# Pareto Optimal Redistribution

By Harold M. Hochman and James D. Rodgers*

The neoclassical approach to public finance identified with Richard Musgrave [10, Ch. 1] divides the process of budget determination and the functions of government into three parts or branches.[1] The allocation branch, justified by the failure of the market to satisfy the demand for public goods, engages in explicit reallocative measures required to rectify this failure and achieve allocative efficiency. The distribution branch is charged with the purely normative responsibility of bringing about the desired size distribution of income, or optimal Lorenz curve, through taxation and transfer payments. The stabilization branch performs the conventional macroeconomic fiscal functions of attaining full employment, price stability, and a satisfactory rate of economic growth.

Though the interdependence of these three branches is generally recognized, their conceptual separation serves both a methodological and heuristic purpose. The distinction between actions designed to promote efficient use of resources and actions designed to make the distribution of income more equitable avoids the "... confusion of the underlying issues at the planning stage" that would result if the budget were viewed " . . . in consolidated terms from the outset" [10, p. 38] and helps the analyst to sort out the diverse issues with which public finance deals.[1] However, the neoclassical approach adopts the tripartite separation not only because it offers the analyst a useful intellectual framework, but because it also serves as a foundation on which a normative theory of the budget based on the value postulate of consumer sovereignty can be constructed. This, as argued elsewhere [4], raises logical difficulties. This normative theory permits only allocation activities, and even here only the provision of public goods that are not merit wants, to be judged in terms of the Pareto criterion. Its implication is that redistribution and stabilization cannot (or should not?) be consistent with consumer sovereignty.

We believe that this line of reasoning is misleading. It implies that redistribution yields no benefits to the parties who finance it, so that from this viewpoint it imposes a simple deadweight loss. The implication is rather unappealing, to say the least. If accepted, redistribution carried out by government institutions can only be explained as legalized Robin Hood activity, and redistribution through private institutions would seem to imply individual irrationality. While it is plausible to assume that some portion of governmental redistribution simply reflects the political power of the recipients, it is also plausible that part of this redistribution is beneficial to the taxpayers as well as to the recipients. The benefit to the former group would appear to stem from two sources, which need not be mutually exclusive. One

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[1] It is, of course, an oversimplification to associate the sharp separation of allocation and distribution problems with Musgrave alone. This treatment is characteristic in neoclassical economics generally, and in particular in the "new welfare economics." See, for example, [1] and [9]. We cite Musgrave because his treatise is a core part of virtually every graduate course in public finance.

is the preference for security against drastic future income fluctuations, and the second is interdependence among individual utility functions. In the analysis which follows, a model is developed to explain redistribution in terms of this latter source of benefit—interdependent preferences.

Given interdependence among individual utility functions, it is possible that some redistribution will make everyone better off.[2] Efficiency criteria can be applied, therefore, to redistribution of income through the fiscal process.[3] If, for example, the utility of individuals with higher incomes depends upon and is positively related to the incomes of persons lower in the distributive scale, tax–transfer schemes which raise the disposable incomes of those in the poorer group may improve everyone's utility level. Where this is true, as we shall assume, Pareto optimality, contrary to the orthodox approach to public finance, is not only consistent with but requires redistribution. Both allocation and redistribution can be dealt with in terms of the same methodology and the same criterion—efficiency. Then it can be argued that the distributive goal of vertical equity is contained within the Paretian concept of efficiency.

A simple example, involving two persons, will clarify this approach to redistribution. Suppose that Mutt, the taller, has an annual income of $10,000 and Jeff, the shorter, an annual income of $3,000. Suppose, further, that Mutt's utility level varies directly with Jeff's income (i.e., $\partial U_M/\partial Y_J>0$ where $U_M$ is Mutt's utility and $Y_J$ is Jeff's income). In determining the appropriate extent of redistribution between Mutt and Jeff, the neoclassical approach, as we interpret it, would not focus on this externality. Instead, it would refer to a social welfare function with a capacity for making interpersonal comparisons. This function would be either a social ordering of the Bergson type or a Benthamite cardinal utility calculus that permits judgments about the equity of distributional adjustments to be couched in terms of objective measures of sacrifice. Indeed, this is inherent in its strictly normative interpretation of redistribution. Our approach, in contrast, implies that redistributive activities can be justified without a social welfare function that makes interpersonal comparisons, provided that utility interdependence is recognized and taken into account in formulating social policy. If, because increases in Jeff's income affect Mutt's utility favorably, gains from trade through redistribution are possible, and if there is no appropriate private vehicle through which Mutt will donate a portion of his income to Jeff,[4] the establishment of collective institutions through which such an income transfer can be processed may increase the welfare of both parties. Redistribution through the fiscal process is just as necessary for the attainment of Pareto optimality in these circumstances, as the collective provision of conventional public goods.[5]

[2] Provided this interdependence takes the form of an external economy.

[3] Similar logic can be applied to the stabilization function. Aggregate targets, too, are public goods, and government action can be justified in terms of the "paradox of isolation." We shall, however, say nothing more about stabilization in this paper.

[4] Voluntary transfers, as within families, would likely occur in the two-person case. In the $N$-person case, however, individuals, unless coerced, may choose to be "free-riders" and it is the incentive to behave in this way that may be viewed as the raison d'etre of government. Since we are interested, ultimately, in the $N$-person case, we rule out voluntary transfers in the present two-person example. And when we turn to the $N$-person case, we assume that the possibility of voluntary redistribution through private charity has been exhausted, thus focusing attention on the incremental redistributive activities carried out under public auspices. For a thorough discussion and analysis of the conditions under which private charity can or will internalize Pareto-relevant interdependence, see David B. Johnson [8].

[5] An alternative way of viewing the problem posed in this paper is in terms of the utility possibility function, a construction frequently employed in welfare

So much for our rationale.[6] Section I examines the possible patterns of utility interdependence in the two-person case and, for one of these, devises a simple model of Pareto optimal redistribution. Section II generalizes this model to the $N$-person case and discusses it in the context of two alternative representations of the size distribution of income. Section III examines the actual pattern of fiscal incidence in the United States, speculates about the conditions under which this pattern might be Pareto optimal, and offers some conjectures as to why actual incidence departs from the hypothetical patterns derived in Section II. Section IV contains some concluding remarks.

