Granular Instrumental Variables

Discussion by James D. Hamilton

These slides available at http://econweb.ucsd.edu/~jhamilto/slides/ASSA_20220107_granular_IV.pdf

All variables measured in deviation from mean

q_{it} = log of oil production in country i

p_t = log of price of oil

supply: $q_{it} = \phi^q p_t + \eta_t^q + u_{it}^q$

ϕ^q = elasticity of oil supply $(\phi^q > 0)$

η_t^q = shock to supply that is common to all countries

u_{it}^q = shock to supply that only affects i

q_t = log of total world oil production

$$= \log\Big(\sum_{i=1}^{N} \exp(q_{it})\Big)$$

$$\simeq \sum_{i=1}^{N} s_i^q q_{it}$$

s_i^q = average share of country i in world production

$$\sum_{i=1}^{N} s_i^q = 1$$

Multiply supply curve for country i

$$q_{it} = \phi^q p_t + \eta_t^q + u_{it}^q$$

by s_i^q and sum over i

$$q_t = \phi^q p_t + \eta_t^q + u_t^q$$

$$u_t^q = \sum_{i=1}^{N} s_i^q u_{it}^q$$

c_{it} = log of oil consumption in country i

demand: $c_{it} = \phi^c p_t + \eta_t^c + u_{it}^c \qquad \phi^c < 0$

η_t^c = shock to demand that is common to all countries

c_t = log of total world oil consumption $\simeq \sum_{i=1}^N s_i^c c_{it}$

s_i^c = average share of country i consumption

Multiply demand curve by s_i^c and sum over i

$$c_t = \phi^c p_t + \eta_t^c + u_t^c$$

$$u_t^c = \sum_{i=1}^N s_i^c u_{it}^c$$

Assumption:

$$E\begin{bmatrix} u_{1t}^{c} \\ \vdots \\ u_{Nt}^{c} \end{bmatrix}\begin{bmatrix} \eta_t^q & u_{1t}^q & \cdots & u_{Nt}^q \end{bmatrix} = \mathbf{0}$$

supply: $q_t = \phi^q p_t + \eta_t^q + u_t^q$

demand: $c_t = \phi^c p_t + \eta_t^c + u_t^c$

equilibrium: $c_t = q_t$

$$\Rightarrow p_t = \frac{\eta_t^c + u_t^c - \eta_t^q - u_t^q}{\phi^s - \phi^q}$$

Cannot estimate ϕ^q by OLS because

$\eta_t^q + u_t^q$ is correlated with p_t

$$c_{it} = \phi^c p_t + \eta_t^c + u_{it}^c$$

$$c_t = \phi^c p_t + \eta_t^c + u_t^c$$

$$z_{it}^c = c_{it} - c_t = u_{it}^c - u_t^c$$

z_{it}^c depends only on $u_{1t}^c, \ldots, u_{Nt}^c$

z_{it}^c is uncorrelated with $\eta_t^q + u_t^q$

z_{it}^c is correlated with p_t

$\Rightarrow$ z_{it}^c is valid instrument for estimating ϕ^q

Could use z_{it}^c for any country i (have N instruments)

Can test the $(N-1)$ overidentifying assumptions

If u_{it}^c has variance σ_{ic}^2 and is uncorrelated with u_{jt}^c,

optimal instrument is $\dfrac{\sum_{i=1}^{N} \sigma_{ic}^{-2} z_{it}^c}{\sum_{i=1}^{N} \sigma_{ic}^{-2}}$

Authors suggest we might instead use

$z_t^c = N^{-1} \sum_{i=1}^{N} z_{it}^c = N^{-1} \sum_{i=1}^{N} c_{it} - c_t$

arithmetic average minus total.

Could in fact also use z_t^c to estimate supply elasticity ϕ_i^q for each country separately:

$$q_{it} = \phi_i^q p_t + \eta_t^q + u_{it}^q$$

and test restriction $\phi_1^q = \cdots = \phi_N^q$

To estimate demand elasticity, use

$$z_t^q = N^{-1} \sum_{i=1}^{N} q_{it} - q_t = N^{-1} \sum_{i=1}^{N} u_{it}^s - u_t^s$$

as an instrument for price in

$$c_t = \phi^c p_t + \eta_t^c + u_t^c$$

In general, these are different instruments

because $\sum_{i=1}^{N} q_{it} \neq \sum_{i=1}^{N} c_{it}$

MLE can be interpreted as IV

gives optimal way to implement

$$\mathbf{y}_t = (q_{1t}, \ldots, q_{N,t}, c_{1t}, \ldots, c_{N-1,t}, p_t)'$$

$$\mathbf{y}_t = \mathbf{c} + \mathbf{\Phi}_1 \mathbf{y}_{t-1} + \cdots + \mathbf{\Phi}_m \mathbf{y}_{t-m} + \boldsymbol{\varepsilon}_t$$

$$E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') = \mathbf{\Omega}(\boldsymbol{\theta})$$

$$\boldsymbol{\theta} = (\phi^q, \phi^c, E(\eta_t^q)^2, E(\eta_t^c)^2, E(u_{it}^q)^2, E(u_{it}^c)^2)'$$

Could take upper $(N \times N)$ blocks of $\mathbf{\Phi}_j$

to be diagonal