Econ 172A, Fall 2003: Problem Set 1

Due: October 12, 2004, in class (no late papers)

The primary purpose of this assignment is to make sure that you can use Excel. (The secondary purpose is to reinforce ideas from the first three weeks of the class.) If you own a copy of Excel (with Solver installed), use it. Otherwise, you may work in the Social Science Computation Lab (Econ 100).

You should be able to do most of the problems now (or shortly after reviewing the lecture notes on Excel available in the reader or on the website). I will not discuss the computer program during lecture. Using Excel can be frustrating. I encourage you to try to do these problems now instead of waiting until the night before they are due.

Notes:

- You do not yet know how to interpret the information that Excel provides on sensitivity analysis. There is no need to include Excel sensitivity reports with your answers.
- You may work with others, but you must submit your own work.
- Clearly label your computer output (you can do this by carefully labeling your Excel spreadsheets or by annotating your printouts).
- In addition to the information from Excel, for each problem separately note the answers.
- Your numerical answers should contain an explanation (usually brief is sufficient) explaining how you arrived at the answer.
- Whenever a question asks for "justification" you **must** provide a complete reason. A numerical answer or a simple 'yes' or 'no' is not sufficient.

1. Consider the linear programming problem:

   Find x_1, x_2, x_3, x_4 to solve: $\max x_1 + x_2 + 10x_3 + x_4$ subject to:

$$\begin{array}{rcrcrcrcr} 10x_1 & + & 3x_2 & - & 4x_3 & + & x_4 & \leq & 45 \\ 2x_1 & - & x_2 & & & + & 3x_4 & \leq & 1 \\ & & x_2 & + & x_3 & & & \leq & 10 \\ & & x & & & & & \geq & 0 \end{array}$$

   (a) Solve the problem using Excel. (You can begin by copying the template as explained in the lecture notes.)

   Parts b and c vary the original problem. To save time, you can save the original problem after you type it into the spreadsheet. There are a few ways to do this. I recommend copying the page. Select the page where you created the spreadsheet for part a, bring down menu: "Edit;" select "Move or Copy Sheet . . . ;" check the make a copy box in the "Move or Copy" dialog box; (you can now decide where you want the copy to go); click ok. The copy will have a name like "Problem 1a (2)". You should right click on the tab with the name of the worksheet and rename it something useful.

   (b) Change the original problem by turning the first constraint (the one that requires $10x_1 + 3x_2 - 4x_3 + x_4 \leq 45$) into an equation. Print out the new problem, the solution, and identify the optimal value of each variable on the printout. Compare the value of the objective function in parts a and b. Explain this relationship intuitively.

(c) Change the original problem by turning the second constraint (the one that requires $2x_1 - x_2 + 3x_4 \leq 1$) into an equation. Print out the new problem, the solution, and identify the optimal value of each variable on the printout. Compare the value of the objective function in parts a and b. Explain this relationship intuitively.

(d) Change the original problem by adding the constraint $x_1 + x_3 \leq 20$. Print out the new problem, the solution, and identify the optimal value of each variable on the printout. Compare the value of the objective function in parts a and d. Could you have guessed this solution in advance? What if you changed the right-hand side to 16? or to 10?

(e) Solve the problem in part a using the simplex algorithm.

2. Consider the problem: Find x_1, x_2 to solve: $\max 12x_1 + x_2$ subject to:

$$\begin{array}{rcrcr} 10x_1 & + & 3x_2 & \leq & 45 \\ 2x_1 & - & x_2 & \leq & 10 \\ x_1 & + & x_2 & \leq & 10 \end{array}$$

(a) Solve the problem using excel (note: variables may take on negative values).

(b) Now solve: $\min 12x_1 + x_2$ subject to:

$$\begin{array}{rcrcr} 10x_1 & + & 3x_2 & \geq & -45 \\ 2x_1 & - & x_2 & \geq & -10 \\ x_1 & + & x_2 & \geq & -10 \end{array}$$

What is the relationship between this problem and the one in part a? What is the relationship between their solutions? Try to describe a general property illustrated by these examples (by general property, I mean one that does not depend on the specific numbers in the problems, just the transformation involved).

(c) Solve the problem in part a graphically.

(d) Suppose that the objective function in (a) was $x_0 = Kx_1 + x_2$ (so that in part (a) $K = 12$). What is the smallest value of K for which the solution you found to part (a) remains a solution. When K is less than this number, what happens to the solution. (With trial and error, you should be able to come up with the answer via excel. I recommend that you use graphical methods, which are simpler.)