## I. *Patterns of Utility Interdependence and Pareto Optimal Adjustments in the Two-Person Case*

It is a fairly simple matter to identify the possible patterns of utility interdependence between two persons with unequal incomes and to select, for further analysis, those which are consistent with realistic distributional adjustments. Consider the utility functions of the two individuals, Mutt and Jeff, who are the only members of our hypothetical community:

$$U_M^0 = f_M(Y_M^0, Y_J^0) \tag{1}$$

$$U_J^0 = f_J(Y_M^0, Y_J^0), \tag{2}$$

where $U_M^0$ and $Y_M^0$ are the initial values of Mutt's utility index and income, respectively, prior to any redistribution, and $U_J^0$ and $Y_J^0$ are the corresponding values for Jeff. As before, we assume that Mutt has the higher income, i.e., $Y_M^0 > Y_J^0$. Interdependence is present because $U_M$ depends on $Y_J$ and because $U_J$ depends on $Y_M$.[7]

TABLE 1—POSSIBLE PATTERNS OF UTILITY INTERDEPENDENCE

| | | $\partial U_J/\partial Y_M$ (Evaluated at $Y_M^0, Y_J^0$) | | |
|---|---|---|---|---|
| | Jeff / Mutt | $>0$ | $=0$ | $<0$ |
| $\partial U_M/\partial Y_J$ (Evaluated at $Y_M^0, Y_J^0$) | $>0$ | I | II | III |
| | $=0$ | IV | V | VI |
| | $<0$ | VII | VIII | IX |

Nine possible pairs of marginal interrelationships between the two utility functions can be identified,[8] and these are given by the cells in Table 1.

Most of these cases can be ruled out, so

economics. The existence of external economies can result in this function having upward sloping portions, positions which cannot be efficient in the Pareto sense. The problem we analyze is one of moving, by means of redistributive transfers, from such an inefficient point to a point where the function no longer slopes upward. See [7, p. 59 ff.] or [13, p. 73 ff].

[6] Since the initial writing of this paper, other research by Becker [2] and Olsen [12] which makes much the same point has come to our attention. Becker's paper, in particular, develops a theoretical apparatus in which the model we use is, in effect, a special case. It deals briefly, in a similar vein, with the fiscal issues on which we focus.

[7] Of course, variables other than income, e.g., wealth' consumption level, or consumption of particular commodities, could be the source of the interdependence. Income is employed here because it simplifies the analysis.

[8] Situations in which externalities are inframarginal are excluded from our consideration. An inframarginal externality exists when

$$\frac{\partial U^i}{\partial Y_j} = 0 \quad \text{and} \quad \int_0^{Y_j} [\partial U^i/\partial Y_j]\, dY_j \gtrless 0.$$

In such cases no transfer is appropriate, though one would be if, given the $i^{th}$ person's income and the assumption that the externality is an external economy, the $j^{th}$ person's income were sufficiently smaller than $Y_j$. In our two-person example, utility interdependence might not be marginally relevant because Mutt's income is too low for his demand for Jeff's income to have become effective or because the initial difference between Mutt's and Jeff's incomes ($Y_M^0 - Y_J^0$), on which we focus, is less than some critical minimum. In this paper, however, we shall apply no restrictions on either far as rationalizing distributive adjustments is concerned, by making a relatively weak assumption and by imposing certain reasonable restrictions. We assume (a) that both Mutt and Jeff, given prevailing prices of goods and services and the prevailing interest rate, have marginal utilities of income for own-consumption greater than zero (i.e., $\partial U_M/\partial Y_M$, $\partial U_J/\partial Y_J > 0$). We require, in addition, that (b) all transfers be Pareto optimal (i.e., harm neither Mutt nor Jeff) and that (c) all transfers flow from the person with the higher income to the person with the lower income. Therefore, since $Y_M$ exceeds $Y_J$, only one-way transfers from Mutt to Jeff are permitted. Furthermore, transfers large enough to reverse the initial distributional ordering are not allowed. For the two-person case, therefore, the transfer can be no greater than $(Y_M^0 - Y_J^0)/2$.

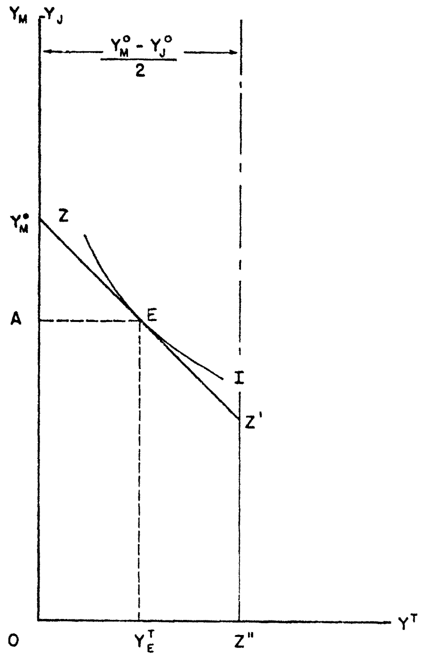
Using assumption (a) and restrictions (b) and (c), all interdependence patterns except those in the top row of Table 1 can immedately be eliminated. Cases IV and VII would require a transfer from Jeff to Mutt, violating restriction (c). Case V represents the situation of utility independence, the orthodox neoclassical assumption; a transfer in either direction, given (c), would harm one of the parties, violating (b). This same conclusion holds also for Cases VI, VIII, and IX. There is no possible transfer, in either direction, that would harm neither Mutt nor Jeff.

Hence only Cases I, II and III remain. The externality patterns of Cases II and III are, for purposes of indicating the Pareto optimal pattern of redistribution, one-way patterns, which imply that only Mutt's preferences need be consulted. So long as Jeff's utility is either independent of Mutt's income (Case II) or varies inversely with it (Case III), his utility function can be ignored; in either case, a transfer to Jeff, given (b), is certain to improve his welfare. In Case I, on the other hand, it is not certain that a transfer from Mutt to Jeff will increase Jeff's utility because the reduction in Mutt's disposable income that it implies makes Jeff feel worse. However, for Jeff to be harmed by a transfer from Mutt, his marginal utility of own-consumption $(\partial U_J/\partial Y_J)$ must be more than offset by the external diseconomy generated by the reduction of Mutt's income $(\partial U_J/\partial Y_M)$. Obviously, this is most unlikely, and in the analysis that follows, we assume that $(\partial U_J/\partial Y_J > \partial U_J/\partial Y_M)$, so that any transfer that Jeff receives, benefits him. It makes no difference, therefore, which of the three interdependence patterns in the top row of Table 1 is assumed. In all of them, transfers, given consumer sovereignty, are entirely a matter of Mutt's volition, and the process of determining a Pareto optimal redistributive transfer can concentrate on his preferences alone.

