A Signaling Explanation for Charity

Amihai Glazer; Kai A. Konrad


Stable URL: http://links.jstor.org/sici?sici=0002-8282%28199609%2986%3A4%3C1019%3AAEFC%3E2.0.CO%3B2-6


Your use of the JSTOR archive indicates your acceptance of JSTOR’s Terms and Conditions of Use, available at http://www.jstor.org/about/terms.html. JSTOR’s Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at http://www.jstor.org/journals/aea.html.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is an independent not-for-profit organization dedicated to creating and preserving a digital archive of scholarly journals. For more information regarding JSTOR, please contact support@jstor.org.
A Signaling Explanation for Charity

By Amihai Glazer and Kai A. Konrad*

The standard model of voluntary provision of public goods considers utility to be a function $u = u(x, G)$ of consumption of a private good $x$ and of the sum of contributions to a public good $G$. Recent theoretical analyses derive the following properties of the Nash equilibrium. 1) In large economies only the very rich contribute to the public good (James Andreoni, 1988; Timothy L. Fries et al., 1991); the share of contributions in total income is negligible. 2) Governmental supply of the public good crowds out private supply. If all individuals donate in the original equilibrium this crowding out is complete (Peter G. Warr, 1982; B. Douglas Bernheim, 1986; Theodore C. Bergstrom et al., 1986).

These theoretical results conflict with empirical evidence on private charity. According to Arthur H. White (1989), nine out of ten Americans report giving, and many studies show that crowding out is only partial (Richard Steinberg, 1989). Such evidence led some theorists to suppose that donations enter directly into a person’s utility function, that is, $u = u(x, g, G)$, where $g$ is his donation (see, for example, Andreoni, 1989, 1990; Richard Cornes and Todd Sandler, 1984, 1994; Bruce R. Kingma, 1989; Robert McClelland, 1989; Russel D. Roberts, 1987; Sandler and John Posnett, 1991; Steinberg, 1986, 1987). In this warm-glow approach the act of giving directly enters the utility function (Kenneth J. Arrow, 1972).

We consider here an additional motive—the desire to demonstrate wealth, perhaps because individuals prefer to socialize with individuals of the same or higher social status. Though we suppose individuals seek to signal high absolute wealth, much of our analysis also applies when individuals signal relative wealth. Over a century ago John Rae (1834) and John S. Mill (1847) considered such a motive. The behavior of consumers who signal income is studied by other economists, and most elegantly in the work of Robert H. Frank (1984a, 1984b, 1985a, 1985b).

Though people may signal status by conspicuously consuming private goods, we do not think they will be used exclusively. First, conspicuous consumption may be banned by social norms\(^1\) when charitable donations are not.\(^2\) Second, ownership of luxury goods may be difficult to observe reliably. A consumer who wants to impress others may therefore buy a zirconium ring instead of a diamond one, an imitation Rolex watch instead of a genuine one, or may rent an expensive automobile for special occasions rather than incur the greater expense of purchasing one.\(^3\) Such behavior can ruin the use of expensive goods as signals.

Third, charitable donations may be especially good signals to people who belong to a peer group but cannot see the big house or luxury car of another member. Impressing former college roommates who may live in other

---

* Glazer: Department of Economics, University of California, Irvine, CA 92717; Konrad: Department of Economics, Free University of Berlin, D-14195 Berlin, Germany. We are grateful for suggestions by Richard Cyert, Roger Gordon, Daniel Klein, Charles Lave, Kjell Erik Lommerud, Richard Romano, Larry Rothenberg, Herbert Simon, Stergios Skaperdas, and three anonymous referees. Glazer is grateful for the support of the Graduate School of Industrial Administration at Carnegie Mellon University.

---

\(^1\) Anton Velleman (1900) gives a historical overview of laws forbidding consumption of luxury goods. Roger D. Congleton (1989) suggests that status seeking through charitable activities may be the result of an evolution towards status-seeking games with positive external effects.

\(^2\) Donations and inter-vivos transfers to children may also signal income, and some of the conclusions on charitable donations may apply.

