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EASY RIDERS, JOINT PRODUCTION, AND PUBLIC GOODS*

Richard Cornes and Todd Sandler

The conventional wisdom on pure public good provision has certain accepted propositions:[1] (1) As group or community size increases, easy (or free) riding[2] and its associated suboptimality also increase (Olson, 1965). (2) The stability of Nash (or Nash–Cournot) equilibrium solely depends on the sign and magnitude of an income effect (see, for example, Breton, 1970, p. 574; Olson and Zeckhauser, 1966; Pauly, 1970, p. 574; Williams, 1966, p. 21). (3) Nash behaviour always leads to inefficiency for public goods (Pauly, 1970). (4) A model with identical individuals has a symmetric equilibrium where everyone provides the same equilibrium quantity of a pure public good (Olson, 1965). (5) There is no measure for the extent of easy riding. These are important propositions that are universally used and accepted in public finance.

This article explores the comparative statics of the demand for a marketed good that jointly provides public *and* private outputs or characteristics. Once joint products are admitted to the analysis of public goods, we demonstrate that propositions (1)–(4) no longer hold. In particular, the analysis shows that the consumption relationship of the jointly produced outputs influences the slope of the expenditure reaction paths, the stability of equilibria, and the departure of Nash equilibria from optimality (i.e. *the extent of easy riding*). When the joint products are complements in a Hicksian sense, particularly interesting results follow including the possibilities of positively sloped reaction paths and of reduced easy riding as the size of the community is increased. Thus, an agent's public expenditures may *increase* in response to increased public expenditures of others, even when all goods are normal with positive income elasticities. Such a result may well apply to an activity like philanthropy, where charitable activities provide private as well as public benefits to contributors (see, Sugden, 1982; Posnett and Sandler, 1983).

Even in the absence of jointness, this article demonstrates that propositions (1) and (4) are not valid. Furthermore, a simple, but useful, geometric technique is presented. This device can generate reaction paths and an index of easy riding, whose existence disproves proposition (5). Additionally, we derive the stability conditions for an n-person public good model.

In Section V the theoretical results are applied to the study of a military alliance, where an arsenal jointly produces private and public outputs. This

* The authors gratefully acknowledge the helpful comments of John Hutton, an associate editor, and a referee. Full responsibility for any remaining shortcomings rests solely with the authors. An earlier version was drafted when Sandler was on leave at Australian National University.

[1] See, for example, the references cited by Breton (1970), and Sandler and Tschirhart (1980).

[2] We prefer the term *easy ride*, because *free ride* implies a zero contribution to public provision; such a corner solution does not typically characterise public good provision. The term 'easy ride' implies a contribution short of the 'right' amount.

application allows for a more accurate specification of an ally's demand for military expenditures (Murdoch and Sandler, 1982, 1984). A second application involves common property resources. Finally, the analysis is applied to the study of aggregate saving and the choice of investment projects, by showing that the distribution of current income is independent of the level of aggregate saving only when saving has no jointly produced private components.

Section I presents the joint product model from the viewpoint of an individual. A two-person community is examined in Section II, while an n-person community is studied in Section III. Section IV compares the joint product model to the orthodox model without jointness, and Section V contains applications. Conclusions follow in Section VI.

I. INDIVIDUAL BEHAVIOUR

Our modelling of individual consumer behaviour uses the characteristics approach developed by Gorman (1980) and Lancaster (1971), and recently expounded by Deaton and Muellbauer (1980, ch. 10). The typical consumer has a utility function, $u(\cdot)$, defined over three characteristics:

$$u = u(c, x, Z). \tag{1}$$

The precise nature of these characteristics will shortly be explained. We note first, though, that $u(\cdot)$ is assumed to be a thoroughly well-behaved utility function. In the analysis which follows, except where stated otherwise, all three characteristics are assumed to be goods and preferences are strictly convex, so that $u(\cdot)$ is strictly increasing and strictly quasiconcave. We also assume that $u(\cdot)$ is continuous and at least twice differentiable.[1] In view of our very orthodox assumptions regarding the utility function, any novelty in the model must come not from (1) but from the nature of the characteristics and the manner in which they are generated.

Each unit of the characteristic, c, is produced by purchasing one unit of a particular marketed commodity. This commodity produces no other characteristic for this individual or for any other, and consequently can be thought of as private. Furthermore, there is no other activity which generates c. We can therefore use c to denote either the individual's consumption of the private characteristic or his purchase of the commodity which produces c. We use c as the numeraire in the rest of the paper, and put its price equal to unity.

Each unit of the second private characteristic, x, is produced by purchasing $1/\beta$ units of a marketed commodity, q. We assume that β is exogenously fixed. The purchase of a unit of q, besides generating β units of x, also generates an exogenously given γ units of the third characteristic, z. Using subscripts on a variable to distinguish individuals, the total consumption of the third characteristic by individual h is

$$Z_h = Z \equiv z_1 + z_2 \dots + z_h + \dots + z_n.$$

In short, Z_h is a public characteristic; the quantity generated by any single individual is automatically made available to all other consumers.

[1] See Deaton and Muellbauer (1980, pp. 25–36) for a discussion of the properties of utility functions.

The typical consumer maximises utility subject to two constraints. In the first place, he has a given money income, I, and faces a given price p for the marketed commodity q. His budget constraint, which holds with equality so long as he prefers more of at least one of the characteristics is

$$c + pq = I. \tag{2}$$

Second, he forms a certain expectation about the level of other individuals' generation of z. We define $\tilde{Z} \equiv Z - z$ as the level of the third characteristic which he expects to be generated among the rest of the community. The consumer's problem may now be summarised as

$$\begin{aligned} &\underset{\{c,x,z\}}{\text{maximise}}\ u(c, x, \tilde{Z}+z) \\ &\text{subject to} \quad x = \beta q, \quad z = \gamma q \\ &\text{and} \quad c + pq = \text{I}. \end{aligned}$$

Our description of individual behaviour began with a utility function defined over characteristics. It is possible, and occasionally proves extremely useful, to express utility as an 'indirect' function of the marketed commodities. Our assumptions concerning the generation of characteristics enable us to write

$$\begin{aligned} u(c, x, \tilde{Z}+z) &= u(c, \beta q, \tilde{Z}+\gamma q) \\ &= v(c, q; \tilde{Z}). \end{aligned}$$