### *Hypothetical Patterns of Pareto Optimal Transfers*

Suppose, now, that an increase in $Y_J$ (as in Case II) augments Mutt's utility. How large a transfer will Mutt desire to make to Jeff? To answer this question, consider Figure 1, which is concerned with Mutt's choice of how much of his income to retain for himself and how much to transfer to Jeff. This choice will obviously depend both on $Y_M^0$ and $Y_J^0$. We assume, largely because it facilitates our examination of redistribution in the $N$-person case, that the size of the transfer depends upon the differential $Y_M^0 - Y_J^0$, rather than, among other specifications, either the absolute levels of $Y_M^0$ and $Y_J^0$ or the initial ratio, $Y_M^0/Y_J^0$. Thus the ordinate of Figure 1

$Y_M^0$ or $Y_M^0 - Y_J^0$, save the requirement that $Y_M^0 - Y_J^0$ exceed zero, in ascertaining whether interdependence is marginally relevant and, therefore, calls for a redistributive transfer. For rigorous definitions of the various types of externalities and their conceptual significance, see J. M. Buchanan and .W C. Stubblebine [5].




OZ: INITIAL INCOME DIFFERENTIAL, $Y_M^0 - Y_J^0$
$Y^T$: TRANSFER TO JEFF

FIGURE 1

measures the excess of $Y_M^0$ over $Y_J^0$, and the abscissa measures transfers from Mutt to Jeff, $Y^T$. The situation in which $Y_M^0 = Y_J^0$ is represented by the origin of Figure 1. (It is labeled "0" because this is where the differential is zero and where the transfer size is zero, not because Jeff's initial income $Y_J^0$ is 0.) The terms on which Mutt is able to exchange own-consumption for increments in Jeff's income is given by the slope of $ZZ'Z''$, Mutt's opportunity locus or "budget line." The budget line becomes vertical at $Z'$ because of the restriction that $Y^T$ must not be so large as to reverse the distributional ordering. The slope of the $ZZ'$ segment is $-1$, since a given size transfer to Jeff reduces the amount of income that Mutt retains for his own use by the same amount. $I$ is one of Mutt's (convex) indifference curves containing points which indicate the terms at which Mutt is willing to exchange own-consumption for increments in $Y_J$.[9] The positive dependence of $U_M$ on $Y_J$ is reflected by the negative slope; if $U_M$ did not depend on $Y_J$, Mutt's indifference map would simply consist of a set of horizontal lines. If the initial incomes are given by $Y_M^0$ and $Y_J^0$, so that the initial differential is equal to $0Z$, transfers to Jeff of any amount up to $Y_E^T$ raise Mutt's utility level and a transfer of $Y_E^T$ allows Mutt to attain equilibrium at $E$, where the marginal utility of a dollar of own-consumption equals the marginal utility of a one dollar increment in Jeff's income. Thus, point $E$, by definition, is a Pareto optimum.[10,11]

Having provided an analysis to determine the size of the transfer that Mutt desires to make to Jeff for a given income differential, the next step is to determine how this amount varies with the differential, so that the structure of a Pareto optimal, explicitly redistributive tax-transfer system can be ascertained.

[9] Because both axes in Figure 1 are measured in terms of units of the numeraire, the only feasible points for Mutt lie on the budget line itself. With no transactions or administration cost and no charitable deductions to reduce Mutt's tax obligations, a dollar increase in $Y^T$ implies a dollar decrease in Mutt's income for own-use.

[10] There are transfers greater than $Y_E^T$ that would reduce Mutt's utility relative to the level implied by $I$ but would leave him better off than he would have been in the absence of any transfer at all, i.e., on the indifference curve (not represented in Figure 1) that cuts the ordinate at $Z$.

[11] Note, however, that although the presence of an external economy is a necessary condition for Pareto optimal transfers, it is not a sufficient condition. If the slope of Mutt's indifference curves were everywhere less than unity in absolute value, he would regard the price of any income transfer, in terms of own-consumption foregone, as excessive. In this situation, there is no transfer to Jeff, either voluntary or coerced, that would be Pareto optimal.

Similarly, concave indifference curves (not represented in Figure 1) would also imply a corner solution at $Z$ or an equilibrium at $Z'$, the kink in the budget line. Whether the equilibrium, in this case, would be at $Z$ (implying that no transfer is Pareto optimal) or at $Z'$ (implying that income equality is required for Pareto optimality) would depend on the precise shapes of the concave indifference loci.

How should the tax on Mutt vary with the income differential? (1) Should it be a constant amount or fixed sum, or should it vary as the initial income differential $Y_M^0 - Y_J^0$ $(=0Z)$ varies? If the latter, should it increase (2) in proportion to $0Z$ or (3) less than proportionately? Or should it vary (4) inversely with the differential? The answer, in the two-person model, depends on the elasticity, with respect to $Y_M - Y_J$, of Mutt's demand for increments in Jeff's income, which we shall refer to as Mutt's transfer-elasticity and denote as $E_M$.[12] Figures 2 through 5 illustrate these four cases. Changes in the size of the initial differential, $0Z$, produce parallel shifts of the budget line, which generate a locus of equilibrium positions. $E_M$, in these diagrams, is the elasticity of this locus, the income-differential consumption ($IC$) line.[13] The $IC$ lines in Figures 2 through 5 require the particular tax-transfer patterns indicated in the four questions posed above, assuming that the equilibria are always to the left of $Z'$. If, for example, $E_M = 0$, a fixed sum transfer is Pareto optimal; if $E_M = 1$, the optimal transfer increases in proportion to $0Z$.

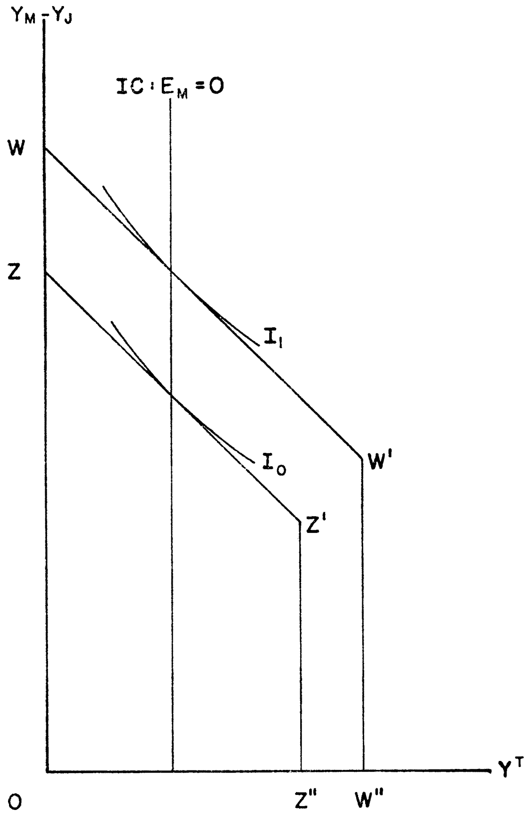



FIGURE 2

## II. *Pareto Optimal Adjustments in the N-Person Case*

### *The N-Person Model*

Must a Pareto optimal structure of redistributive taxes be progressive, propor-

[12] Transfer-elasticity differs only slightly from the more familiar income-elasticity of demand. Income-elasticity would measure the responsiveness of Mutt's demand for transfers to Jeff to changes in $Y_M$ itself. Transfer-elasticity, on the other hand, measures its responsiveness to changes in $0Z = Y_M^0 - Y_J^0$, the initial differential, regardless of whether these are due to a change in $Y_M^0$ with $Y_J^0$ constant, a change in $Y_J^0$ with $Y_M^0$ constant, or changes in both $Y_M^0$ and $Y_J^0$.