\(^3\) This argument is loosely related to that of Bernheim (1991) and Laurie Simon Bagwell and Bernheim (1992) who show that agents may have to choose the more wasteful signal to generate separating.
parts of the world may require a notice in the alma mater’s alumni magazine. Such contributions are large. In 1991 U.S. universities received $10.2 billion in voluntary support. Of this, $5 billion was from individuals, and of the $5 billion, $2.3 billion was from alumni. Thus, about one fourth of private support for higher education came from alumni (Statistical Abstract of the United States, 1993). Similarly, Britain’s upper ten thousand find that London’s charities provide perfect services in publicizing donations in a discreet but still observable way. Thus, a person may want to signal income to several different peer groups. He may use conspicuous luxury consumption in some status games and may use donations to signal income to those who cannot see his consumption of luxury goods.

Fourth, when people receive a warm glow from charitable donations, the donations generate intrinsic utility like luxury goods do. If the rich have particularly strong warm-glow feelings then giving reveals high wealth. Some less rich but particularly status motivated people could then try to pool with this group of rich people by making donations. Thus, if warm glow is important to some persons, it can stimulate donations by persons who care little about warm glow but much about status.

Fifth, when at least some people make donations to increase the supply of a public good (that is, when we observe private provision of a public good), then crowding out effects may make signaling of income by donations more effective than signaling by conspicuous consumption of private goods. To see how, recall that a central result in the standard game of private provision of a public good is that, with a large but finite group of possible contributors, and with homogeneous preferences, only those with incomes above a critical level contribute to the public good (Andreoni, 1988). Therefore, if initially only people who do not care about status make contributions, the contribution equilibrium would reveal income. Signaling can also appear for other observable goods, but the effect is more pronounced for private provision of a public good: rich people’s donations crowd out the donations of the less wealthy. That is, any donations by people seeking status would cause a decline in donations by those who care only about provision of the public good. But the reduced donations occur not uniformly among initial donors, but instead among the least wealthy of them, thereby shifting upward the income distribution of donors. For some utility functions and distributions of wealth it is possible that a signaling equilibrium appears with donations to public goods, but not with purchases of private goods.

We will not model these interactions because we simplify matters by assuming infinitely many individuals: in our framework people have no motivation to contribute for the purpose of increasing provision of the public good. This allows us to show most clearly that status reasons for contributions can lead to excessive voluntary contributions.

The paper proceeds as follows. Section I gives evidence for status-motivated giving. Section II presents a fully revealing signaling game. Extensions of this model in Section III show that status-motivated giving can lead to excessive private provision of a public good and that the model applies to a very general concept of status preferences. Section IV concludes.

I. Empirical Evidence for Conspicuous Giving

The data on charitable giving support the hypothesis that donors donate at least partly for signaling purposes rather than only to aid
the recipient or to obtain satisfactions unrelated to status.

Individuals who donate to signal their income will not make anonymous donations. In contrast, both the standard and the warm-glow models are consistent with anonymous donations. The data we collected show that anonymous donations are rare. Here are some examples. The Pittsburgh Philharmonic received 2,240 donations from individuals in 1991. Only 29 (1.29 percent) were anonymous. The fall 1991 Yale Law Report (sent to all alumni of Yale Law School) names donors to the Yale Law School Fund. Of the 1,950 entries, only four are anonymous. Incidentally, to ease the task of discovering which classmates donated how much, the names of donors are listed by graduation year in each category. Donations to Harvard Law School show the same pattern: donations are listed by size, and fewer than 1 percent of donations were anonymous. Similarly, in 1989–1990 Carnegie Mellon University received donations from 5,462 individuals. Only 14 (0.3 percent) were anonymous. Perusal of all reports by nonprofit organizations on file at the Pittsburgh Business Library found no institution with rates of anonymous donations higher than in these examples.

Donors may have several motivations for giving. The paucity of anonymous donations indicates that concern about status is one of them. We can further distinguish the motivations of individuals by examining semipublic donations—those where the names of donors are public, but where the amount each donated is only approximately inferable. Consider donations to the Cameron Clan at Carnegie Mellon University. The university’s annual report names all people who gave between $500–$999, but does not separately report each donation amount. A person motivated solely by status who gives in this category should give exactly $500; a prediction the standard and warm-glow models do not make. The average gift in 1988–1989 was $525, which suggests that most gifts were very close to $500. Similar distributions appear in other categories. The 1993–1994 report of the Harvard Law School Fund shows the same pattern. For example, 980 people contributed in the category of $500–$999. Contributions of exactly $500 would constitute 93 percent of the total raised in this category.

