Ultimatum Bargaining

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Econ 182

1 Demonstration:

The *Proposer-Responder Game*

2 Background: Nash Equilibrium

Example

Let's think about how we make a prediction in this game:

- Each Player can choose to **Push 3** to the other player or **Pull 1** from the other player.
  - We call "Push" and "Pull" the *strategies*
- This is how we show this in a game matrix:

		Column Player	
		Push	*Pull*
Row Player	*Push*	3,3	0,4
	Pull	4,0	1,1

- What's the equilibrium?
- To make a prediction, we need a model of how each person "thinks" about the game.
- How will the other person *react* to my decision?

The Nash assumption

- Players would say, “If my opponent chooses strategy X, I am best off choosing strategy Y.”
  - Call this a *Best Reply*
  - “Y is a best reply to X.”
  - Note, Y may be one thing or a list of several that are tied.
- If Y is a best reply to X for the Row player, and X is a best reply to Y for the Column player, then (X, Y) is an *equilibrium.*

		Column Player	
		Push	*Pull*
Row Player	*Push*	3,3	0,4
	Pull	4,0	1,1

Solution:

- Suppose Row plays *Push*. What's the Best Reply for Column?
- Suppose Row plays *Pull.* What's the Best Reply for Column?

		Column Player: *Push*	Column Player: *Pull*
Row Player	*Push*	3,3	0,4
	Pull	4,0	1,1

Solution:

- Suppose Row plays *Push*. What's the Best Reply for Column? A: *Pull*
- Suppose Row plays *Pull.* What's the Best Reply for Column? A: *Pull*
- This is called a *dominant strategy* – The best reply to every strategy of the opponent is always the same.
- Since both Row and Column have the same dominant strategy to *Pull,* the *Nash Equilibrium is (Pull, Pull)* and payoffs are (1,1).
- NOTE: This equilibrium is not efficient–each would make more if they both chose *Push.*
- NOTE: This game is called the *Prisoners' Dilemma*.

3 Today's Demonstration: Proposer-Responder

- This game is called the **Ultimatum Game** (for good reason)
- Objective:100 points to split.
- Player 1: Proposes $X \in \{0, 1, 2, ..., 100\}$ for Player 2, the rest for him/herself.
- Player 2:
  - Accept the proposal and earnings are $(100 - X, X)$ or
  - Reject the proposal and earnings are $(0, 0)$.

What's the Equilibrium?

4 What's the Equilibrium?

- What is Player 2's BR to Player 1's offer of...
  - $X = 100?$
  - $X = 99?$
  - ...etc...
  - $X = 1?$
  - $X = 0?$

5 What's the Equilibrium?

- What is Player 2's BR to Player 1's offer of...
  - $X = 100?$ Accept, since it is higher than 0.
  - $X = 99?$ Accept since it is higher than 0.
  - ...etc...
  - $X = 1?$Accept since it is higher than 0.
  - $X = 0?$ Accept or Reject, since they both give 0..
- What is Player 1's BR to Player 2's strategy?

6 What's the Equilibrium?

- What is Player 2's BR to Player 1's offer of...
  - $X = 100?$ Accept, since it is higher than 0.
  - $X = 99?$ Accept since it is higher than 0.
  - ...etc...
  - $X = 1?$Accept since it is higher than 0.
  - $X = 0?$ Accept or Reject, since they both give 0..
- What is Player 1's BR to Player 2's strategy?
  - Offer 1. It will be Accepted and I'll earn 99! or....
  - Offer 0, assuming Player 2 accepts and earn 100!

Well, actually, that's not the only equilibrium

- Go back to Nash's definition of equilibrium.
- Suppose a responder's (Player 2's) strategy is to accept $X = 50$, but reject all others.
  - This isn't necessarily a "reasonable" strategy, but Nash didn't define what's reasonable.
- Then the Proposer's (Player 1's) best reply to the responder's strategy is to offer $X = 50$.
- These two strategies are best replies, so are Nash Equilibria.
- This means if any threat to reject any offer but some $X > 0$ becomes part of an equilibrium, *if player 1 believes player 2 will carry out the threat.*
- Why isn't this "reasonable"?
  - It's stupid for Player 2 to reject an offer of $X = 49$, for instance.
  - Hence Player 1 shouldn't believe player 1 if he threatens to reject anything but 50.
  - In fact, the only threat Player 1 should believe is one to reject 0.
- In other words, after player 1 chooses $X > 0$, any threat to reject this is "not credible" so we will use this as our definition of not "reasonable."
- We call this *refinement* of Nash Equilibrium *Subgame Perfection.*

Restating it intuitively..