The transfer-elasticity concept, indeed our use of $Y_M^0 - Y_J^0$ as the key variable, is clearly a simplification of reality. Our choice of the specific form that this formulation implies to attach to the utility functions specified earlier in general terms is based on intuition and convenience. To look at Figure 1 as a subset of $Y_M$, $Y_J$ space with the axes shifted by the amount of $Y_J^0$ would yield a more general analysis, but one which would be much less manageable than ours, which is general enough to enable us to make the points in which we are interested. Our argument is illustrative rather than definitive, and adoption of the differential as the crucial variable simplifies the illustrations in the $N$-person case of Section II, by allowing us to abstract from abolute levels of income in our calculations.

However, the implications of this simplification should be pointed out. Under our assumption, equal absolute increases in Mutt's and Jeff's incomes would leave the optimal transfer to Jeff unchanged. Nor does the response to a change in the differential depend on the starting income levels. If, instead, the optimal transfer were an increasing function of, say, the ratio of $Y_M$ to $Y_J$, rather than the difference between them, it would decrease if $Y_M$ and $Y_J$ increased by the same absolute amount.

[13] The $IC$ line is analogous to the income-consumption line. The difference is that $Y_M^0 - Y_J^0$ is variable here, whereas $Y_M$ varies in the case of the income-consumption line. Because of the choice of axes on which to measure the transfer and initial differential, $E_M$ varies inversely with the absolute slope of $IC$.

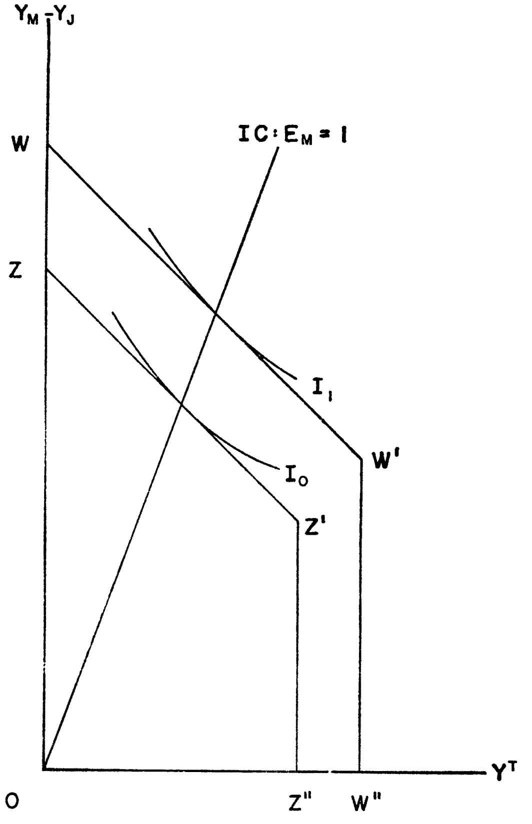


FIGURE 3

tional, or regressive? What pattern of fiscal residuals does such a tax structure imply? Answers to such questions require that our analysis be extended to the $N$-Person case.

We assume an institutional setting in which free-riding, i.e., strategic behavior, is precluded so that the political mechanism through which interdependence is internalized accurately reflects the distributional preferences of individuals in this regard. To secure the gains from trade that are possible because of interdependence, individuals choose to compel themselves to make redistributive transfers, just as they compel themselves to pay taxes to finance the provision of other collective goods. As in the two-person case, it is assumed that the tax transfer process does not change the initial distributional ordering, so that relative positions in the income scale are unaffected. It is assumed, further, that individuals have identical tastes[14] (so that all would exhibit the same consumption patterns at any given income level), and that income taxation produces no incentive effects or excess burden (i.e., the supply of labor or demand for leisure are perfectly inelastic).

Only two of the $IC$ configurations are considered, the cases in which $E_M=0$ (implying that Pareto optimality requires fixed-sum transfers) and $E_M=1$ (implying that transfers proportional to $(Y_M^0-Y_J^0)$ are optimal). It is assumed that each individual (1) makes a transfer to (permits himself to be taxed on behalf of) every person with a lower income (in a lower income bracket) and (2) receives a transfer from each individual with a higher income. Except for those in the lowest and highest income brackets, then, all individuals pay some redistributive taxes (are in Mutt's status relative to some persons) and receive some redistributive transfers (are in Jeff's status relative to others). Each individual's net outcome is the algebraic sum of the outcomes in the pairwise equilibrium relationships that emerge with all persons who have initial incomes different from his. Thus, in the $N$-Person model, Pareto optimal tax payments and transfers received depend on both $E_M$'s

[14] Obviously, this assumption is unrealistic. Any real-world blanket redistributive tax would, of course, deviate from Pareto optimality not only because of differing transfer elasticities on the part of different individuals (Mutts) whose preferences exhibit row 1, Table 1 interdependence, but also because the preferences of some Mutts are characterized by row 2 or row 3. It does not follow from this, however, that interpersonal utility comparisons, in terms of a crude cardinal utility calculus or a more refined social welfare function, are needed to justify all redistribution. But it does raise the question of what governmental unit should intermediate redistributive transfers, and more broadly, of the optimal redistributive areas in a fiscal federalism, a question analogous in some respects to that of determining optimum currency areas in the theory of international trade.

(one's own and others') *and* the shape of the size distribution of income. $E_M$ and $(Y_M^0 - Y_J^0)$ determine the Pareto optimal transfer between each pair of individuals. One's position in the income scale determines the number of persons to whom he will make transfers and the number from whom he will receive them. Each individual's aggregate tax payments (summed over all Jeffs), transfer receipts (summed over all Mutts), and fiscal residual (receipts minus payments) depend, therefore, on both considerations.[15,16]

*Pareto Optimal Patterns of Redistribution*

Pareto optimal distributional adjustments are derived for two distributional settings, a rectangular distribution ($D_r$) and a summary representation of the actual income distribution in the U.S. in 1960 ($D_a$). This is done twice for each distribution; once on the assumption that the $E_M$'s of all $N$ individuals are zero and once assuming that the $E_M$'s are unity. Results in the four cases examined are summarized in the tables indicated in Table 2.