For any given value of $\tilde{Z}$, $v(\cdot)$ is a well-behaved continuous, strictly increasing, and strictly quasiconcave utility function defined on the variables c and q. Fig. 1 shows how the consumer's problem can be expressed as the orthodox problem of choosing c and q so as to maximise $v(c, q; \tilde{Z})$ subject to (2). The three axes radiating from the origin O measure quantities of the three characteristics on which u depends. In the resulting 3-dimensional space, one can imagine indifference surfaces, all convex to the origin O. The distance OO' measures the total level of generation of the public characteristic among the rest of the community, $\tilde{Z}$. Starting from the point O', the individual may purchase units of q, each of which generates β units of x and γ units of z and moves him along $O'Q$. Alternatively, he may purchase units of c, which move him along $O'C$. He is therefore constrained to be in the 2-dimensional space defined by the axes $O'Q$ which measures q, and $O'C$ which measures c. The interaction of the resulting plane with the indifference surfaces generates indifference curves which are convex to the origin O'. Within the same space, the figure also shows the budget constraint, BB. In view of the standard nature of the resulting problem, we can immediately say that, for any given $\tilde{Z}$, a consumer optimum implies that $p = \text{MRS}_{qc} = (\partial v/\partial q)/(\partial v/\partial c)$. By explicitly comparing $v(\cdot)$ and $u(\cdot)$, we can relate p to the marginal rates of substitution (MRS) in characteristics space, and show that

$$\text{MRS}_{qc} = \beta\text{MRS}_{xc} + \gamma\text{MRS}_{zc}. \tag{3}$$

This result simply says that at any point in (c, q) space the MRS between the two marketed commodities equals a weighted sum of the MRSs between, on the one hand, each of the two jointly produced characteristics and, on the other

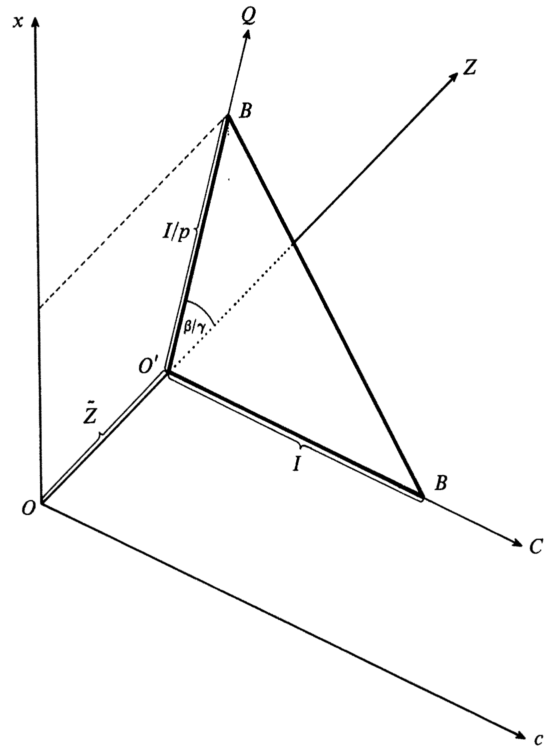

Fig. 1. The consumer constraint plane in three-dimensions.

hand, the numeraire characteristic. The weights reflect the relative importance of the two characteristics as outputs of the joint production process.[1]

Now observe that in any equilibrium, given the values of x, Z and u, the consumer acts so as to minimise his expenditure on c. This defines the restricted, or conditional, cost function as expounded in Deaton and Muellbauer (1980, p. 110):

$$c(x, Z, u) = \min_{\{c\}} [c|u(c, x, Z) \geqq u],$$

where the variables x, Z and u take their equilibrium values. We assume that $c(\cdot)$ is everywhere twice continuously differentiable. The partial derivatives of $c(\cdot)$ with respect to x and to Z yield, respectively,

$$\partial c(x, Z, u)/\partial x = -\text{MRS}_{xc} = -\pi_x(x, Z, u),$$
$$\partial c(x, Z, u)/\partial Z = -\text{MRS}_{zc} = -\pi_z(x, Z, u).$$

The functions $\pi_x(\cdot)$ and $\pi_z(\cdot)$ define the MRSs appearing on the right-hand side of (3) as functions of x, Z and u. This cost function approach provides a simple and convenient way of analysing the comparative statics of demand for the commodity q, which jointly produce the private and public characteristics x and z. Of particular interest is the response of the individual's demand for q to

[1] It is possible to generalise (3) by interpreting x and z, and hence the appropriate terms on the right-hand, as vectors rather than scalars.

changes in the quantity of the public characteristic generated by the rest of the community, $\tilde{Z}$. Before deriving this, we should note certain properties of $\pi_x(\cdot)$ and $\pi_z(\cdot)$.

To begin with, the function $c(\cdot)$ is convex in the quantities x and Z. We assume that it is twice continuously differentiable. The Hessian matrix,

$$\begin{bmatrix} \dfrac{\partial^2 c}{\partial x^2} & \dfrac{\partial^2 c}{\partial x \partial Z} \\ \dfrac{\partial^2 c}{\partial Z \partial x} & \dfrac{\partial^2 c}{\partial Z^2} \end{bmatrix} = \begin{bmatrix} -\pi_{xx} & -\pi_{xz} \\ -\pi_{zx} & -\pi_{zz} \end{bmatrix},$$

where π_{xz}, for example, denotes $\partial \pi_x(\cdot)/\partial Z$, is positive semidefinite. This implies the following inequalities:

$$\pi_{xx} \leqslant 0, \quad \pi_{zz} \leqslant 0, \quad \begin{vmatrix} \pi_{xx} & \pi_{xz} \\ \pi_{zx} & \pi_{zz} \end{vmatrix} \geqslant 0.$$

We assume in what follows that the determinant, at least, is strictly positive.

The cross-partial $\pi_{xz} (= \pi_{zx})$, the sign of which cannot be determined by *a priori* considerations alone, provides the criterion by which Hicks (1956, p. 156) classifies commodities as q-substitutes or q-complements. Suppose x is held constant while Z is increased. Suppose further that, as Z increases, c is taken away from the individual so as to keep his utility unchanged. If, as a result of this experiment, the willingness to pay for x increases (i.e. $\pi_{xz} > 0$), then x and Z are q-complements. If, on the other hand, $\pi_{xz} < 0$, then x and Z are q-substitutes. In the next section, the sign and magnitude of π_{xz} is seen to be a crucial factor determining the slope of the reaction path. In particular, a high positive value of π_{xz} can produce an upward-sloping reaction curve, so that the individual's demand for q rises in response to larger expected values of $\tilde{Z}$.