Further analysis is possible with a smaller data set from a unit at Carnegie Mellon University which listed the dollar amount of each contribution. We find, for example, that of 82 contributions made in the range $1,000–$4,999, 56 (or 68 percent) were exactly $1,000. Also, people that give primarily for signaling purposes should be more likely to give a bit more than the minimum in a category rather than a bit less; no such motivation appears under the warm-glow and private-provision models. As one measure of such a bias, we tabulate the number of donations that lie within 10-percent above the minimum in a category, and the number that lie within 10-percent below the minimum of a category. We find, for example, that for the $1,000 minimum 17 people gave just above the minimum, while only 4 gave just below. Together with the 56 who exactly gave $1,000, this is 73 of 82 individuals in this category. Tabulations with different cutoffs yield similar results.

II. A Signaling Equilibrium

Though charitable organizations usually report donations within categories, thus not allowing perfect revelation, for analytical tractability we shall consider an equilibrium with perfect revelation and a continuum of types, as characterized in George J. Mailath (1987) and Norman J. Ireland (1994). Consider a set $I$ of individuals $i \in I$. The income distribution is described by a density function $f(y)$ on the interval $[y_{\text{min}}, y_{\text{max}}]$; a consumer gets utility from a private good $x$, consumption of which is unobservable, and from his income status. Status of individual $i$ is determined by his signaled net income $\hat{y}_i - g_i$, defined as the belief by other individuals about $i$'s income net of donations. This belief

---

* As in other models with signaling, multiple equilibria may exist. In particular, some belief functions lead to a pooling equilibrium with $g^*(y) = 0$, some belief functions lead to partially revealing equilibria, and some different fully revealing equilibria arise if the distribution of income has discrete jumps.
is derived from observable donations, \( g_i \), to a charity. The utility function

\[
(1) \quad u^i = u(x_i, \hat{y}_i - g_i) = u(x_i) + w(\hat{y}_i - g_i)
\]

is the same for all individuals. It is twice differentiable. Marginal utility of each argument is strictly positive and decreasing. An individual gets no direct utility from aggregate donations to charity.\(^7\) Each cares only about the signaling effect of his own donation. Also, we assume \( u(y_i, y_{\min}) > u(0, y_{\max}); \) nobody would ever want to donate all his income. The individual’s budget constraint is

\[
(2) \quad y_i = x_i + g_i.
\]

Income \( y_i \) is exogenous, and differs across individuals. Let \( g_i \) be observable or be made observable by the charitable institution, whereas \( x_i \) and \( y_i \) are not observable by others. Then beliefs about individual \( i \)’s income is a function only of the observable donation, \( \hat{y}_i = \hat{y}_i(g_i) \).

In our model the equilibrium is defined by individuals’ choices of donations \( g^*(y) \) as a function of their incomes, and by a strictly monotonic function of beliefs \( \hat{y}_i(g_i) \) such that, for each \( i \) with income \( y_i \), the vector \( (y_i - g^*(y_i), \hat{y}_i(g^*(y_i)) - g^*(y_i)) \) maximizes (1) subject to the budget constraint (2), to \( g_i \geq 0, x_i \geq 0 \), and subject to

\[
(3) \quad \hat{y}_i(g^*(y_i)) = y_i,
\]

that is, beliefs are correct in the equilibrium. Strict monotonicity of the belief function and of \( g^*(y) \) in combination with (3) makes the equilibrium a fully revealing equilibrium.

**PROPOSITION 1:** A perfectly revealing equilibrium exists.

\(^7\) If individuals also get intrinsic utility from making donations, we could add a "warm-glow" element to our model. The extension would not invalidate our analysis; the warm-glow model would become a special case—one in which status signaling is absent. But to work out the new aspects of the signaling approach it is more useful to concentrate on the element that is distinct from the warm-glow model.

The proof is in the Appendix. Some properties of the equilibrium are as follows.

**PROPERTY 1:** A replication of the economy has no effect on any individual’s donation \( g^*(y_i) \).

**PROPERTY 2:** Let government fund a charity providing a public good, and let revenue for this funding be raised by a lump-sum tax that does not change the support of the income distribution. Then crowding out is partial.