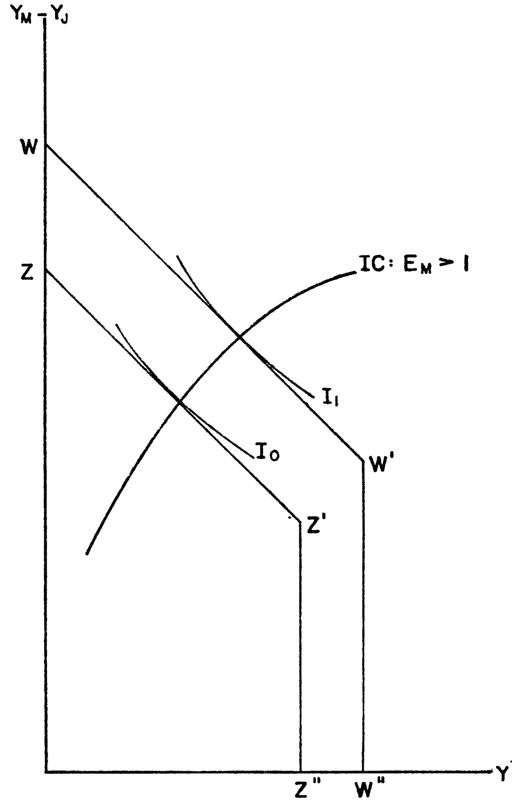



FIGURE 4

$D_r$, which is a useful benchmark case, is described numerically in the first two columns of Tables 3 and 4. It is a simple

TABLE 2—PARETO OPTIMAL REDISTRIBUTIONS CLASSIFIED BY TRANSFER ELASTICITY AND INCOME DISTRIBUTION

| | Income Distributions | | |
|---|---|---|---|
| | $E_M$ \ $D$ | $D_r$ | $D_a$ |
| Transfer-Elasticity | $E_M=0$ | Table 3 | Table 5 |
| | $E_M=1$ | Table 4 | Table 6 |

[15] Our efforts to identify the incidence of Pareto optimal redistributive adjustments under different assumptions about $E_M$ should not be confused with the problem of determining the appropriate incidence of the overall tax structure. We assume that the costs of allocative activities are distributed on a benefit basis, before redistribution is contemplated at all, and, therefore, deal only with the marginal incidence of distributional adjustments, ignoring the feedbacks of redistribution that might confound this prior application of the benefit principle. An overall Pareto optimum is, obviously, a matter of transfer-elasticity (or some analogous measure of distributional preferences) and these income-elasticities. Hence, we are implicitly assuming away any changes in evaluations of conventional public goods that the Pareto optimal transfers might bring about. Another way of putting the matter is to say that we are assuming that individuals, in choosing their consumption mixes, fully anticipate the transfers they are to receive.

[16] $N$, the absolute size of the community, is of no significance in our calculations. We can either assume that $N$ is constant or that the fiscal residuals of individuals are unaltered, if it changes. This assumption requires that (1) changes in $N$ are spread proportionately among all income classes, preserving the relative distribution; (2) the levies on individual Mutts are varied in inverse proportion to the number of Jeffs concerned; and (3) administration of the tax-transfer process is subject to constant returns to scale.

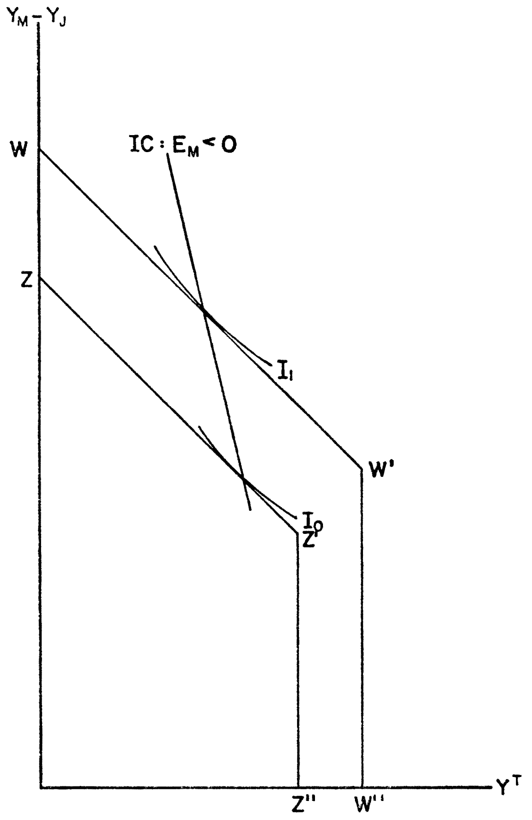



FIGURE 5

rectangular distribution for a community of five persons and contains five income classes of identical width. Each class contains one individual having an income equal to the mean of the class limits.

$D_a$, the second distribution, is described in the first two columns of Tables 5 and 6. It is the summary distribution, for the U.S. in 1960, which Gillespie [6] used in reporting his estimates of fiscal incidence in the U.S. (discussed in Section III). In using $D_a$, we assume, for convenience, that the community contains one hundred individuals or families, an assumption that permits us to use relative frequencies instead of the absolute distribution in our analysis. $D_a$ contains seven income classes of varying width, with an open-ended class ($10,000 and over) at the top. Within income classes, families are treated as if they were identical in size. All incomes in the five intermediate classes are assumed to be equal to the mean of the class limits, referred to in the tables as the "representative class income."[17] To specify representative class incomes for the "under $2,000" and "$10,000 and over" classes, a linkage procedure was used, producing estimates of $800 for the first bracket (under $2,000) and $15,000 for the top bracket ($10,000 and over).[18]

[17] This simplifies our calculations and assures that modest redistributive adjustments cannot reverse the distributional ordering.