II. THE 2-PERSON COMMUNITY

This section looks at the behaviour of a community consisting of two individuals, each of whom behaves like the individual of the last section. We assume that each regards the other's purchase of q, and hence their contribution to the public characteristic, as given. Such quantity-taking or Nash behaviour, while difficult to justify as a literal description of behaviour for the 2-person community, is easier to accept in the large numbers case, to which the present example is a useful prelude.

Each individual, then, maximises $v_h(c_h, q_h; q_j)$ subject to his budget constraint and an exogenous value for q_j, representing the other individual's contribution. Before analysing the properties of equilibria and optima in the 2-person system, it is instructive to consider the response of h's demand for q_h to exogenous changes in q_j. The public goods literature tends to suggest that, if an individual's expectation concerning the provision of a public good by others rises, his own contribution will fall (see, for example, Sugden, 1982).

To explore this presumption, rewrite (3) using the functions $\pi_x(\cdot)$ and $\pi_z(\cdot)$

and recall that MRS_{qc} equals the market price, p. For the individual whose behaviour we are examining – say individual 1 – this equation must hold in any equilibrium. The arguments x, and Z, may be replaced by $\beta_1 q_1$ and $\gamma_1 q_1 + \gamma_2 q_2$, respectively. Holding the price constant, we wish to solve for the response dq_1 to an exogenous change, dq_2, by taking a total differential of

$$p = \beta_1 \pi_x(\beta_1 q_1, \gamma_1 q_1 + \gamma_2 q_2, u) + \gamma_1 \pi_z(\beta_1 q_1, \gamma_1 q_1 + \gamma_2 q_2, u). \tag{3'}$$

After rearranging the resulting differential,[1] we get

$$\frac{dq_1}{dq_2} = \left[\frac{\gamma_2(\beta_1 \pi_{xz} + \gamma_1 \pi_{zz})}{\Phi}\right] + \left[\frac{(\beta_1 \pi_{xu} + \gamma_1 \pi_{zu})}{\Phi}\right] \frac{du}{dq_2}, \tag{4}$$

where

$$\Phi \equiv -(\beta_1 \gamma_1) \begin{bmatrix} \pi_{xx} & \pi_{xz} \\ \pi_{zx} & \pi_{zz} \end{bmatrix} \begin{pmatrix} \beta_1 \\ \gamma_1 \end{pmatrix} > 0.$$

The term appearing in the first set of square brackets in (4) gives the response dq_1/dq_2 when utility is held constant, and thus represents the substitution effect. The sign of the denominator is known to be positive. The numerator consists of a weighted sum of partial derivatives. π_{zz} is known to be non-positive; however, π_{xz} may be of either sign. Moreover, the parameter β_1 is independent both of π_{xz} and of γ_1. Theory cannot therefore rule out the possibility that q-complementarity between x and Z, associated with a high value of β_1 relative to γ_1, makes $\beta_1 \pi_{xz} > -\gamma_1 \pi_{zz}$.

The total response dq_1/dq_2 includes a real income term, represented by the second term in square brackets in (4). The partial derivative π_{xu} denotes the change in the willingness to pay for x as utility is increased through an increase in c, holding x and Z constant. Intuitively, there is a presumption that this is positive. For a given value of Z, one can draw a well-behaved indifference map in (c, x) space. The statement that $\pi_{xu} > 0$ is equivalent to the statement that x is a normal good when Z is held constant. Similar comments apply to π_{zu}.

There are, then, two circumstances which can produce a positive response dq_1/dq_2. If either x or Z is a superior good in the precise sense given above, and if the associated coefficient, β_1, γ_1, is relatively large, then the resulting positive income effect may dominate the right-hand side of (4). Less familiar is the circumstance in which strong q-complementarity between x and Z may dominate the overall outcome.

In order to analyse the properties of the community equilibrium, we wish to exploit the 'reaction curves' implied by (4). To facilitate comparison with the Pareto-optimal allocations we develop the reaction curve diagram by first generating iso-utility, or indifference, curves in (q_1, q_2) space. Consider the first individual. His utility function may be written as

$$v_1(c_1, q_1; q_2) = v_1(I_1 - pq_1, q_1, q_2),$$

[1] The resulting differential is

$$0 = \beta_1[\beta_1 \pi_{xx}\, dq_1 + \pi_{xz}(\gamma_1 dq_1 + \gamma_2 dq_2) + \pi_{xu}\, du] + \gamma_1[\beta_1 \pi_{zx}\, dq_1 + \pi_{zz}(\gamma_1\, dq_1 + \gamma_2 dq_2) + \pi_{zu}\, du].$$

Solving for dq_1 in terms of dq_2 and du, and dividing the resulting expression on both sides by dq_2, yield (4).

where I_1 and p are exogenously fixed throughout the analysis. Now consider any pair of values for q_1 and q_2. To this pair of values, there corresponds a particular utility level V_1^*. Consider the locus of all pairs of values of q_1 and q_2 giving rise to this same utility level. The resulting indifference curve, together with many others, each corresponding to a particular level of utility, form an indifference map in (q_1, q_2) space with the following properties: First, since the public characteristic is assumed always to be a good, indifference curves associated with higher values of q_2 for a given value of q_1 represent a higher value of utility for individual 1; in Fig. 2, $V_1^{**} > V_1^*$. Second, since we assume that for any given q_2 there is only one optimal solution for q_1, each curve has a unique minimum point. Again, this is shown in Fig. 2, where a moment's reflection should persuade the reader that the reaction curve whose slope we derived in

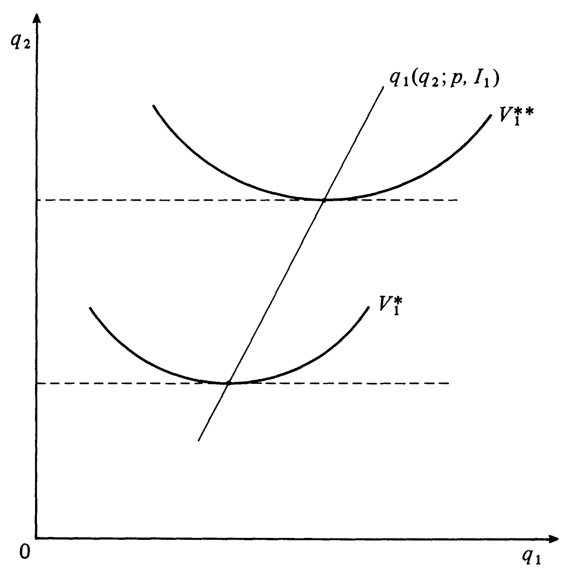

Fig. 2. Iso-utility curves and Nash reaction path.

equation (4) is the locus of minima of the indifference curves. At each of these points individual 1, taking q_2 together with his budget constraint as given, is doing the best he can. We have drawn 1's reaction curve to reflect the possibility already raised of a positive slope.

