the first constraint must be an equation: $I13 = K13$. There is no need to delete $I13 \leq K13$ since this constraint is implied by the equation (there is also no harm in deleting this constraint.) $x_1 = 3.833333333, x_2 = 6.666666667, x_3 = 3.333333334, x_4 = 0$ with the maximum value $x_0 = 43.83333334$. The constraint makes the previous solution infeasible, so the value of the problem goes down.

   (c) The answer does not change (from part a). This is because when we make the constraint an equation, we shrink the feasible set. So the value of the problem cannot go up. On the other hand, the original solution satisfied the constraint as an equation, so that it remained feasible. Hence the solution cannot go down. By the way, I set up the problem by simply adding the restriction that the first constraint is an equation.

   (d) If I add a constraint of the form $x_1 + x_3 \leq K$ the solution does not change as long as the x from part a satisfies the constraint (that is, $K \geq 10.5$). This is true for the same reason as parts b and c: if you can shrink the feasible set without removing the solution, then the solution cannot change. When $K < 10.5$ the solution must change and the value will go down. I attach a spreadsheet for this computation. The new solution is: $x_1 = 0, x_2 = 0, x_3 = 10, x_4 = .333333$ with the maximum value $x_0 = 41.36363636$.

   (e)

Row	$Basis$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	$Value$
(0)	x_0	-1	-1	-10	-1	0	0	0	0
(1)	x_5	10	3	-4	1	1	0	0	45
(2)	x_6	$< 2 >$	-1	0	3	0	1	0	1
(3)	x_7	0	1	1	0	0	0	1	10

Row	$Basis$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	$Value$
(0)	x_0	0	-1.5	-10	.5	0	.5	0	.5
(1)	x_5	0	8	-4	16	1	5	0	50
(2)	x_1	1	$-.5$	0	1.5	0	.5	0	.5
(3)	x_7	0	1	$< 1 >$	0	0	0	1	10

Row	$Basis$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	$Value$
(0)	x_0	0	8.5	0	.5	0	.5	10	10.5
(1)	x_5	0	12	0	16	1	5	4	90
(2)	x_1	1	$-.5$	0	1.5	0	.5	0	.5
(3)	x_3	0	1	1	0	0	0	1	10

This yields the solution. Notice that I could have pivoted in different columns: in the first iteration, any of the first four columns; in the second iteration either x_2 or x_3. Since I knew the answer, I picked a way that led to a solution in two pivots.

2. (a) Solution: $x_1 = 4.6875, x_2 = -0.625$ and the value is 55.625.

(b) Solution: $x_1 = -4.6875, x_2 = -0.625$ and the value is -55.625.

If the first problem was: $\max x_0$ subject to $Ax \leq b$, then the second problem was $\min x_0$ subject to $Ay \geq -b$. Replace y by $-x$ and we see that the problems are the same, so all the transformation does is change the sign of the solution and the value.

(c) The feasible set has two corners: $(\frac{15}{7}, \frac{55}{7})$ and $(\frac{75}{16}, -\frac{5}{8})$. It is bounded by the three constraints (although it includes arbitrarily small points – it is unbounded in the south west).

(d) When the objective function is parallel to the constraint $10x_1 + 3x_2 \leq 45$ any point on the north east boundary of the feasible set is a solution; if there is more weight on x_1, the solution is the same as in (a). Therefore we need $K \geq \frac{10}{3}$. When K is smaller the solution shifts to $(x_1, x_2) = (\frac{15}{7}, \frac{55}{7})$.