[18] We calculated the total income received by each unit of one percent of all families in the two bottom and two top income brackets in $D_a$ from Gillespie's distribution of aggregate income by size class [6, p. 174, Table 13, line 1]. We then computed the ratios of these figures for the two bottom brackets (.32) and the two

TABLE 3—PARETO OPTIMAL REDISTRIBUTIONS
Rectangular Income Distribution ($D_r$): Transfer-Elasticity ($E_M$) = 0

| | | Fixed-Sum Transfers of $100 | | | Tax Structure | |
|---|---|---|---|---|---|---|
| Income | Number of Individuals | Tax Paid ($) | Transfer Received ($) | Pareto Optimal Fiscal Incidence ($) | Marginal Rate | Average Rate |
| 1,000 | 1 | — | 400 | +400 | — | — |
| 2,000 | 1 | 100 | 300 | +200 | .10 | .050 |
| 3,000 | 1 | 200 | 200 | 0 | .10 | .067 |
| 4,000 | 1 | 300 | 100 | −200 | .10 | .075 |
| 5,000 | 1 | 400 | — | −400 | .10 | .080 |

TABLE 4—PARETO OPTIMAL REDISTRIBUTIONS
Rectangular Income Distribution ($D_r$): Transfer-Elasticity ($E_M$) = 1

| Transfer 5 Percent of Income Differential | | | | | Tax Structure | |
|---|---|---|---|---|---|---|
| Income | Number of Individuals | Tax Paid ($) | Differential Transfer Received ($) | Pareto Optimal Fiscal Incidence ($) | Marginal Rate | Average Rate |
| 1,000 | 1 | — | 500 | +500 | — | — |
| 2,000 | 1 | 50 | 300 | +250 | .05 | .025 |
| 3,000 | 1 | 150 | 150 | 0 | .10 | .050 |
| 4,000 | 1 | 300 | 50 | −250 | .15 | .075 |
| 5,000 | 1 | 500 | — | −500 | .20 | .100 |

Let us consider $D_r$. If $E_M$ is zero for all individuals making transfers, the Pareto optimal tax structure is degressive, as Table 3 indicates. In our example, each of the five persons in the community, one at each income level, transfers \$100 to each individual in a lower bracket. Thus, the individual with an income of \$1,000 is exempt, while those with higher incomes are taxed at a constant marginal rate of 10 percent. The average rate increases, monotonically, from zero to 8 per cent, and the Pareto optimal fiscal residuals (distributional transfers received less taxes paid, after accounting for conventional public goods on a benefit basis) are symmetrical, by virtue of $D_r$'s symmetry.

This outcome suggests that Pareto optimality requires more progressivity if, with a rectangular distribution, $E_M>0$. This conclusion, with its implication that progressive taxation can be justified without interpersonal utility comparisons, is illustrated by Table 4, which summarizes the outcomes for the case in which all $E_M$'s are unity. In this case the implied marginal rates of tax rise from zero to 20 percent. In our example, the factor of proportionality, $k$, is assumed to be .05, making the implied transfer between any pair of individuals in different income brackets .05 $(Y_M^0-Y_J^0)$.

The results can be recast in terms of the $IC$ lines of Figures 2 and 3. In Figure 2, $E_M=0$ and Pareto optimality requires a degressive tax (degressivity). In Figure 3, $E_M=1$, and the $IC$ line has a zero intercept and a slope equal to the reciprocal of $k$. In this case, the Pareto optimal tax structure is clearly more progressive than it is with a vertical $IC$ line; and, in general, implied progressivity is greater, the smaller the slope of the $IC$ line.

The outcomes with $D_r$ tell us something about the incidence of Pareto optimal redistributive adjustments in the context of any distribution in which frequencies vary monotonically with income. With declining frequencies, the ratio of Jeffs (to whom transfers must be made) to Mutts (from whom transfers are received) increases more rapidly with income than it does with a rectangular distribution. Thus, if $E_M$ is the same for all individuals, the Pareto optimal tax structure is necessarily more progressive than it is with $D_r$. In the unlikely case in which frequencies increase with income, the converse would hold true.

We turn now to $D_a$. (See Tables 5 and

top brackets (1.77). Our estimate of average income in the bottom bracket (\$800) was derived by multiplying the representative class income of \$2,500 in the second (\$2,000 to \$2,999) bracket by .32 and rounding the product to the nearest \$100. Our estimate of average income in the top bracket (\$15,000) was obtained by multiplying the representative class income of \$8,750 in the \$7,500 to \$9,999 bracket by 1.77 and rounding.

TABLE 5—PARETO OPTIMAL REDISTRIBUTIONS
Actual Income Distribution ($D_a$): Transfer-Elasticity ($E_M$) = 0

| Representative Class Income[a] | Percent of Families[b] | Fixed-Sum Transfer of $5: Tax Paid ($) | Transfer Received ($) | Pareto Optimal Fiscal Incidence ($) | Tax Structure: Marginal Rate | Average Rate |
|---|---|---|---|---|---|---|
| 800 | 14 | — | 430 | +430 | — | — |
| 2,500 | 9 | 70 | 385 | +315 | .042 | .028 |
| 3,500 | 9 | 115 | 340 | +225 | .045 | .033 |
| 4,500 | 11 | 160 | 285 | +125 | .045 | .036 |
| 6,250 | 28 | 215 | 145 | − 70 | .032 | .034 |
| 8,750 | 15 | 355 | 70 | −285 | .056 | .040 |
| 15,500 | 14 | 430 | — | −430 | .011 | .028 |

[a] Class mid-points for all but bottom and top brackets. Procedure for obtaining "representative class incomes" for bottom and top brackets is discussed in fn. 18.
[b] Implies a community consisting of 100 families or individuals.

6.)[19] Unlike $D_r$, which is symmetrical, $D_a$ is skewed to the right.[20] Where $E_M=0$, the implication of this asymmetry is that the structure of Pareto optimal taxes is not uniformly progressive throughout the distribution. With "fixed-sum" transfers of $5 (Table 5) this tax structure is progressive up to, but not including, the model income bracket ($5,000–$7,499); in this bracket the marginal tax rate decreases from 4.5 to 3.2 percent. This decline in the marginal rate occurs because the percentage change in "representative class income" between the fourth and fifth income brackets exceeds the percentage change in the number of individuals entitled to

[19] The size of the fixed-sum transfer (for Table 5) and the value of $k$ (for the computations underlying Table 6) make no difference in the shape of the pattern of residuals and are thus analytically irrelevant. The values used in our computations were chosen for their convenience, to facilitate subsequent comparisons of our hypothetical residuals and the actual residuals reported by Gillespie.

[20] Columns (1) and (2) of Tables 5 and 6 show 71 percent of all families with incomes under $7,499, which is less than half the assumed mid-point of the top bracket, $15,500.

TABLE 6—PARETO OPTIMAL REDISTRIBUTIONS
Actual Income Distribution ($D_a$): Transfer-Elasticity ($E_M$) = 1

| Representative Class Income | Percent of Families | Transfer 0.1 Percent of Income Differential ($k=.001$): Tax Paid ($) | Transfer Received ($) | Pareto Optimal Fiscal Incidence ($) | Tax Structure: Marginal Rate | Average Rate |
|---|---|---|---|---|---|---|
| 800 | 14 | — | 553 | +558 | — | — |
| 2,500 | 9 | 24 | 412 | +388 | .014 | .010 |
| 3,500 | 9 | 47 | 335 | +288 | .023 | .013 |
| 4,500 | 11 | 79 | 267 | +188 | .032 | .018 |
| 6,250 | 28 | 154 | 167 | + 13 | .043 | .025 |
| 8,750 | 15 | 333 | 94 | −239 | .072 | .038 |
| 15,500 | 14 | 914 | — | −913 | .086 | .059 |

receive transfers. After this decline, the marginal rate increases to 5.6 percent in the sixth bracket ($7,500–$9,999) and then declines again to 1.1 percent in the "$10,000 and over" class.