To analyse the community equilibrium, we begin by superimposing the second individual's reaction curve, thereby obtaining a diagram familiar to all students of duopoly. The use of the reaction curve technique allows us to broach a number of issues which have received rather scanty attention from public goods theorists. To begin with, we can ask about the conditions for local stability of an equilibrium. We show in an Appendix that, under a very simple but widely used adjustment mechanism, the local stability of an equilibrium requires that, in the neighbourhood of that equilibrium, $(\partial q_1/\partial q_2)_1 \cdot (\partial q_2/\partial q_1)_2 < 1$, where $(\partial q_1/\partial q_2)_1$ is the slope of the first individual's reaction path and $(\partial q_2/\partial q_1)_2$ is the slope of the second individual's reaction path. This condition means that if the reaction curves have slopes of the same sign, individual 1's curve should have the greater slope in absolute terms.

Suppose we restrict attention to a community of identical individuals. Since tastes and incomes are the same, the two individual's reaction curves will be reflections of one another about the 45° line through the origin. Even this special case contains some interesting possibilities. Indeed, we intend to restrict our attention even further, to the case of monotonic reaction curves. Non-linear reaction curves may have multiple intersections, some of which will be locally stable. If the reaction curves are upward-sloping everywhere, all equilibria will be symmetrical in the sense that our two clones will be choosing identical allocations.

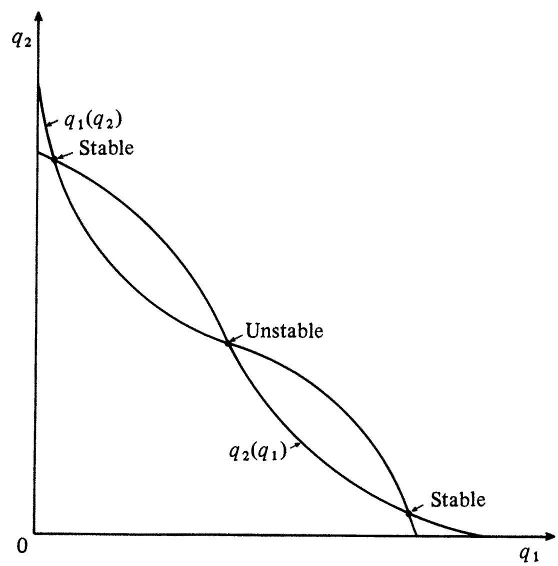

Fig. 3. Stable asymmetric equilibria for two identical individuals.

For the weak complement or substitute case, an intriguing and largely neglected possibility exists. Suppose that, over the range that we are considering, the reaction curves are both downward-sloping. If conditions are such that an equilibrium exists – which we assume – there will be a symmetrical equilibrium; however, there is no reason to suppose that it is necessarily locally stable. In Fig. 3 the two stable equilibria are asymmetric in the sense that, in each one, the two clones are choosing different allocations. This result even holds in the absence of joint products, since q_2 and q_1 could represent the amounts of a pure public good rather than a marketed activity producing joint products. There seems, therefore, to be no guarantee that a community of identical individuals will equilibrate towards a symmetric public goods equilibrium in which each of the members behaves like 'the representative' citizen. In each of the stable asymmetric equilibria in Fig. 3, one of the individuals is expecting his companion to make a relatively small contribution to the public characteristic, and consequently is encouraged to substitute into q and generate the lion's share of

Z; while his companion, expecting generous social provision of Z, is thereby led to take a relatively easy ride. Nevertheless, each clone has the same expectations for any *given* level of provision by the other. They, however, hold different expectations at the stable asymmetric equilibria, since each clone faces different *initial* contributions by the other individual. Thus, the asymmetric behaviour regarding public good contributions, noted by Olson (1965, pp. 35–6), does not require income differences among the participants.

Fig. 4 reinstates some indifference curves to facilitate an explicit comparison between equilibrium and Pareto-optimal allocations. In the 2-person community, Pareto optima are simply the points of tangency between the two sets of indifference curves; their locus is the dotted line PP. The properties already

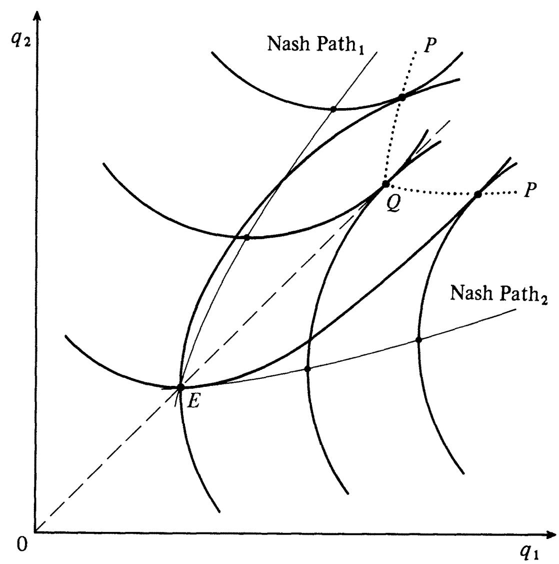

Fig. 4. Index of easy riding.

established for the indifference curves ensure that the independent-adjustment, or Nash, equilibrium lies below and to the left of the locus PP in the 2-person model. The reader may confirm that this is true whatever the shapes of the reaction curves, and in the face of multiple equilibria.