**PROPERTY 3:** Income redistribution among donors is generically not neutral. If \( g^*(y) \) is convex (concave), then an income redistribution that reduces income inequality but does not change \( y_{\min} \) reduces (increases) donations. The function

\[
(4) \quad g^*(y) \text{ is concave if and only if} \quad \left[ (-v''/v') - (-w''/w') \right] < 0.
\]

**PROPERTY 4:** Assume that poor people are added to a population such that the support of incomes increases from \([y_{\min}, y_{\max}]\) to \([y_{\min} - \theta, y_{\max}]\), with \( \theta > 0 \). Then in the new equilibrium the original donors make higher donations.

Proofs of these properties are in the Appendix.

The crowding-out properties of our model, as given by Property 2, are the same as in the warm-glow model (see Andreoni, 1989 p. 1453): both models predict less than perfect crowding out. Empirical results show that crowding out depends—among other things—on the type of charity that receives governmental contributions (Steinberg [1989] surveys studies of crowding out). The differing relevance of the signaling approach for different types of charities may partially account for this fact.

Properties 3 and 4 most notably distinguish our approach from other models. The relations between the convexity of \( g^*(y) \), income redistribution, and aggregate contributions, and between the support of the income distribution and aggregate donations are specific to our signaling model.\(^8\) Property 3 can be rephrased:

\(^8\) For a related result in a different model see Frank (1985a p. 105).
suppose that \((-v''/v') < (-w''/w')\). Individuals are less "risk averse" with respect to utility from intrinsic consumption, \(v(y_i - g_i)\), than with respect to utility from signaling income, \(w(\tilde{y}_i - g_i)\). Then a mean-preserving spread reduces per capita contributions. Property 4 considers a situation in which the smallest possible income becomes smaller: donations for any given income increase. If lower-income individuals enter the group, all other individuals contribute more.

Though we do not have the data to test the effects of additional consumers on donations, the prediction does have empirical applications. For signaling wealth, people will make donations only when income in the community is heterogeneous. If, for example, everyone at a university has the same income after graduation, then none will make any donations as alumni. A university may be able to elicit greater aggregate contributions by increasing the heterogeneity of its student body. This can motivate a university to give scholarships to the middle class: they may increase future donations by rich alumni. Similarly, management of some clubs may want to provide lower fees to poorer members (without making public the incomes of such members). The increased heterogeneity of members may make each more willing to signal his income by donating to that club.

We now turn to considering a subsidy for donations.

**PROPERTY 5:** Suppose government subsidizes individuals' donations \(g_i\) with a proportional subsidy rate, that is, if an individual spends a dollar on the public good, government reimburses the fraction \(\alpha/(1 + \alpha)\). Let the subsidy be financed by exogenous sources or by a reduction of the government's direct contributions to the public good. Then each individual contributes
\[
g**(y) = (1 + \alpha) g*(y)
\]
in the new signaling equilibrium.

For the proof see the Appendix.

In the warm-glow model, a dollar spent on subsidies stimulates charity more than does a dollar of direct grants (Andreoni, 1990 p. 470). Intuitively, with matching grants, an additional dollar on public goods buys more of the public good, and, hence yields higher marginal utility. In the signaling model matching grants reduce the signaling value of an increased donation: the cost of each dollar donation is reduced to \(1/(1 + \alpha)\), so that a donor who wants to generate an equivalently strong signal as without reimbursement, must make a gross contribution of \((1 + \alpha)\).

Property 5 has some implications. First, if government finances the matching grants by reducing its funding for the public good, then the per capita provision of the public good in the economy is unchanged. Second, private contributions net of reimbursement are independent of \(\alpha\), provided that the government finances the matching grants in a way that does not affect the individuals' own budgets.

**III. Extensions**

The results in this paper can be extended in different ways.

A) Suppose the donations are used to provide a public good, \(I\), which is a function \(\Gamma = \Gamma(g, I)\), where \(I\) is the set of individuals normalized to the unit interval, and \(g\) is the per capita donation.

\[
\Gamma(g, I) = \Gamma\left(\int_{i \in I} g_i, di, I\right).
\]

Andreoni and Bergstrom (1992) show in the standard model that whether matching grants induce higher contributions depends on whether the government chooses a clever tax scheme for financing these grants. If not, neutrality of matching grants may indeed apply.