When optimal incidence patterns are derived for $D_a$ under the assumption that $E_M=1$ (see Table 6) this complex rate structure is not obtained. The Pareto optimal tax structure is, rather, uniformly progressive. With $k$, the factor of proportionality, equal to .001, marginal rates of tax rise from 1.4 percent in the first bracket ($2,000–$2,999) to 8.6 percent for families with incomes of "$10,000 and over."

These examples have demonstrated a means of determining Pareto optimal redistributive adjustments, albeit under highly restrictive assumptions and only for certain special cases.[21] The structure of the Pareto optimal redistributive taxes, whether progressive, proportional, regressive, or lacking uniformity, depends on the values of the transfer-elasticities and the shape of the initially existing distribution of income, as determined by initial endowments and the operation of the market economy.

### III. *Actual Incidence and Pareto Optimal Redistribution*

Granting all of the other assumptions we have made, under what assumptions about utility interdependence would the actual fiscal structure be Pareto optimal? To answer this question, among others, Table 7 compares the Pareto optimal fiscal residuals computed in Section II, for $D_a$, with one of the sets of residuals estimated by Gillespie [6, p. 162, Table 11]. Columns (1) and (2) of Table 7 describe $D_a$. Column (3), which reports the actual residuals, is derived from Gillespie's estimates of the consolidated fiscal incidence of federal, state, and local taxes and expenditures in the U.S. for 1960.[22] To obtain the figures in Column (3), we multiplied Gillespie's estimates of fiscal incidence in each income bracket, which he reported in proportional terms, by the "representative class income." Thus Column (3) indicates, in absolute terms, the fiscal residuals (benefits of expenditures and transfers received, less taxes paid) which accrued to average individuals in each bracket. For comparison we report, in Columns (4) and (5), the Pareto optimal fiscal residuals with $D_a$ in the "fixed-sum" ($E_M=0$) and "proportional transfer" ($E_M=1$) cases of Tables 5 and 6, respectively.[23]

[21] We have considered only two values of the transfer-elasticities and two income distributions, and have assumed identical utility maps. While the analysis could be generalized by a mathematical formulation, there seemed to us to be virtue in simplicity.

[22] Needless to say, Gillespie faced many difficult problems, requiring essentially arbitrary choices, in compiling these estimates. One such problem was that of choosing an income base. The base for which the estimates in Column (3) of Table 5 are derived is adjusted broad income, money income adjusted for transfers, government expenditures, and taxes. Other problems included (1) distribution of the burdens of specific taxes, (2) imputation of the benefits of specific expenditures to beneficiary groups, and (3) distribution of these beneficiary groups among income classes. For taxes, Gillespie could use published material, e.g., Musgrave's study [11]. For expenditures, he had less to go on in the way of prior research. Since demand prices for public goods are not revealed, the only practicable alternative was to allocate benefits on the basis of some measure of cost undertaken on behalf of individuals. Perhaps the most intractable problem was the distribution of general (nonallocable) expenditures, e.g., national defense, among individuals and income groups. In Table 11 [6, p. 126], which we used as our source of Column (3), such expenditures are distributed on an income (rather than, say, on a per capita) basis.

[23] The appropriate interpretation of Column (3) differs slightly from that of Columns (4) and (5). The latter indicate the net gain or loss, in strictly monetary (i.e., not welfare) terms, that would accrue to each individual from a Pareto optimal redistributive process. In addition to redistributive adjustments, the figures in Column (3) include imputations of the aggregate benefits accruing to individuals from public goods, minus the taxes paid to obtain these benefits. From our viewpoint, however, this difference in interpretation is of little importance. This may be seen by assuming (as we have) that the political mechanism, in providing public goods, accurately reflects individual preferences, and that both redistribution in kind and in money terms are consistent with such preferences, in the same sense in which monetary transfers alone internalize the general externalities taken into account in our simpler model.

Both of the hypothetical patterns of residuals, Columns (4) and (5), vary inversely with income. The real-world residuals in Column (3), however, do not fully conform with this pattern. The most obvious differences between the actual and hypothetical residuals occur in the first, second, and sixth income brackets. Instead of decreasing between the first and second brackets, the fiscal residual actually increases, almost in proportion to income, i.e., from \$441 to \$1,110. Furthermore, in the "\$7,500–\$9,999" bracket, the fiscal residual is positive, not negative. In terms of the absolute deviation from either of the hypothetical residuals (for $E_M=0$ or 1), the first of these abberations is more significant,[24] especially when compared to the level of income in the bracket in which it occurs. We choose, consequently, to ignore the abberation in the sixth bracket and discuss only the one in the second.

The fact that the fiscal process seems to subsidize the 14 percent of all families in the "under \$2,000" bracket less heavily than the 9 percent in the \$2,000–\$2,999 bracket does not coincide with our hypothetical computations in which the Pareto optimal residuals decrease monotonically as income increases. It is worthwhile to explore alternative explanations of this outcome.

One possibility is that the first bracket may well consist, to a greater extent than those just above it, of rural poor. In rural areas, communities are smaller and social pressure to interact is consequently greater. Payment of income in kind is likely to be more common, and simple bilateral or multilateral transfers through private charity are more likely to be feasible, reducing dependence on the fiscal process as a redistributive mechanism. In urban areas social conditions may not fit this model as well. Urban poverty, moreover, is readily apparent to more individuals with relatively high incomes, and general interdependence among individuals in different income groups is by virtue of proximity even more pronounced. Fiscal machinery is more likely to enjoy a clear advantage as the mechanism of redistribution, because the social group is large and private arrangements that can overcome "free rider" behavior to the degree required are more difficult to devise.

A second, less benign, explanation is that those who are really poor, i.e., families with incomes under \$2,000, may be almost devoid of political power. Their welfare counts for less than that of individuals with higher incomes in the calculations of politicians, just as their preferences count for less in the market sector. This argument, implying that political power and effective demand go hand in hand, may be extended to a hypothesis that the actual fiscal structure reflects a coalition among middle income groups, that is, among families whose incomes lie between the bottom and top brackets. It should be noted, however, that this hypothesis is diametrically opposed to the notion that the actual fiscal structure comes at all close to being Pareto optimal; for, if it were Pareto optimal, families in the top bracket would, by definition, prefer the disproportionate tax burden they now bear, according to Gillespie's estimates, at least to a situation without redistribution, so no such coalition would be necessary.

