Econ 172A, Fall 2003: Midterm Examination II

Instructions.

1. Please check to see that your name is on this page. If it is not, then you are in the wrong seat.
2. The examination has three questions. Answer them all.
3. If you do not know how to interpret a question, then ask me.
4. You must justify your answers, with one exception. In Question 1, provide a brief explanation of the steps in your reasoning. You must invoke Complementary Slackness. In Question 2, the formulation parts require that you define the variables in words (complete sentences, please) and assign them appropriate units. Parts (b), (d), and (e) require brief reasons. You can, for example, show that a problem is feasible by identifying a point in the feasible set. No justification is required for the first part of Question 3. On the second part (parts (d) - (g)) you must briefly explain why your answer is correct.
5. The table below indicates how points will be allocated on the exam.

	Score	Possible
I		30
II		35
III		35
Exam Total		100

1. Consider the linear programming problem:

   Find x_1, x_2 and x_3 to solve:

$$\begin{array}{rrcrcrcl} \max & x_1 & + & 3x_2 & + & 4x_3 & & \\ \text{subject to} & x_1 & + & x_2 & + & x_3 & \leq & 5 \\ & 2x_1 & - & x_2 & - & 4x_3 & \leq & 4 \\ & & & x_2 & + & 2x_3 & \leq & 3 \\ & & & & & x & \geq & 0 \end{array}$$

   (a) Write the dual of the problem.

   (b) Use Complementary Slackness to determine which of the vectors (x_1, x_2, x_3) below is a solution to the problem.

      i. $(x_1, x_2, x_3) = (2, 3, 0)$

      ii. $(x_1, x_2, x_3) = (4, 1, 0)$

      iii. $(x_1, x_2, x_3) = (3, 1, 1)$

   (c) Use your answer to the previous question to find a solution to the dual.

2. BENCO makes three types of toys: trains, trucks, and cars. Trains earn a profit of \$3 each; trucks earn \$2 each; and cars earn \$5 each. The toys are made from wood, paint, and labor. To make a train you need one minute of labor; three ounces of paint; and 1 square foot of wood. To make a truck you need two minutes of labor; one ounce of paint; and four square feet of wood. To make a car you need one minute of labor; two ounces of paint; and one square foot of wood. BENCO has available 430 minutes of labor; 460 ounces of paint; and 420 square feet of wood. BENCO wishes to determine what to produce in order to maximize profits.

   (a) Formulate a linear programming problem that will determine BENCO's profit maximizing production plan. Your formulation should include: a definition of variables (including units); an algebraic expression for the objective function; and an algebraic expression for the constraints.

   (b) Is the problem you wrote in part (a) feasible? If it is, explain why it is. If it is not, explain why it is not. If you cannot tell, explain what additional information you would need to know in order to determine whether it is feasible.

   (c) Write the dual of the problem you found in part (a).

   (d) Is the problem you wrote in part (c) feasible? If it is, explain why it is. If it is not, explain why it is not. If you cannot tell, explain what additional information you would need to know in order to determine whether it is feasible.

   (e) Does the problem in part (a) have a solution? If it does, explain why. (It is not necessary to solve the problem.) If it does not, explain why not. If you cannot tell, explain what additional information you would need to know whether they problem had a solution.

   (f) Interpret the dual that you found in part (c). Your interpretation should define the dual variables in words (and provide units for the dual variables) and explain what the dual objective function and constraints mean in your problem.

3. I solved a linear programming problem written in the form:

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0.$$

Attached find the Excel Answer and Sensitivity report. (I deleted some irrelevant information.) In these reports, I replaced several values with letters ((a) through (ff)). Using the information in the table, replace as many question marks as possible with the correct information. You need not justify these answers (simply write the answers in the appropriate spaces, next to the question mark). If you do not have enough information to figure out one or more of the values, write "NOT ENOUGH INFORMATION" next to the question marks.

In addition to completing the tables, please answer the following questions. For these questions, short justifications are required.

(a) How many variables are in the original problem?

(b) How many variables are in the dual?

(c) What is the objective function of the original problem?

(d) Would the solution to the problem change if the coefficient of x_2 in the objective function were decreased by 3 (and the rest of the problem remained unchanged)?

(e) What would the value of the problem be if the right hand side constant on the first constraint were 224 (and the rest of the problem remained unchanged)?

(f) What would the value of the problem be if the right hand side constant on the second constraint were 50 (and the rest of the problem remained unchanged)?

(g) What would the solution to the problem be if the coefficient of x_1 in the objective function was 5 (and the rest of the problem remained unchanged)?