

ECON 441: ANALYTICAL PUBLIC FINANCE

Homework 2

1. When Harry met Sally, he knew they would have many disagreements. For example, Harry likes to watch baseball. Let x be the amount private goods and y be hours per day spent watching baseball. Harry's utility is $U_H = x_H y_H$. However, when Harry consumes y_H , it makes Sally worse off. Sally's utility is $U_S = 2x_S - y_H$. Both Harry and Sally are endowed with 10 units of x and 18 waking hours in a day.
 - a) If we were to impose a Pigouvian tax on Harry's consumption of y , what would be the appropriate tax? Explain.
 - b) Suppose instead that we were to rely on the Coase analysis, and that we awarded Harry the property rights. Illustrate the situation in an Edgeworth box, and indicate the efficient trades that Harry and Sally can make. (NOTE: Be precise about how you draw Sally's indifference curves, and approximate Harry's.)
2. Tex and Spuds are two Texas oil drillers. The land owned by Tex is right next to the land owned by Spuds. There is one pool of oil beneath BOTH properties. The more oil wells that Tex sets up, the less oil Spuds can get from any well that he drills. Likewise, the more oil wells that Spuds sets up, the less oil there will be for Tex to drill.

Let x be the total number of wells drilled by both Tex and Spuds. The total revenue (to BOTH Tex and Spuds) from drilling oil is $R(x) = 40x - 2x^2$. Hence, the revenue *per oil well drilled* is $R(x)/x = 40 - 2x$. The marginal revenue from drilling an *additional* well is $dR(x)/dx = 40 - 4x$. It costs \$20 to set up and operate an oil well.

 - a) Suppose that Tex and Spuds act competitively. What is the total number of oil wells, x , that will be drilled? Explain verbally why this is not Pareto efficient.
 - b) What would be the Pareto Efficient total number of wells, x ?
 - c) Suppose that the government wanted to put a well-tax on each well drilled. What tax should the government put on wells?
3. Thelma has utility function $U_T = x_T G$ and Louise has utility function $U_L = x_L G$. The resource constraint in the economy is $x_T + x_L + cG = W$, where $c > 0$ is the cost of G in terms of x .
 - a) Give an intuitive explanation of the *Free Rider Problem* in terms of the above model.
 - b) Find the Samuelson Condition for the economy with Thelma and Louise. Give an intuitive interpretation of this condition.
 - c) What is the Pareto Efficient level of G in this economy?

4. Each time that the steel mill produces a ton of steel, the smoke it generates ruins one bushel of apples at a nearby orchard.
 - a) Describe verbally why the situation is not Pareto efficient.
 - b) Let $c_s(s)$ be the cost of producing steel, and let $c_a(a)$ be the cost of producing apples. Suppose the price of steel is \$5 per ton, and the price of apples is \$2 per bushel. Write an expression for the profits of the steel producer.
 - c) Write an expression for the profits of the apple producer. (Don't forget to include the externality.)
 - d) What is the appropriate Pigouvian tax (or subsidy) in this situation, and who should pay (or receive) it? (HINT: apply your description in (a) to parts (b) and (c).)
5. Suppose a sardine cannery is located next to a bakery. Each acts as a competitive firm in their markets. Let S be the amount of sardines canned, and let B be the loaves of bread baked. The cost function of the cannery is $c_s(S) = S^2/50$, and the cost function of the bakery is $c_b(B) = B^2/50 + 2S$. The price of S is $P_s = 4$ and the price of B is $P_b = 2$
 - a) If each firm operates independently, how much S and B will be produced? Are these levels Pareto efficient? Explain.
 - b) If you were to encourage Pareto efficiency with a Pigouvian tax, which firm would you tax, and how much would the tax be? Explain.
6. Consider an economy consisting of only two people, Bill and George. There is one private good, corn, and one public good, Mega-missiles. Let Bill's preferences be $U_B = (1 + M)x_B$ and let George's preferences be $U_G = (2 + M)x_G$ where x is the amount of the private good, and $M = 1$ if the Mega-missile is built and $M = 0$ if it is not built. Suppose Bill is endowed with 4 million dollars, while George is endowed with 15 million dollars.
 - a) What is Bill's willingness to pay for the Mega-missile? George's willingness to pay?
 - b) Suppose the Mega-missile costs 6 million dollars. If the Mega-missile is built, Bill and George will each be assessed 3 million dollars in taxes. Who will favor the Mega-missile and who will oppose it. Explain.
 - c) George says that "We must build the Mega-thing. After all, it's Pareto improving to do so!" Bill sternly replies, "George, I've known Pareto improvements. I've heard Professor Andreoni's lectures on Pareto improvements. And George, that is no Pareto improvement!" Is Bill correct or was that comment uncalled for?
 - d) Could you propose a division of the costs of the Mega-missiles that both Bill and George could agree on? Give an example.