A third, less provocative, though possibly more realistic explanation might attribute the apparent plight of the "under \$2,000" group to quirks in the statistical procedures underlying Gillespie's estimates and our own calculations. Many difficulties are encountered in imputing the

[24] For example, in the second bracket the deviation of the actual residual from the computed residual in Column (5) is \$1,110–\$388; relating this deviation to income in the second bracket, we obtain (\$1,110–\$388)/\$2,500 = .31. This proportional deviation is much greater than that in the sixth bracket: (\$148 + \$239)/\$8,750 = 0.04.

TABLE 7—COMPARISON OF FISCAL INCIDENCE UNDER ALTERNATIVE TAX-TRANSFER ASSUMPTIONS

| (1) Representative Class Income ($) | (2) Percent of Families | (3) U.S. Fiscal Structure, 1960[a] | (4) $E_M=0$ Fixed-Sum Transfers of $5[b] | (5) $E_M=1$ Transfer 0.1 Percent of Income Differential ($)[c] |
|---|---|---|---|---|
| 800 | 14 | + 441 | +430 | +558 |
| 2,500 | 9 | +1,110 | +315 | +388 |
| 3,500 | 9 | + 648 | +225 | +288 |
| 4,500 | 11 | − 58 | +125 | +188 |
| 6,250 | 28 | − 131 | − 70 | − 13 |
| 8,750 | 15 | + 148 | −285 | −239 |
| 15,500 | 14 | −2,046 | −430 | −913 |

[a] U. S. Fiscal Structure, 1960: Gillespie [6, p. 162, Table 11, line 11]. Reported figures are the product of "representative class incomes" and effective rate (expenditure benefits and transfers received minus taxes paid) of fiscal incidence.
[b] Table 5.
[c] Table 6.

burdens of taxation and the benefits of expenditures. Thus, the increase in residuals between the first and second brackets might, at least to some degree, be attributed to such factors as the imputation of the benefits of general expenditures, e.g., national defense, on an income-related rather than per capita basis.

Despite aberrations and ambiguity, it is interesting to ask, "If the actual residuals in Table 7 are Pareto optimal, what are the implied patterns of utility interdependence?" If we suppose that the actual residuals reflect the exact amount of redistribution required to internalize such interdependence, so that the actual fiscal structure is by implication, Pareto optimal, we can infer something about the values of transfer-elasticities at various income levels and the shapes of individuals' *IC* lines. In general, the residuals with $E_M=1$, the "proportional transfer case," seem to be better correlated with the actual pattern of incidence than the residuals in the "fixed-sum" case ($E_M=0$). This is particularly clear in the top income bracket, which seems, according to Column (3), to finance the lion's share of any redistribution that actually occurs.[25] Residuals in the second through the seventh brackets, taken as a group, suggest that individuals in high brackets have larger transfer-elasticities than those in lower brackets; thus, instead of remaining constant, as Column (5) assumes, $E_M$ appears to increase with income.[26] Furthermore, since the middle income groups (brackets four through six) seem, more or less, to break even in the fiscal process, it would appear that utility interdependence increases in significance as income increases and becomes really significant only when income reaches a level of $10,000 or more.[27]

[25] Thus, so far as vertical equity is concerned, our observations suggest that typical discussions of the U.S. fiscal structure might overstate the normative significance of erosion of the tax base.

[26] Except for the minor lapse in the $7,500–$9,999 class.

[27] If, however, the actual residuals are not Pareto optimal, an alternative explanation of the disproportionate fiscal burden on the $10,000 and over group is required. The obvious alternative, that this burden reflects political weakness, controverts the generally held belief in the correlation of political and economic power. Our analysis, unfortunately, provides neither an answer to this riddle nor a criterion for choosing between such polar explanations of observed fiscal incidence.

In terms of the diagrams, this suggests that the $IC$ line coincides with the ordinate (and has a slope equal to or only slightly different from infinity) until the income differential begins to exceed, say, \$6,000–\$8,000. At this point its slope becomes finite ($E_M$ begins to exceed zero). Thereafter, as $0Z=(Y_M^0-Y_J^0)$ increases, $E_M$ increases and the $IC$ line, as in Figure 4, bends downward to the right. Thus, to return to our general example, where individuals are in Mutt's status, the income or consumption levels of those with Jeff's status appear to be normal goods. Stated a bit differently, the general implication is that the ratio of the marginal utility of own-consumption to the marginal utility of others' consumption declines over the income range considered.

## IV. *Conclusion*

In trying to reconcile income redistribution (e.g., through a negative income tax) with consumer sovereignty and an individualistic interpretation of the fiscal and political processes, we have experimented with alternative hypotheses about utility interdependence. In the presence of such interdependence, Pareto optimality may not only be consistent with redistribution, but may require it. If so, the necessary fiscal adjustments depend on the implicit transfer-elasticities and the shape of the size distribution of income.

Pursuing this line of thought, we have calculated, for several situations, the patterns of tax burdens, transfers, and fiscal incidence that would be Pareto optimal. We have also tried to determine the type of utility interdependence that is implied by the actual fiscal residuals prevailing in the U.S. in 1960, on the assumption that those residuals were Pareto optimal.

An important implication of our analysis is the finding that the case for progressive taxation, aimed at redistributing income, may be far less "uneasy"[28] than most of us have come to believe. Quite to the contrary, progressive taxation, for explicit redistributive purposes, may be fully consistent with the Pareto criterion under quite reasonable conceptual assumptions. Progressivity, given such assumptions, may be interpreted as a matter of revealed preference, which does not require interpersonal utility comparisons for its justification. Whether these assumptions are empirically valid is, of course, another question, but one that should yield to empirical investigation.

All this does not pretend to claim that fiscal reality does not deviate from the requirements of Pareto optimality. It does. But the fact that it does may be a technical matter and not a conceptual necessity. Departures of the actual fiscal structure from the Pareto ideal may simply reflect an operational inability to correctly juxtapose individual preferences and fiscal incidence. We are suggesting, therefore, that if more could be learned about utility interdependence through empirical investigation of private and public choice patterns and processes, it might turn out to be possible to utilize this information to achieve a fiscal structure more in accord with the individualist ethic that underlies the economist's model of resource allocation.

Of course, one might personally feel that the amount of redistribution dictated by the Pareto criterion will not be "enough." We are not saying that society should necessarily follow only the Pareto rule. It is possible, however, to develop a theory of redistribution based on such a rule, and considerable redistribution might be indicated if it were operationally possible to

[28] Blum and Kalven [3] is the classic discussion of progression. Their treatment, which examines progression as traditionally justified in terms of sacrifice and interpersonal comparisons, reinforced the pre-existing skepticism of the profession.

devise a fiscal structure consistent with this criterion.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