Econ 172A, Fall 2003: Problem Set 2, Suggested Answers

Comments: Grades were high on this assignment (mean 90; median 93). There were significant deductions on question 1 for students who only submitted excel output; a large number of students gave incorrect answers to 2(g) – deductions were small, but you might consider asking the "Econ Tutor" for a rebate; students who did not provide excel worksheets for question 2 lost 10 points. Please note: while it was acceptable to solve problem 2 by using excel over and over again, many of the questions can be answered using the sensitivity table. You will need to be able to find answers using the table on the exam. Learn how to do it.

1. I denote by x_i the number of cases of each variety of wine produced in a week, for i = red, white, and blue. Taking into account the costs of production, the objective function is

$$\max 5x_R + 8x_W + 4x_B.$$

In addition to non-negativity constraints, the problem has four other constraints:

   (a) $x_R + 2x_W + 3x_B \leq 220$ (processing time)

   (b) $6x_R + 8x_W + 8x_B \leq 480$ (bottling time)

   (c) $1.5x_R + 1.5x_W + 2x_B \leq 3000$ (warehouse capacity)

   (d) $x_R \geq 60$ (contracted delivery of red wine)

When I solve this, I find that it is right to produce 60 cases of red and 15 of white, earning \$420 each week.

2. (a) This change is within the allowable range. Production doesn't change, but profits rise by \$1 per case, a total of \$60. (So profits are now 480.)

   (b) If Blue's price doubles, then it goes up by \$21. This is a \$21 reduction in the profit per case of blue. Since you were not producing Blue before, you should not start now. Production and profits do not change.

   (c) If the production cost of blue is cut in half, the profit goes up by 11.5. This is not an allowable increase, so the solution changes. Profit increases to \$517.50. The new production plan is to producte 60 red and 15 blue.

   (d) This change is not within the allowable range (it is larger than 1.33), so you need to resolve the problem. The new solution is to produce only white (80 cases) and make 400 (lower than the original 420). (See excel.)

   (e) Nothing. Pallo only needs 90 hours of processing.

   (f) Now things change. You produce 70 cases of white, nothing else, and earn 350. (See excel.)

   (g) Nothing happens. Pallo has excess warehouse capacity. Additional warehouse space is worth nothing, so Pallo doesn't buy the new warehouse at any price.

   (h) Profits go up by \$10, because the dual variable for the wine constraint is equal to \$1, a reduction of the constraint by 10 to 50 is in the allowable range, so Pallo saves \$1 for each of the 10 cases that no longer needs to be supplied.

(i) Since Pallo makes 15 cases of white wine, this change saves 15 hours of bottling time. Since bottling time has a dual price of $1, this should increase profit by at least $15. The change in technology may also cause other changes in the production process, so it is useful to redo the problem using excel. When you solve the problem, it turns out that you increase production of white wine to 17.14 cases (red and blue production remain the same). Profit goes up to $437.14, an increase of slightly more than $15.

(j) The new product has a profit contribution of $8 - $5 = $3. Using the dual variables, you can figure out the cost of the ingredients. There is an excess supply of warehouse space and processing time, so these inputs cost nothing. Bottling time costs $1 per hour (shadow price of bottling constraint), so a case of yellow wine will add $2 in additional bottling cost. Since this cost is less that the profit per case, it is worthwhile to product yellow wine (Pallo net earnings increase by $1 per case produced).

3. (a) The problem as written is unbounded. x_3 can be made arbitrarily large, increasing the objective function without violating constraints. Excel just tells you that cell values do not converge (and does not enable sensitivity analysis). This problem is more interesting when the objective is minimize!

   (b) The transformed problem is $\max c \cdot x$ subject to $Ax \leq b$, where $c = (3, 1, 1, -1)$, $b = (-50, 8)$, and

$$A = \begin{bmatrix} 1 & 0 & -5 & 5 \\ 2 & -3 & 0 & 0 \end{bmatrix}$$

   (c) The answer is $\min b \cdot y$ subject to $yA \geq c$, where A, b, c come from the previous part. Written out in detail:

$$\begin{array}{lrcrcr} \text{min} & -50y_1 & + & 8y_2 & & \\ \text{subject to} & y_1 & + & 2y_2 & \geq & 3 \\ & & - & 3y_2 & \geq & 1 \\ & -5y_1 & & & \geq & 1 \\ & 5y_1 & & & \geq & -1 \\ & & & y & \geq & 0 \end{array}$$

   (d) You should still obtain a message that cell values do not converge – the problem remains unbounded.

   (e) You do not have any information to compare, except for the solver's report that the problem is unbounded. There are no differences between the two problems or their solutions.

4. For the fourth problem, I include the complete Excel file. Filling in the blanks required understanding that certain information is repeated (for example, the value of the variables and the left-hand sides of constraints) and complementary slackness, which allows you to conclude that when a dual variable is positive, then there is no slack (and so on). Finally, if you know the solution and the objective function coefficients, then you can figure out the value of the problem. For this problem, the variables are SS, AS, SC, and DC. When you solve the problem these variables take on values: 1000, 39000, 12000, and 5333.333. The respective coefficients are given: .6,.6,12,14. Hence the value is

$$.6(1000) + .6(39000) + 12(12000) + 12(5333.333) = 242666.333.$$

I do not know how to figure out the allowable increase for SC market limit and SS. The correct answer to these questions is "I don't know."