This way of modeling a public good in an economy with an uncountable set of individuals is borrowed from Jean-Jacques Laffont (1975) who uses this setup to model a macroeconomic externality.
Let the public good enter the utility function in an additively separable way: \( u' = v(x_i) + w(\tilde{y}_i - g_i) + z(\Gamma) \). In (5) \( d\Gamma/dg_i = 0 \) and, therefore, \( d\Gamma/dg_i = 0 \), since a single individual has zero measure. This fact together with additive separability implies that the term \( z(\Gamma) \) does not affect donation behavior. Accordingly, in the signaling equilibrium \( \Gamma \) is strictly positive, but may be larger or smaller than the socially optimal level.

How does this extended model relate to the standard model in which concern for income status is absent? To consider a continuum of contributors we had to use Laffont’s description of a public good as in (5), and we need to compare our results to a modified standard model that applies a similar description of the public good. With a finite number, \( n \), of contributors, the analogue to (5) has utility depend on private consumption and on \( \Gamma = (1/n)G = \sum_{i=1}^{n} g_i/n \). For this modified standard model, if \( n \) is constant, the results concerning existence and uniqueness of equilibrium, redistribution neutrality and perfect crowding out are identical to those in the standard model. For the modified standard model, where we can speak of \( n \to \infty \), the sum of contributions converges to zero. Hence in both the standard model and in the modified standard model, contributions are an infinitesimally small share in aggregate expenditure if the number of possible contributors is large. In our model, in contrast, charitable contributions can constitute a significant share of expenditures.

In the warm-glow model, the “warm glow” of giving yields an additional private incentive to donate. However, in the warm-glow equilibrium a joint increase in donations by all individuals that makes everyone better-off is always feasible. The effect of additional donations on welfare is ambiguous in our signaling model: the negative externalities of status signaling via donations have to be weighed against the positive externalities of donations. The outcome can be under-provision or over-provision of the public good.

B) Suppose that individuals are interested in signaling not a high absolute income, but instead a high relative income. The utility function is then the functional

\[
(6) \quad u' = v(x_i) + w(\tilde{y}_i - g_i, F(y - g)),
\]

where \( F(y - g) \) describes the aggregate distribution of net income. How does this change our results?

Proposition 1 holds without change. Taxes and redistribution of income, however, usually change \( F(y - g) \). As a result, \( g*(y) \) may (but need not) change as \( \partial w/\partial (\tilde{y}_i - g_i) \) changes. If, for example, utility is affected only by properties of \( F(y - g) \) that do not vary with income redistribution, then most of the properties of Section II hold. That is, the results can apply when individuals care about relative rather than about absolute status.

IV. Discussion

This paper examines a novel but plausible motive for charitable giving. Charitable donations which are observable can signal wealth or income. The signaling equilibrium of charitable donations has attractive properties. Donations increase proportionally with population size. Governmental funding for public goods affects private charity only through an income effect. Crowding out need not be complete. Income redistribution is generically nonneutral. Increasing the spread between the poorest and richest person in a community tends to increase private donations.

Our theory is based on the assumption that people are willing to make charitable donations even if they will not increase provision of the public good. Why then do the organizations receiving these donations provide any service at all? An answer is given in our model. We assume that donors want others to know about their donation. In other words, a consumer is more willing to donate to an organization the more likely is the intended audience to hear about that donation. Successful organizations must therefore attract many patrons. A good orchestra will have its published program read by more people, and will therefore give a wider audience to the names of its donors published in that program. Thus, though we do not require organizations to provide services with the money they raise, mar-

\[13\] This is frequently assumed in the literature on status seeking, and is particularly true when relative income is defined as a function of one's income and of others' average income (see, for example, Richard Layard, 1980).
ket forces lead them to provide services. Under the standard view of private provision of a public good, consumers should prefer to donate to organizations with low costs of raising money. Yet we observe that many successful nonprofit organizations have high fund-raising costs. In our model this is understandable to the extent that such fund-raising activities publicize the amounts donated by others. Dinners, benefit concerts, and promotional literature can fall into this category.