One final, but important, observation should be made before we leave the 2-person world. Fig. 4 suggests a natural index of the extent of easy riding. The ratio OE/OQ expresses the equilibrium consumption of q – hence the equilibrium level of generation of the public characteristic – as a proportion of its required level at the optimum Q. This index lies between 0 and 1 if Z is a good, and low values of the index reflect the presence of extensive easy riding. While this index should not be interpreted as a welfare measure, it gives precision to an interesting and widely discussed aspect of public goods problem. One obvious determinant of the extent of easy riding springs to mind. Suppose the reaction curves are as shown, but the indifference curves of both individuals are drawn so that, in the neighbourhood of their minima, they exhibit a high degree of curvature. Then the optimum Q will be relatively close to E. The extent of easy riding depends upon the degree of curvature of indifference curves in (q_1, q_2) space. This, in turn, depends upon the rate of change of the own-substitution and cross-substitution terms discussed above as we move along indifference curves. In short, to make statements about the degree of easy riding we have to go one derivative further than is usual and take account of terms such as $\partial^2\pi_x/\partial Z\partial x$ and $\partial^2\pi_x/\partial Z^2$.

III. THE n-PERSON COMMUNITY

Stemming from the seminal work of Olson (1965), there has been widespread concern in the public goods literature that, as the number of individuals in a community rises, the problem of underprovision of public goods will become more acute. Our aim here is to explore the implications for this concern of introducing joint production of x and z into the analysis. Our graphic technique enables us to consider the effect of changes in the size of the community on both equilibrium and optimum allocations, thereby allowing a precise measure of the degree of easy riding. Our principal conclusion is agnostic: In the presence of jointness, an increase in the size of the community may either exacerbate or mitigate the problem of easy riding. For very large values of n, it is not necessarily true that individuals tend to become completely free riders in either an absolute or a relative sense.[1]

In order to exploit our diagrammatic technique, we make some simplifying assumptions. We continue to assume that individuals are identical in all relevant respects, and for convenience set $\gamma_h = 1$ for all individuals. Additionally, stable symmetric equilibria are assumed for all values of n, and attention is confined to a comparison of these equilibria with symmetric Pareto optima. This enables us to model the 'representative citizens' consumption of q in the light of the aggregate behaviour of the rest of the community.

In Fig. 5, h's consumption of q is measured along the horizontal axis, while the aggregate consumption of q by all his fellow citizens, $\tilde{Q}_h$, is measured along the vertical axis. We already know that, if $n = 2$, the symmetric equilibrium will be at E_2, where h's reaction curve intersects the 45° ray through the origin. The symmetric optimum will be at Q_2, where one of h's indifference curves is tangential to that same ray. At Q_2, neither individual can be made better off without his fellow being made worse off.

Now suppose a third individual joins. To locate the equilibrium and optimum for the trio, we first draw the ray marked $n = 3$ with a slope of 2. The new equilibrium is at E_3, where the representative citizen's reaction curve intersects the $n = 3$ ray. At E_3, each representative citizen is consuming q_h'' in response to an aggregate consumption of $2q_h''$ among the rest of the community. The construction of the $n = 4$ ray with a slope of 3, and the generation of E_4, proceed in an obvious manner, as does the generation of equilibria as the com-

[1] We call an individual a free rider in the absolute sense if his individual contribution tends to zero as n increases. He is a free rider in a relative sense if the ratio of his Nash equilibrium to his socially optimum contributions – OE/OQ in Fig. 4 – tends to zero as n increases.

munity grows further. The Pareto optima are equally straightforward to locate. For, say, $n = 3$ the optimum is at Q_3, where the $n = 3$ ray is tangent to an indifference curve.

In Fig. 5 the reaction curve is upward sloping from E_2 to E_4, so that as n increases so, too, does the equilibrium contribution to the public characteristic. Moreover, the locus of Pareto optima bends back, which is more than sufficient to make $OE_4/OQ_4 > OE_3/OQ_3 > OE_2/OQ_2$; hence, the extent of easy riding is reduced as n increases from 2 to 4.

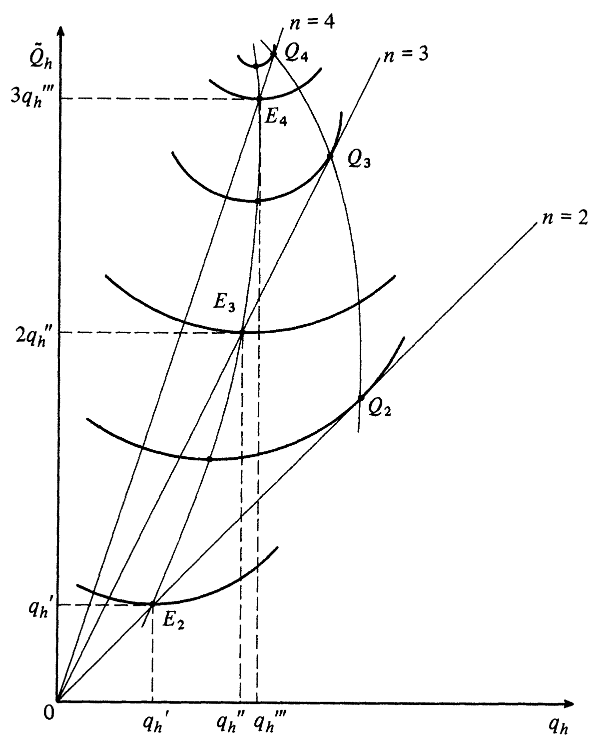

Fig. 5. Nash and Pareto equilibria for an n-person model.

We are not, of course, denying the possibility that easy riding may be more prevalent in larger communities, but simply pointing out that, even in the absence of powerful income effects, it is not an inevitable feature of public goods models (see Chamberlin, 1974). One can construct various possible shapes for the equilibrium and optimum loci in Fig. 5, and it is instructive to do so. One special case we believe to be particularly important. Suppose that, beyond a certain point, there is satiation with respect to the public characteristic. Fig. 6 reflects this by having closed loops for indifference curves. As n increases from 2 up to 4, the equilibrium and optimum allocations converge. If one believes in eventual satiation, then for values of n below the satiation level there is a presumption that E and Q will converge as n increases; thus, a Nash equilibrium need not imply inefficiency in an n-person model.

An interesting interpretation can be given to the locus of optima. Suppose that each individual, instead of taking his fellow citizen's present behaviour as given, acts on the assumption that they will act as he does. The resulting 'Kantian' behaviour, which is discussed by Laffont (1975), implies an equilibrium which is Pareto optimal. The locus of symmetric optima may be interpreted as the locus of Kantian equilibria. An unexplored area of analysis concerns the effect of group size on the appropriate equilibrium concept. If smaller group size engenders Kantian beliefs, then suboptimality may only involve groups beyond a critical size.

