Econ 172A, F2003: Midterm Examination I

Instructions.

1. Please check to see that your name is on this page. If it is not, then you are in the wrong seat.
2. The examination has 4 questions. Answer them all.
3. If you do not know how to interpret a question, then ask me.
4. You must justify your answers to the first three questions. No justification is needed on the last question.
5. The table below indicates how points will be allocated on the exam.

	Score	Possible
I		30
II		25
III		25
IV		20
Exam Total		100

1. Consider the following linear programming problem. Find x_1 and x_2 to solve:

$$\begin{array}{rrcrcr} & \max x_0 & & & & \\ \text{subject to} & 5x_1 & + & 3x_2 & \leq & 45 \\ & x_1 & - & x_2 & \geq & 1 \\ & x_1 & & & \geq & 2 \end{array}$$

(a) Graphically represent the feasible set of this problem.

(b) Graphically solve the problem for the following values of x_0:

i. $x_0 = x_1 - x_2$.

ii. $x_0 = -x_1 + x_2$.

iii. $x_0 = 3x_1 + 4x_2$.

In each case, graphically identify the solution (explain why the graph tells you that you have a solution); write down the values for x_1 and x_2 that solve the problem; write down the value of the problem. If the solution to the problem is not unique, then give two solutions. If the solution does not exist, then explain why it does not exist.

(c) Which of the following points can be a solution to a linear programming problem for some (linear) choice of x_0 and the constraint set given above?

i. $(x_1, x_2) = (6, 5)$.

ii. $(x_1, x_2) = (2, 0)$.

iii. $(x_1, x_2) = (2, 2)$.

iv. $(x_1, x_2) = (9, 0)$.

v. $(x_1, x_2) = (3, 2)$.

Justify your answers.

(d) Which of the points in Part (c) can be a **unique** solution to a linear programming problem for some (linear) choice of x_0 and the constraint set given above? Justify your answers.

2. Consider the linear programming problem:

   Find x_1, x_2 and x_3 to solve:

$$\begin{array}{rrcrcrcr}
\text{min} & x_1 & - & x_2 & - & x_3 & & \\
\text{subject to} & 5x_1 & + & 3x_2 & - & 3x_3 & \geq & 45 \\
 & x_1 & - & x_2 & & & = & 1 \\
 & & & x_2 & & x_3 & \geq & 0
\end{array}$$

   (a) Write the problem in the form: $\max c \cdot x$ subject to $Ax \leq b$, $x \geq 0$.

   (b) Write the dual of the problem.

3. Consider the linear programming problem:

$$\begin{array}{rrcrcrcrcl} \max & 2x_1 & - & 4x_2 & - & 6x_3 & + & 5x_4 & & \\ \text{subject to} & x_1 & + & 4x_2 & + & 8x_3 & - & 2x_4 & \leq & 2 \\ & -x_1 & + & 2x_2 & + & 4x_3 & + & 3x_4 & \leq & 1 \\ & & & & & & & x & \geq & 0 \end{array}$$

(a) Write the initial simplex array for the problem. (That is, write the problem in a form suitable for a simplex algorithm pivot.)

(b) Perform two steps of the simplex algorithm starting with the array you provided in part (a). If it is not possible to perform two steps, explain why not. If it is possible, state the feasible "guess" provided by the second step of the algorithm. Confirm that this guess satisfies the constraints of the problem. Explain whether this guess solves the optimization problem.

Use the tables below (which may contain extra rows and or columns) for your answers.

Row	*Basis*	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0									
(1)										
(2)										
(3)										

Row	*Basis*	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0									
(1)										
(2)										
(3)										

Row	*Basis*	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0									
(1)										
(2)										
(3)										

4. For each of the statements below, circle **TRUE** if the statement is always true, circle **FALSE** otherwise. No justification is required.

These problems refer to the linear programming problem (P) written in the form:

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

and its dual

$$\min b \cdot y \text{ subject to } yA \geq c, y \geq 0.$$

(a) **TRUE** **FALSE**
If (D) is not feasible, then (P) is not feasible.

(b) **TRUE** **FALSE**
Let u be a vector of ones (with the same number of components as b). If (P) has a solution, then

$$\max c \cdot x \text{ subject to } Ax \leq (b + u), x \geq 0$$

has a solution.

(c) **TRUE** **FALSE**
If (P) has a solution and $\overline{c} \leq c$, then

$$\max \overline{c} \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

has a solution ($\overline{c}$ may be different from c, but $\overline{c}$ has the same number of components as c).

(d) **TRUE** **FALSE**

If a linear programming problem is infeasible, then it will continue to be infeasible if the objective function changes.