APPENDIX

PROOF OF PROPOSITION 1:

It is sufficient to show that a belief function \( \hat{y}_i(g_i) \) exists that is one-to-one, and a donation function \( g^*(y) \) exists, such that: i) for any individual with income \( y_i \in [y_{\text{min}}, y_{\text{max}}] \) the value \( g^*(y_i) \) solves the problem of maximizing (1) globally subject to the belief function; ii) \( g^*(y) \) is one-to-one; iii) \( g^*(y_i) \) is feasible [that is, \( g^*(y_i) \in [0, y_i] \forall y_i \]; and iv) \( g^*(y_i) \) fulfills \( \hat{y}_i(g^*(y_i)) = y_i \).

We try the following belief solution. Let \( \hat{y}_i(g_i) \) be the definite solution of the first-order differential equation\(^{14}\)

\[
(A1) \quad \hat{y}_i'(g_i) = \frac{v'(\hat{y}_i(g_i) - g_i)}{w'(\hat{y}_i(g_i) - g_i)} + 1
\]

\[= \phi(\hat{y}_i(g_i), g_i) \]

with the initial condition \( \hat{y}_i(0) = y_{\text{min}} \). This belief function is strictly increasing. Hence, \( \hat{y}_i(g_i) \) is one-to-one. The function has finite slope \( \phi > 1 \) as \( \infty > v' > 0 \) and \( \infty > w' > 0 \) is assumed. Further,

\[
(A2) \quad \hat{y}_i''(g_i) = \frac{v''(\hat{y}_i - g_i) \times w'(\hat{y}_i - g_i)}{(w'(\hat{y}_i - g_i))^2} \times (\hat{y}_i' - 1)
\]

Consider the inverse of the belief function \( \hat{y}_i(g_i) \); call it \( g^*(y) \). By the slope \( \phi \) in (A1), this inverse exists with \( g^*(y_{\text{min}}) = 0 \) and slope \( 0 < g^* < 1 \). We show that \( g^*(y) \) fulfills i), ii), iii) and iv).

If individuals with income \( y_i \) choose \( g^*(y_i) \) then iv) is fulfilled by definition of \( g^*(y) \). The slope \( 0 < g^* < 1 \) implies ii). For iii), by construction, \( g^*(y_{\text{min}}) = 0 \). Further, \( 0 < g^* < 1 \) for all \( y_i \in (y_{\text{min}}, y_{\text{max}}) \), and, hence, \( 0 < g^*(y_i) < y_i \).

It remains to show that the individuals’ choice of \( g^*(y_i) \) yields a unique global maximum of utility. Individual \( i \)'s choice of \( g^*(y_i) \) fulfills \( i \)'s first-order condition

\[
(A3) \quad -v'(y_i - g_i) + (\hat{y}_i'(g_i) - 1) 
\]

\[\times w'(\hat{y}_i(g_i) - g_i) = 0, \]

since for the choice \( g^*(y_i) \), we get \( y_i = \hat{y}_i \) and, hence (A3) becomes identical with (A1). The second-order condition for a local maximum is also fulfilled:\(^{15}\) using (A1) and (A2) we obtain

\[
(A4) \quad \frac{d^2u}{(dg_i)^2}_{y_i = \hat{y}_i} = v''(\hat{y}_i - g_i) 
\]

\[\times \hat{y}_i'(g_i) < 0. \]

Further, \( g^*(y_i) \) is the only local maximum for the given belief function \( \hat{y}_i(g_i) \) that is implicitly defined in (A1); suppose for some \( y_i \) that some other \( \tilde{g}(y_i) \neq g^*(y_i) \) fulfills (A3). Then, by (A1),

\[
-v'(y_i - \tilde{g}(y_i)) + w'(\tilde{y}_i(\tilde{g}(y_i)) - \tilde{g}(y_i)) 
\]

\[\times [\tilde{y}_i'(\tilde{g}(y_i)) - 1] = -v'(y_i - \tilde{g}(y_i)) 
\]

\[+ w'(\tilde{y}_i(\tilde{g}(y_i)) - \tilde{g}(y_i)) 
\]

\[\times v'(\tilde{y}_i(\tilde{g}(y_i)) - \tilde{g}(y_i)) 
\]

\[\times w'(\tilde{y}_i(\tilde{g}(y_i)) - \tilde{g}(y_i)) = 0. \]

\(^{14}\) The global solution of the first-order differential equation in (A1) exists and is unique for a given initial value since, in particular, \((\hat{y}_i(g_i), g_i)\) is differentiable with respect to \( \hat{y}_i \) (Heinrich Behnke et al., 1962 pp. 302, 316).