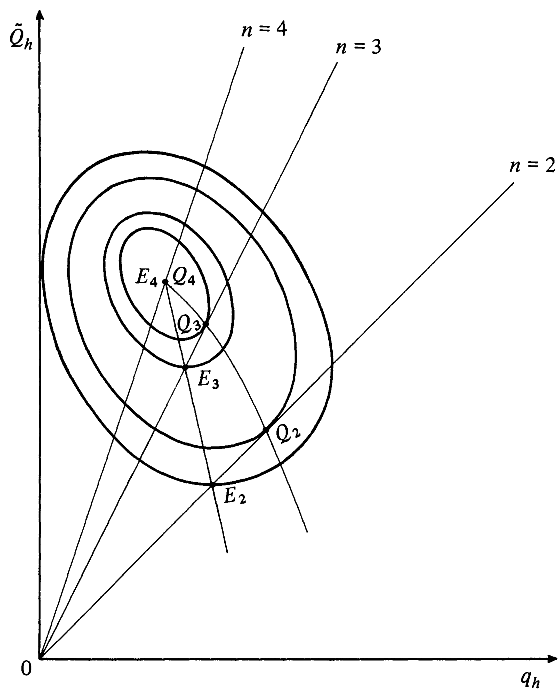

Fig. 6. Nash and Pareto equilibria for an n-person model: the case of a bliss point.

IV. THE ORTHODOX MODEL WITHOUT JOINTNESS

The orthodox model, from which joint production of characteristics is absent, is easily derived as a special case of our model. One can assume either that the representative consumer has no taste for the private characteristic, x, or that the marketed commodity, q, generates no units of x, so that $\beta = 0$. In either case, the MRS_{zc} can be written as a function of Z and u alone, so that in equilibrium the typical individual's allocation satisfies the condition

$$p = \gamma \pi_z(Z, u).$$

Consider the response of individual 1's demand for q to a change in q_2 in the 2-person example. Equation (4) simplifies to

$$dq_1/dq_2 = -(\gamma_2/\gamma_1) - (\pi_{zu}/\gamma_1 \pi_{zz})(du/dq_2).$$

If the real income effect of a change in dq_2 is zero, either because compensation is paid or because $\pi_{zu} = 0$, the remaining compensated response implies a linear negatively sloped reaction curve in (q_1, q_2) space, with a slope of $-\gamma_2/\gamma_1$. Moreover, if the curve is redrawn in (z_1, z_2) space, it has a slope of 45° to the axes, indicating a constant value for $Z = z_1 + z_2$.[1] In a community of identical individuals whose tastes are such that $\pi_{zu} = 0$, the Nash equilibrium provision of Z is therefore independent of the size of the community. So long as Z is a public good, its equilibrium level will either rise or fall as n increases according to whether Z is a normal or an inferior good. In the orthodox model, any such change is wholly the consequence of the income effect.

V. APPLICATIONS

The joint production model and the diagrammatic representation of public goods problems introduced above have a number of applications, of which we draw attention to three: common property resources, aggregate savings, and alliance behaviour. The analysis can also be applied to philanthropy and the theory of congestion. In the latter case, utilisation of a shared good yields private benefits and public benefits (congestion). These other applications are examined elsewhere (see, for example, Posnett and Sandler, 1983).

Common Property Resource

Common property problems involving a scarce factor which is not imputed a rent, such as the fishery problem discussed by Dasgupta and Heal (1979, pp. 55–73), may be represented by a somewhat modified version of Figs. 4 and 5. Measure the size of a typical individual firm's fleet, r, along the horizontal axis and the size of the rest of the fleet, $\tilde{R}$, along the vertical axis. The resulting diagram, in which isoprofit curves and a reaction curve for the typical firm may be drawn, looks similar to the familiar duopoly diagram (see, for example, Malinvaud, 1972, p. 151; or Dixit, 1979). The difference is essentially one of interpretation: In the duopoly model, the interdependence between firms works through the price mechanism, since the actions of one firm influence the demand curve faced by the other. In the present example, the fishermen are price-takers but impinge on one another by depleting the stock of fish. Using Scitovsky's (1954) well-known distinction, duopoly involves a pecuniary externality, while the common property resource problem involves a technological externality.

Elsewhere, we have exploited the methods introduced here to develop an 'index of tragedy' along the lines of the index of easy riding (Cornes and Sandler, 1983) so as to compare Pareto optima and Nash equilibria for the commons. Thus, we are able to show the influence of group size on the extent of suboptimality. The diagram can also be used to examine non-Nash conjectures for the commons, where one exploiter recognises that his optimal fleet choice may affect the optimising choice of the other firms in the industry. Such conjectures are reasonable when few firms comprise the industry and these firms

[1] This result is identical to Sugden's (1982, p. 344, equation (6)) findings when real income effects are zero; thus, our 'orthodox' results replicate his conclusions.

have had long-term interactions with one another. Tangencies between the isoprofit curves and a set of expectations contours represent the non-Nash path. The index of tragedy can then be used to compare non-Nash and Pareto optima. Such a comparison identified the type of conjecture that can reduce inefficiency. Additionally, we have demonstrated that a Nash conjecture can never be consistent (i.e. realised in equilibrium) and that only one form of conjecture, implying a worse tragedy of the commons than usually supposed, can be consistent (see Cornes and Sandler, 1983, pp. 791–2 for details). These are novel results, easily displayed with tools developed here.

Aggregate Savings and Choice of Techniques

Our second example concerns the literature on the alleged suboptimality of aggregate savings and its implications for project selection in developing countries. We relax the assumption of identical individuals, allowing differences in tastes and in incomes. The resulting analysis highlights properties of the orthodox model which are simple, robust, and which have important practical policy implications.

The argument, associated with Sen (1972), runs along the following lines. The concern of members of the present generation for the well-being of future generations is in the nature of a public good. To the extent that individuals take an easy ride, relying on their citizens to contribute, the level of aggregate savings will tend to be suboptimal. The argument goes on to suggest that, in view of the paucity of other policy instruments for correcting this problem, the burden of raising the aggregate level of saving out of current income should be borne, at least in part, by the choice of investment projects. Specifically, if two projects are identical in all respects, except that the recipients of the income generated by project A tend to have a higher marginal propensity to save out of current income than do the recipients of the income generated by project B, Sen (1972) would have us rank A above B.

