Econ 172A
Fall 2003

Possible Answers to Problem Set #1

Comments: There are 100 points for possible. The class median was 85.

Grading: In the first question, you typically lost five points for incorrect numerical values. On parts (d) and(e) there were reductions for incomplete or incorrect intuition. In the second question, there were also five point deductions for incorrect numerical answers. We made larger deductions for incomplete justification in the last two parts.

Substance:
1. (b) Most people did not state that the feasible set shrinks when the inequality changes to an equation.
   (d) Many identified the trade-off relationship between x_2 and x_4 but failed to see that the problem was unbounded.
   (e) Few correctly discussed the technical issues of slack and binding constraints, but could provide an intuitive answer.

2. (c) Some correctly identified the relative effects of the changed coefficients in objective function, but most simply stated the changes as reason without intuition.
   (d) Many drew the graphs correctly but did not provide justification for solution (i.e. direction of increase of level sets). Please provide justification on the exam!

1. All Excel output is on the accompanying Excel Spreadsheet.
   a. $x_1 = 0$, $x_2 = 20$, $x_3 = x_4 = 0$. The value of the objective function is 200.
   b. The answer does not change.
   c. $x_1 = x_2 = x_3 = x_4 = 0$. The value of the objective function is 0.
   d. The problem is unbounded (my spreadsheet is what Excel provided; the solver gave the message: “The cell values do not converge.”)
   e. The answer is the same as part a when the right-hand side is at least 20. When it is less than 20, the answer changes. The spreadsheet provides the answer when the right-hand side is 16: $x_1 = 0$, $x_2 = 16$, $x_3 = x_4 = 0$. The value of the objective function is 160.

When you go from part a to part b, you shrink the feasible set (by turning the second constraint into an equation). This change cannot make the maximum value go up because it reduces the set of feasible choices. In this case, shrinking the feasible set does not hurt you because the constraint that you turned into an equation held as an equation in part a. In part c solution is to set $x_1 = x_2 = x_3 = x_4 = 0$. The minimum is zero. You may have been able to guess this without computation because the objective function is increasing in each variable, so since the variables are all constrained to be non negative, zero is the lowest conceivable value for the objective function. Since setting all variables equal to zero is feasible, it must solve the minimization problem. In part d, the change allows x_4 to be negative. It should be clear that doing this makes it possible to increase the value of the objective function: making x_4 negative allows you to increase x_2, which (because of the relative values of the coefficients in the objective function and the constraints) increases the value of the objective function. Excel tells you that you can increase the value of the objective function without bound. Compare part a to part e. When you solve part a, you have some of the second resource “left over” (you use up 20 of your 24 units). Increasing the amount available can’t help you. Decreasing it, but leaving at least 20 units doesn’t hurt you. Once you decrease the right-hand constant below 20 the solution you found in part a is no longer feasible. The value of the problem decreases.

2. a. The solution is to set x = 0 and y = 20. The optimal value is 80.

b. i. The solution is to set x = 0 and y = 20. The optimal value is now 60.

b. ii. The solution is to set x = 0 and y = 5. The optimal value is now -5.

c. The difference between the problems in Part b and the problem is Part a is that the coefficients of the objective function are different.   The solution to b. i. does not change because the coefficients are “relatively” similar. When we reduce the coefficient of y in the objective function from 3 to –1 (going from b.i to b.ii), using y becomes sufficiently unattractive than it is appropriate to decrease y. The reason that we don’t decrease y to zero is that doing so would violate the second constraint

d. I will leave this to you. You should be able to duplicate the answers of the earlier parts.