\(^{15}\) The discussion of sufficiency of the first-order condition below relates to the single-crossing property in Maitalh (1987).
or,
\[ v'(y_i - \bar{g}(y_i)) = v'(\tilde{y}_i(\bar{g}(y_i)) - \bar{g}(y_i)), \]
or,
\[ y_i = \tilde{y}_i(\bar{g}(y_i)), \]
or
\[ \bar{g}(y_i) = g^*(y_i). \]

Finally, this unique local maximum is also the global maximum. To confirm this we have to check the corner solutions. It follows from
\[ u(y_i, y_{min}) > u(0, y_{max}) \]
that \( g_i = y_i \) is not a maximum. Except for \( y_i = y_{min} \) for which \( g^*(y_{min}) = 0 \) is also the interior maximum, \( g_i = 0 \) is also not a maximum. To confirm this we note that \( du/dg_i = 0 \) at \( g_i = 0 \) and \( y_i = y_{min} \), and \( d^2u/(dg_i dy_i) = -v''(y_i) > 0 \).

PROOF OF PROPERTIES 1–5:

Property 1 follows from the fact that donations \( g^*(y_i) \) depend on \( y_{min} \) but not on the size of \( I \).

For Property 2 note first that governmental contributions have no effect on \( g^*(y) \). Hence, if government contributes some amount \( c \) and finances this by a lump-sum tax that is imposed on some individual \( i \) such that \( i \)'s income becomes \( y_i - c > y_{min} \), this individual reduces his contributions by \( \Delta g_i = g^*(y_i) - g^*(y_i - c) \). Crowding out is partial because \( g'' < 1 \).

Moreover, \( \Delta g_i \) differs for different initial incomes \( y_i \), and is a decreasing (increasing) function of income \( y_i \), if \( g^*(y) \) is concave (convex). This yields the intuition for Property 3: consider redistributing some amount \( c \) from \( i \) to \( j \), with \( y_{min} < y_j < y_i + c < y_i - c < y_j \). This redistribution reduces donations if and only if \( (d/dy)(g^*(y_j)) > (d/dy)(g^*(y_i)) \). Hence, a sufficient condition for a redistribution of income, that reduces income inequality (in the sense of a mean-preserving contraction) to reduce donations, is that \( g^*(y) \) be convex. Further, \( g' = w'/v' + w' \) by (A1), and hence \( g'' = (w''v' - v''w')/(v' + w')^2 \) which is negative if and only if \( (w''/w') < (v''/v') \).

To prove Property 4 consider an initial distribution of incomes on the support \([y_{min}, y_{max}] \). Add a set of consumers with incomes \([y_{min} - \theta, y_{max}] \) where \( \theta > 0 \). This changes \( g^*(y) \) to \( g^{**}(y) \), the inverse of the solution of (A1) for the initial condition \( \tilde{y}_i(0) = (y_{min} - \theta) \). Hence, the new solution has \( g^{**}(y_i) > g^*(y_i) \) for all \( y_i \in [y_{min}, y_{max}] \).

Finally we prove Property 5. Consider the new belief function \( \tilde{y}_i \) with \( \tilde{y}_i((1 + \alpha)g_i) = \tilde{y}_i(g_i) \). Without subsidy, \( g^*(y_i) \) fulfilled the first-order condition (A3). Given the subsidy, the new maximization problem is to maximize
\[ v(y_i - g_i/(1 + \alpha)) + w(\tilde{y}_i(g_i) - g_i/(1 + \alpha)) \]
and yields a first-order condition \(-v' - w' + (1 + \alpha)w\tilde{y}_i' = -v' - w' + w\tilde{y}_i' = 0 \) that is fulfilled for \( g^{**}(y_i) = (1 + \alpha)g^*(y_i) \), since this choice yields
\[
(A5) \quad x_i = y_i - g^{**}(y_i) + \frac{\alpha}{1 + \alpha} g^{**}(y_i)
\]
and signaled net income
\[
(A6) \quad \tilde{y}_i(g^{**}) - g_i^{**}(y_i) + \frac{\alpha}{1 + \alpha} g_i^{**}(y_i)
\]

REFERENCES


Andreoni, James and Bergstrom, Theodore C. "Do Government Subsidies Increase the Private Supply of Public Goods?" Working