However, Warr (1981) has recently pointed out that if indeed the provision of investible resources for future generations is a pure public good as in the orthodox model, then the distribution of current income has no effect on the level of aggregate savings. We briefly examine the basis of Warr's results, and then suggest a possible way of saving Sen's argument.

Consider the representative individual facing the orthodox problem of Section IV. Putting $\gamma = 1$ for simplicity, his problem may be written as

$$\underset{\{c,\, z\}}{\text{maximise}}\ u(c,\ z+\tilde{Z})$$

$$\text{subject to } c+p_z z = I, \text{ and } \tilde{Z} \text{ given.}$$

The interpretation of the public good as the level of aggregate savings may be accommodated by putting $p_z = 1$.

To understand Warr's observation, observe that the budget constraint may be used to remove z from the utility function, which then becomes

$$u(\cdot) = u(c,\ -c/p_z+I/p_z+\tilde{Z}). \tag{5}$$

The variables I/p_z and $\tilde{Z}$ appear in exactly the same way in (5), so that if they change in such a way that $I/p_z + \tilde{Z}$ remains constant, the real situation of the individual is unchanged. Specifically, the optimum c and Z remain constant. The full demonstration of Warr's invariance of savings result, when savings is a pure public good, is derived in an Appendix.

If one believes that the appropriate model of savings behaviour is not the orthodox one, but rather one which incorporates jointness, then Sen's argument for using project selection to generate additional savings through distributional effects *may be rescued*. If the individual's level of savings enters his utility function both as a private good s_h and through its contribution to aggregate savings, $S = \sum_{j=1}^{n} s_j$, the distribution of income will then influence aggregate savings through its effect on the private motive for savings. When, in particular, s_h and S are complements in consumption, the private motive (i.e. s_h) may induce an individual to contribute more in spite of the contributions of others. Investment projects offering such complementary joint products should be favoured.

NATO Allies Behaviour

The theory developed above has been applied by Murdoch and Sandler (1982, 1984) to explain NATO allies' defence expenditure changes after 1973, changes that cannot be explained by existing models (e.g. Olson and Zeckhauser, 1966). Murdoch and Sandler have characterised an ally's arsenal as producing joint products of varying degrees of publicness. In particular, military expenditures yield deterrence and protective (or damage-limiting) outputs. Deterrence, as provided by strategic nuclear forces, produces nonrival benefits to an alliance, since the addition of an ally does not diminish the deterrence provided to the current allies; the benefits derived from the threat of punishment are unaffected by new allies. Moreover, if an attack upon an ally inflicts unacceptable damage on the other allies in terms of fallout or the loss of military personnel, then deterrence is nonexcludable and satisfies both characteristics of publicness.

In contrast, many of the benefits derived from protective (e.g., conventional) weapons are impurely public or private owing to rivalry and excludability. When, for example, conventional weapons are deployed to protect a larger front or perimeter as a new ally joins, a *thinning of forces* results from a spatial rivalry, which detracts from the protection of the original allies. Since protective forces can be withdrawn and redeployed elsewhere for private motives, the benefits of these weapons are subject to exclusion at the will of the provider.

Murdoch and Sandler (1984) have argued that the Olsen and Zeckhauser's (1966) prediction of free riding worked well for the 1960s, because NATO then relied on deterrent, nuclear arsenals owing to NATO's inferior conventional strength. When purely public, deterrent outputs are the primary benefits shared by an alliance, one ally's defence expenditures should decline as the rest of the allies increase their expenditures; i.e. $\partial q/\partial \tilde{Q}$, the slope of the defence expenditure reaction curve, is negative, where $\tilde{Q}$ represents the other allies' defence expenditures. In short, NATO allies' behaviour up until the early

1970s should follow the orthodox model of Section IV, provided that defence expenditure is a normal good.

An anomaly in the slope of an ally's defence expenditure reaction curve is predicted to have occurred after the introduction of the doctrine of flexible response in 1973 (see Murdoch and Sandler, 1984). Under this doctrine, NATO can react to Warsaw Pact challenges in multiple modes, so that all-out nuclear conflict can be avoided when provocation requires a military response. This doctrine changes fundamentally the consumption relationship among the different weapons classes – those of conventional weapons and strategic nuclear weapons. With this doctrine, the European allies must be prepared to defend themselves against conventional aggression on European soil, since the initial stages of warfare are expected to involve conventional exchanges. No longer can these nations rely entirely on nuclear deterrence for their security; thus, jointly produced military outputs become relevant. By tying warfare to a sequence of measured responses involving the deployment of all classes of weaponry, this doctrine makes nuclear and non-nuclear arsenals (and their produced attributes) complementary; thus, $\partial q/d\tilde{Q}$ is now predicted to become positive or less negative after 1973. Such a hypothesis has been tested for nine NATO allies, and the results strongly supported the hypothesis (see Murdoch and Sandler, 1984).

The theoretical analysis in Sections I–III have therefore led to a framework for relating the *signs* of the coefficients in regression estimates of military expenditures to the nature of the goods provided in the alliance. Thus, allies' expenditures for collective defence outputs can be distinguished from their contribution for private defence goods.

VI. CONCLUSIONS

This article has examined the comparative statics of the demand for a marketed good that jointly provides public and private outputs. The introduction of joint products has a significant effect on the conventional wisdom concerning public good provision. In particular, an increase in group or community size need not lead to increased suboptimality when joint products are present. This follows because the jointly produced private output can serve a privatising role, not unlike the establishment of property rights. Complementarity between the joint products brings out this privatising aspect. In addition, we have shown that, in the case of joint products, the stability of a Nash equilibrium is dependent on both a substitution and income effect; hence, the income effect is no longer the sole determining factor for stability. Moreover, Nash behaviour need not imply suboptimality for jointly produced public goods. We have also demonstrated that a public goods model with identical individuals may have an asymmetric equilibrium. Additionally, we have introduced a geometric device, capable of comparing Nash and Pareto equilibria in the same diagram. Finally, the analysis has been applied to topics of current interest.

