Practice Formulation Problems

Here are some practice formulation problems. By the end of the quarter you will be able to solve (or at least get Excel to solve) these problems (except for the one that does not specify numerical values for the data). At this point, the exercise is simply to translate the story into the mathematical form of a linear programming problem. (You should be able to figure out the numerical solution to the second question, however.) While formulating these problems you should reflect on what the linearity assumptions mean (that is, what restriction does assuming linearity place on the problem). After you come up with a formulation, read it over to see whether it makes sense. (It is much easier to check to see whether you have incorrectly formulated a problem than to come up with a correct formulation.)

1. A nut packer has on hand 150 pounds of peanuts, 100 pounds of cashews, and 50 pounds of almonds. The packer can sell three kinds of mixtures of these nuts: a cheap mix consisting of 80% peanuts and 20% cashews; a party mix with 50% peanuts, 30% cashews, and 20% almonds; and a deluxe mix with 20% peanuts, 50% cashews, and 30% almonds. If the 12-ounce can of the cheap mix, the party mix, and the deluxe mix can be sold for \$.90, \$1.10, and \$1.30 respectively, formulate a linear programming problem that determines the number of cans of each type the producer should produce to maximize her return.

2. Ingredients are listed on food containers in order of their percentage contribution to the weight of the product. If an ingredient is listed kth out of N (for example, third out of ten), find the minimum and maximum percentage contribution of that ingredient. Formulate this problem as an LP.

3. A computer firm manufactures two models of microcomputers, the Lemon and the Banana. The firm employs five workers; each works 160 hours per month. Management insists on maintaining full employment during the next month. It requires 18 hours of labor to assemble a Banana and 25 hours to assemble a Lemon. The company wants to produce at least 10 Bananas, but no more than 20 Lemons, during the next month. Bananas generate \$1200 profit per unit, and Lemons yield \$1800 each. Formulate a linear programming problem that describes the firm's optimization problem.

4. Suppose that the *minimum* number of buses required at the ith hour of the day is b_i, $i = 1, 2, \ldots, 24$. Each bus runs 6 consecutive hours. If the number of buses in period i exceeds the minimum required, an excess cost c_i per bus hour must be paid. Formulate an LP that determines how to minimize the total excess cost incurred.

5. A businesswoman has the option of investing her money in two plans. Plan A guarantees that each dollar invested will earn 70 cents a year later. Plan

B guarantees that each dollar invested will earn \$2 two years later. In Plan B, only investments for periods that are multiples of 2 years are permitted. Suppose that the businesswoman has \$100,00 and seeks to maximize the earnings at the end of 3 years. Formulate an LP problem that will do this.

6. (a) A fertilizer company has decided to manufacture a large supply of various plant foods to be sold during the upcoming planting season. The company can invest up to \$25,000 in the three basic ingredients: nitrates, which cost \$800 per ton; phosphates, which cost \$400 per ton; and potash, which costs \$1000 per ton. Three standard grades of plant food will be produced from these ingredients: regular, in which nitrates, phosphates, and potash are combined, respectively, in a 3:6:1 ratio by weight; extra is a 4:4:1 mixture; super is a 6:4:3 mixture. Regular can be sold for \$750 per ton. Extra can be sold for \$800 per ton. Super can be sold for \$900 per ton. The company's objective is to maximize profits (total sales minus total expenditures for ingredients). Its production capacity permits it to manufacture no more than 40 tons of plant food overall. Formulate an LP that would determine how much of each ingredient it should buy and how much of each grade of plant food it should produce.

   (b) Repeat part (a) subject to the additional condition that the firm can somehow earn an immediate 10% on all capital **not** invested in nitrates, phosphates, and potash. Hence, the firm can earn \$1 on each \$10 not spent on the three ingredients.

7. A gambler plays a game that requires dividing her money among four different choices. The game has three outcomes. The following table gives the corresponding gain (or loss) per dollar deposited in each of the four choices for the three outcomes. Assume that the gambler has a total of \$500, with which she may play only once. The exact outcome of the game is not known. In the face of this uncertainty, the gambler decided to make the allocation that would maximize her minimum return. Formulate the problem as a linear programming problem.

Gain (or Loss) per Dollar Risked in Given Choice

OUTCOME	1	2	3	4
1	-3	4	-7	15
2	5	-3	9	4
3	3	-9	10	-8

8. An investor has three investment opportunities available at the beginning of the next 5 years and has a total of \$50,000 available. The characteristics of the investments are:

Investment	*InitialBound*	*Return*(%)	*Term*
1	\$100,000	9	1
2	no limit	6	2
3	\$50,000	10	3

Formulate a linear programming problem that determines the investment plan that maximizes the amount of money that can be accumulated by the beginning of the sixth year.

9. An insurance company developed a list of seven investment instruments with corresponding financial factors. The table below presents these investments and their financial factors. The term of the investment is the expected number of years required for the annual rate of return to be realized (these figures take into account the possibility of reinvestment). The return is the expected rate of return over the 10-year investment horizon. The risk column describes the subjective estimate that represents the portfolio manager's appraisal of the relative safety of each alternative. The risk coefficient is based on a scale of ten. The growth column gives a subjective appraisal of the potential (percentage) increase in the value of the investment for the ten-year period.

Investment	*Term*	*Return*(%)	*Risk*	*Growth*(%)
Treasury Bills	4	3	1	0
Common Stock	7	12	5	18
Corporate Bonds	8	9	4	10
Real Estate	6	20	8	32
Mutual Fund	10	15	6	20
Savings and Loan	5	6	3	7
Cash	0	0	0	0

The company wants to maximize the return on its portfolio of investments, subject to the following restrictions.

(a) The average length of the investment for the portfolio should not exceed 7 years.

(b) The average risk should not exceed 5.

(c) The average growth potential should be at least 10%.

(d) At least 10% of all available funds must be in cash.

(e) The proportion of funds invested must sum to one.

Formulate the company's problem as a linear program.