**Econ 172A, Fall 2003: Problem Set 3, Suggested Answers**

1. (a) Up is a dominant strategy for Row. The value of the game is 5. Row plays Up and Column can do anything in equilibrium.

   (b) Equilibrium strategies are (Down,Left), the value is 2. (Pure strategy security levels are equal.)

   (c) Same as (b).

   (d) In all four parts Column's second strategy is dominated. Here it is useful to delete the strategy. What is left is a game like matching pennies. Both players must randomize 50-50. The value is one.

   (e) Here you need to do some work. Since Row has only two strategies, you can graph his payoffs. When you do, you find that Column will want to play left if Row plays Up with probability of greater than $\frac{2}{3}$; Column will want to player middle if Row plays Up with probability between $\frac{4}{9}$ and $\frac{2}{3}$; and Column will want to play right if Row plays Up with probability less than $\frac{4}{9}$ (draw the picture). Hence the security level of Row occurs when row plays Up with probability of $\frac{4}{9}$ and he earns expected payoff $\frac{24}{9}$. If you draw the picture, you will see that Column can hold Row to security level using either middle or right. So you can find column's security level using only her middle and right strategies. When you do so you get her equilibrium mixture: $(0, \frac{2}{3}, \frac{1}{3})$

2. (a) This game has a pure strategy equilibrium in which both players use their third strategy. The value is eight. I attach an excel spreadsheet, but you could figure this out by computing the pure-strategy security levels.

   (b) Increasing payoffs is good for player 1. Since all numbers in row 1 are positive, then doubling the numbers in the first row is good for player one. I resolved the problem (using excel) to obtain Column's best strategy: $(0.215384615, 0, 0.330769231, 0.453846154)$ and the value for row: 10.41538462. I got row's best strategy by looking at the sensitivity table. Row's optimal mixture is the negative of the dual variables: $(0.446153846, 0.261538462, 0.292307692, 0)$.

   (c) Decrease payoffs by five is just like charging the row player 5 to play the game. Equilibrium strategies do not change. The value (to the row player) goes down by 5.

   (d) This does not change the answer from (a) since making a strategy that Column didn't like even less attractive is irrelevant to the solution of the game.

   (e) In (b) increasing payoffs is good for row (in zero-sum games). In (c) adding a constant to all payoffs never changes equilibrium payoffs, but changes the value by the amount of the constant. In (d) increasing the number can't hurt row. If column wasn't playing the strategy before, then she'll avoid it after the change.

3. In the game, Blotto has three strategies: to attack with 0, 1, or 2 companies. The enemy has two strategies: to attack with 0 or 1 company. If Blotto attacks with 2 and his enemy attacks with 1, then both leave their camp undefended, so both camps fall, so the payoff is zero. If Blotto attacks with 2 and his enemy attacks with 0, then Blotto wins. Blotto also wins if both attack with one. Otherwise, neither side wins. The payoff matrix is:

| | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 2 | 1 | 0 |

The security level for Blotto is zero. The security level for the enemy is $-1$ (that is, if Colonel Blotto knew the enemy's strategy, then he would certainly be able to defeat the enemy).

It makes no sense for Blotto not to attack. That is, attacking with 0 companies is dominated.

Eliminating that strategy, it should be clear that it is best for both players to randomize equally over their remaining strategies.

If the enemy could spy, that means the enemy could make its choice of how to attack contingent on what Colonel Blotto does. One strategy would be: Attack with one company whenever Blotto leaves his camp undefended. Otherwise, don't attack. This strategy yields a payoff of zero. The payoff matrix for the game (with a spy) is quite large if you include all three of Blotto's original strategies (the enemy's strategy specifies whether to attack if Blotto attacks with 0, 1, or 2 companies. There are a total of 8 such strategies). The game simplifies if you take into account that Blotto will always either attack with 1 or 2 companies. In this case, the enemy has four strategies (00: never attack; 01: attack only if Blotto attacks with 2 companies; 10: attack only if Blotto attacks with 1 company; 11: always attack). The payoff matrix is:

| | 00 | 01 | 10 | 11 |
|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 |
| 2 | 1 | 0 | 1 | 0 |