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APPENDIX

Stability Condition in the 2-Person Community

Following the discussion by Cornes (1980) of reciprocal externalities, we consider the following simple adjustment mechanism:

$$dq_1/dt = u_1\{q_1(q_2) - q_1[t]\}, \quad dq_2/dt = \mu_2\{q_2(q_1) - q_2[t]\},$$

where μ_1 and μ_2 are positive constants, $q_h[t]$ is the actual value of q_h at time t, and $q_h(q_j)$ is the uncompensated demand or Nash reaction function for q_h, the statement of which suppresses the constants, p and I_h.

Linearising the system in the neighbourhood of the equilibrium $(\bar{q}_1, \bar{q}_2)$, we obtain

$$\begin{bmatrix} dq_1/dt \\ dq_2/dt \end{bmatrix} = \begin{bmatrix} -\mu_1 & \mu_1\, \partial q_1/\partial q_2 \\ \mu_2\, \partial q_2/\partial q_1 & -\mu_1 \end{bmatrix} \begin{bmatrix} q_1 - \bar{q}_1 \\ q_2 - \bar{q}_2 \end{bmatrix},$$

where $\partial q_1/\partial q_2$ and $\partial q_2/\partial q_1$ are the slopes of the first and second agent's Nash reaction path, respectively.

Stability requires that the determinant associated with the 2×2 matrix be negative definite. Since the μ's are positive, stability requires

$$\mu_1 \mu_2 \cdot \left(1 - \frac{\partial q_1}{\partial q_2} \cdot \frac{\partial q_2}{\partial q_1}\right) > 0$$

or

$$\left(\frac{\partial q_1}{\partial q_2} \cdot \frac{\partial q_2}{\partial q_1}\right) < 1.$$

If individuals are identical, then in the neighbourhood of a symmetric equilibrium we can define $\omega \equiv \partial q_1/\partial q_2 = \partial q_2/\partial q_1$. A necessary and sufficient condition for local stability is that

$$-1 < \omega < 1.$$

Stability Condition in the n-Person Community

The assumed adjustment mechanism is

$$\begin{aligned} dq_h/dt &= \mu_h\{q_h(\tilde{Q}_h) - q_h[t]\} \quad (h = 1, 2, \ldots, n) \\ &= \mu_h\{q_h(\sum_{j \neq h} q_j) - q_h[t]\}. \end{aligned}$$

As before, we assume identical individuals and linearise in the neighbourhood of a symmetric equilibrium. Denoting the quantity responses $\partial q_h/\partial q_k$ by ω, the dynamic system can be written as

$$\begin{bmatrix} dq_1/dt \\ dq_2/dt \\ \vdots \\ dq_n/dt \end{bmatrix} = \mu_1 \mu_2 \ldots \mu_n \begin{bmatrix} -1 & \omega & \ldots & \omega \\ \omega & -1 & & \\ \vdots & & \ddots & \vdots \\ & & & \omega \\ \omega & \ldots & \omega & -1 \end{bmatrix} \begin{bmatrix} q_1 - \bar{q}_1 \\ q_2 - \bar{q}_2 \\ \vdots \\ q_n - \bar{q}_n \end{bmatrix}.$$

A necessary and sufficient condition for local stability is that the matrix of coefficients be negative definite. To locate the implied bounds on the value of ω, it is helpful to apply elementary operations to the determinant of the matrix in order to make all terms below and to the left of the main diagonal equal to zero. If this is done, the value of the determinant, Δ, is seen to be

$$\Delta = |[(n-1)\,\omega-1][-(\omega+1)^{n-1}]|.$$

The determinants associated with the lower-order cofactors can be evaluated in the same way, and the requirement that they alternate in sign may be summed up by the condition that

$$\text{sign}\,\{[(h-1)\,\omega-1][-(\omega+1)^{h-1}]\} = \text{sign}\,(-1)^h \quad (h = 1, 2, \ldots, n).$$

$$\text{Let } A \equiv [(h-1)\,\omega-1] \quad \text{and} \quad B \equiv [-(\omega+1)^{h-1}].$$

Now consider an even value h. The stability condition requires the product AB to be positive. If $\omega < -1$, then $A < 0$ and $B > 0$, and instability results. Therefore, we must have $\omega > -1$. If this is satisfied, then $B < 0$, so that we must have $A < 0$. But this is equivalent to requiring that $\omega < 1/(h-1)$.

A similar argument establishes the same bounds for ω when h is an odd integer. Clearly, also, if $\omega < 1/(h-1)$ when h takes on the value n, the same inequality is satisfied for smaller values of h. Hence, the required stability condition is

$$-1 < \omega < 1/(n-1).$$

Since $dq_h/d\tilde{Q}_h = (n-1)\,\partial q_h/\partial q_k$, the stability condition may be written as

$$(1-n) < dq_h/d\tilde{Q}_h < 1.$$

This shows that local stability is consistent with an upward-sloping reaction curve, so long as the representative quantity response is not too large. In Fig. 5, for example, an equilibrium is stable if, in its neighbourhood, the locus of Nash equilibria has a slope greater than -1. If quantity responses are negative, they can be quite large numerically without upsetting stability.

Derivation of Warr's Invariance of Savings Result

Consider the Nash reaction function $Z(p_z, I, \tilde{Z})$. Take a total differential of this reaction function, while keeping $I/p_z+\tilde{Z}$ constant as in the text. Upon simplification, we have

$$dZ = 0 = p_z(\partial Z/\partial I)-(\partial Z/\partial \tilde{Z}).$$

But $Z(p_z, I, \tilde{Z}) = z(p_z, I, \tilde{Z})+\tilde{Z}$. Therefore, we have

$$\partial Z/\partial I = \partial z/\partial I \quad \text{and} \quad \partial Z/\partial \tilde{Z} = \partial z/\partial \tilde{Z}+1.$$

Substitution gives

$$p_z(\partial z/\partial I) = (\partial z/\partial \tilde{Z})+1. \qquad (*)$$

This relationship is the key to the inability of income redistribution to influence the equilibrium value of Z in the pure public goods model. Starting at an equilibrium in the 2-person economy, consider the effect of transferring income between individuals. For individual 1,

$$z_1 = z_1(I_1, z_2).$$

The differential of this last expression is

$$dz_1 = (\partial z_1/\partial I_1)\, dI_1 + (dz_1/\partial z_2)\, dz_2.$$

A similar relationship holds for individual 2. Using the key relationship (*), together with the fact that $dI_1 = -dI_2$, it is easily shown that $dZ = dz_1 + dz_2 = 0$. This result, it should be emphasised, does not depend on identical tastes in any sense.

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