- The idea is that to be *subgame perfect* it must be a best reply *at the time you make the decision,* and it cannot depend on “what could have been."
- A strategy that rejects positive offers is a good idea *only* if it changes player 1’s behavior. But, once player 1 has made an offer, it never makes sense to reject it.

So what’s are the equilibria, and which among them is Subgame Perfect?

- A *Nash equilibrium* is :
  - Any responder strategy saying: “I will reject any offer but these...."
  - and any Proposer strategy “I will offer the smallest acceptable offer."
- A *subgame perfect Nash equilibrium* is
  - Any responder strategy saying “I will accept any offer" or “I will accept any positive offer"
  - and any Proposer strategy “I will offer the smallest acceptable offer."

7 What did you do in the demonstration?

- Did you choose the equilibrium?
- Why not?

8 Today's Papers

- Guth, Werner, R. Schmittberger, and B. Schwartz. "An Experimental Analysis of Ultimatum Bargaining" *Journal of Games and Economic Behavior*, December 1982, 3(4), pp. 367-88.
- **Forsythe, Robert, Joel Horowitz, N.S. Savin and Martin Sefton. "Fairness in Simple Bargaining Games." *Games and Economic Behavior*, May 1994, 6(3), pp. 347-69.
- Ochs, Jack and Alvin E. Roth. "An Experimental Study of Sequential Bargaining." *American Economic Review*, June 1989, 79(3), pp. 355-84.
- Roth, Alvin E., V. Prasnikar, M. Okunofujiwara and S. Zamir. "Bargaining and Market Behavior in Jerusalem, Ljubljana, Pittsburgh, and Tokyo: An Experimental Study." *American Economic Review*, December 1991, 81(5), pp. 1068-95.
- **Slonim, Robert and Alvin E. Roth, "Learning in High Stakes Ultimatum Games: An Experiment in the Slovak Republic." *Econometrica*, 66, 3, May 1988, 569-96.

(** = assigned readings)

9 The Ultimatum Game

This game is meant to be a simple test of alternating offers bargaining, with costly delay. This is sometimes called Rubinstein Bargaining.

- There is a pie of size M to divide between two players
- Player one offers x to player 2.
- If player two rejects the offer, the pie shrinks to $M_2 < M$.
- When $M_2 = 0$, then this is a final “Take-it-or-leave-it” ultimatum.

10 Early Experiments

Guth, et al. are credited with the first tests of this game. Subgame perfection failed miserably as many offers of 50-50 are made, and many offers, especially those of 20% or less, are rejected.
Why? Is it failure of game theory? This is only true if utility is defined as money earned in the experiment. What if people care about the earnings of both players in the game, and they care about the fairness of the allocation.
Forsythe, et al. published an extremely important test of one side of this hypothesis, that is, proposers care to be nice to recipients.
They designed a simple game that *subtracted* the concerns for rejection, but kept the same prediction under the hypothesis of selfish preferences. They called this the *Dictator Game.* Here, a responder cannot respond – all offers must be accepted. Now there are only two reasons for proposers to make positive offers, intentions and mistakes.
This figure shows what happened:

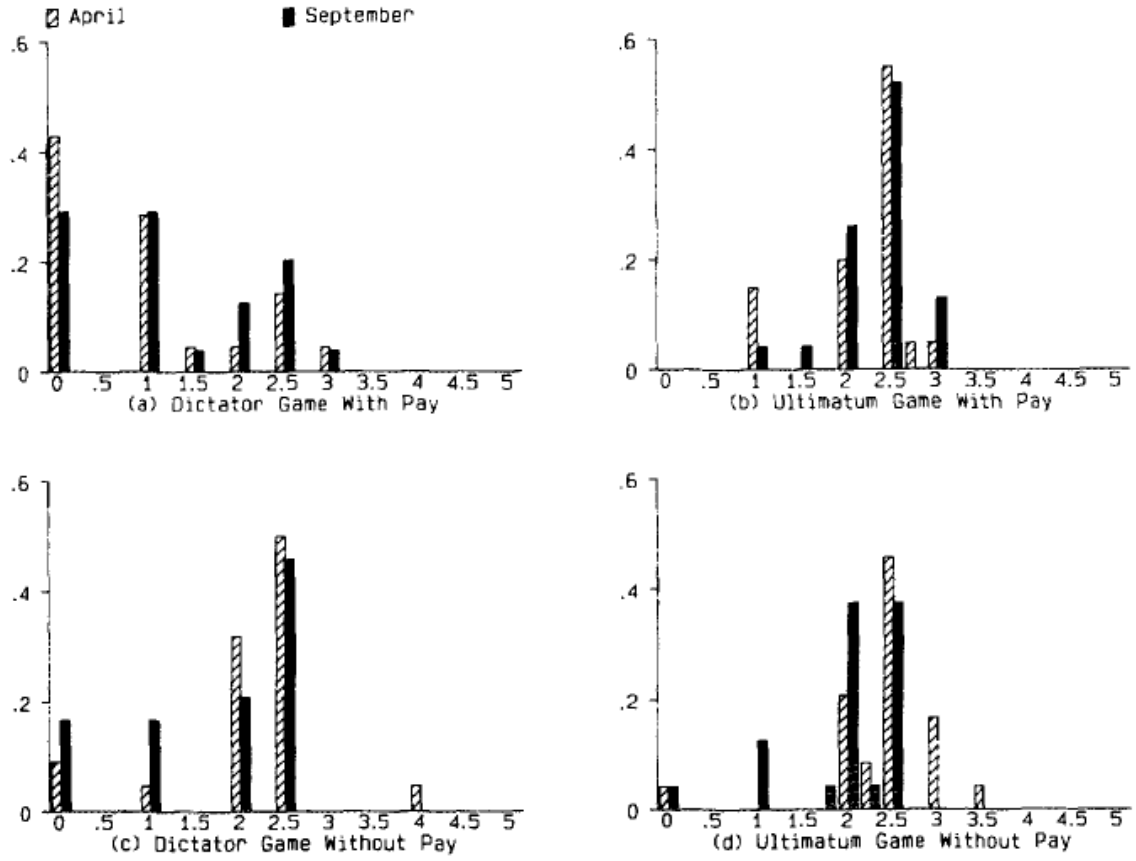


FIG. 1. Histograms of April and September proposals. Each histogram measures the amount of the proposal in dollars on the horizontal axis and the fraction of proposals of this amount on the vertical axis.

Clearly there is some fear of rejection, but also a lot of altruism.

This result raised many interesting questions.

- How do these preferences respond to the parameters of the bargain? If they are errors, they won't move in systematic and predictable ways.
- Are these preferences based in cultures? Sex? Race?
- What about large stakes? Won't bargaining results resemble the game-theoretic prediction with large stakes?
- Are there real and predictable preferences for giving, and what model would generate these results?
  Over the next several lectures, we will explore these and questions.

11 Changing Parameters

Ochs, Jack and Alvin E. Roth. "An Experimental Study of Sequential Bargaining." *American Economic Review*, June 1989, 79(3), pp. 355-84.
These authors looked at games with two or three stages. If player 2 rejects and offer, the pie shrinks and player 2 gets to make an offer to player 1, and so on. The equilibrium is again a factor or backward induction, starting at the end and factoring in the discount rates. By varying the number of stages and the discount rate they can identify how fairness and subgame perfection are interacting.

TABLE 1—EXPERIMENTAL DESIGN, AND RANGE OF EQUILIBRIUM DIVISIONS

	Two-Period		Three-Period	
	Chips	Money	Chips	Money
$\delta_1 = .4, \delta_2 = .4$	Cell 1: (59,41) to (61,39)	($17.70,$12.30) to ($18.30,$11.70)	Cell 5: (76,24)	($22.80,$7.20)
$\delta_1 = .6, \delta_2 = .4$	Cell 2: (59,41) to (61,39)	($17.70,$12.30) to ($18.30,$11.70)	Cell 6: (84,16)	($25.20,$4.80)
$\delta_1 = .6, \delta_2 = .6$	Cell 3: (39,61) to (41,59)	($11.70,$18.30) to ($12.30,$17.70)	Cell 7: (77,23) to (76,24)	($23.10,$6.90) to ($22.80,$7.20)
$\delta_1 = .4, \delta_2 = .6$	Cell 4: (39,61) to (41,59)	($11.70,$18.30) to ($12.30,$17.70)	Cell 8: (65,35)	($19.50,$10.50)

Results:

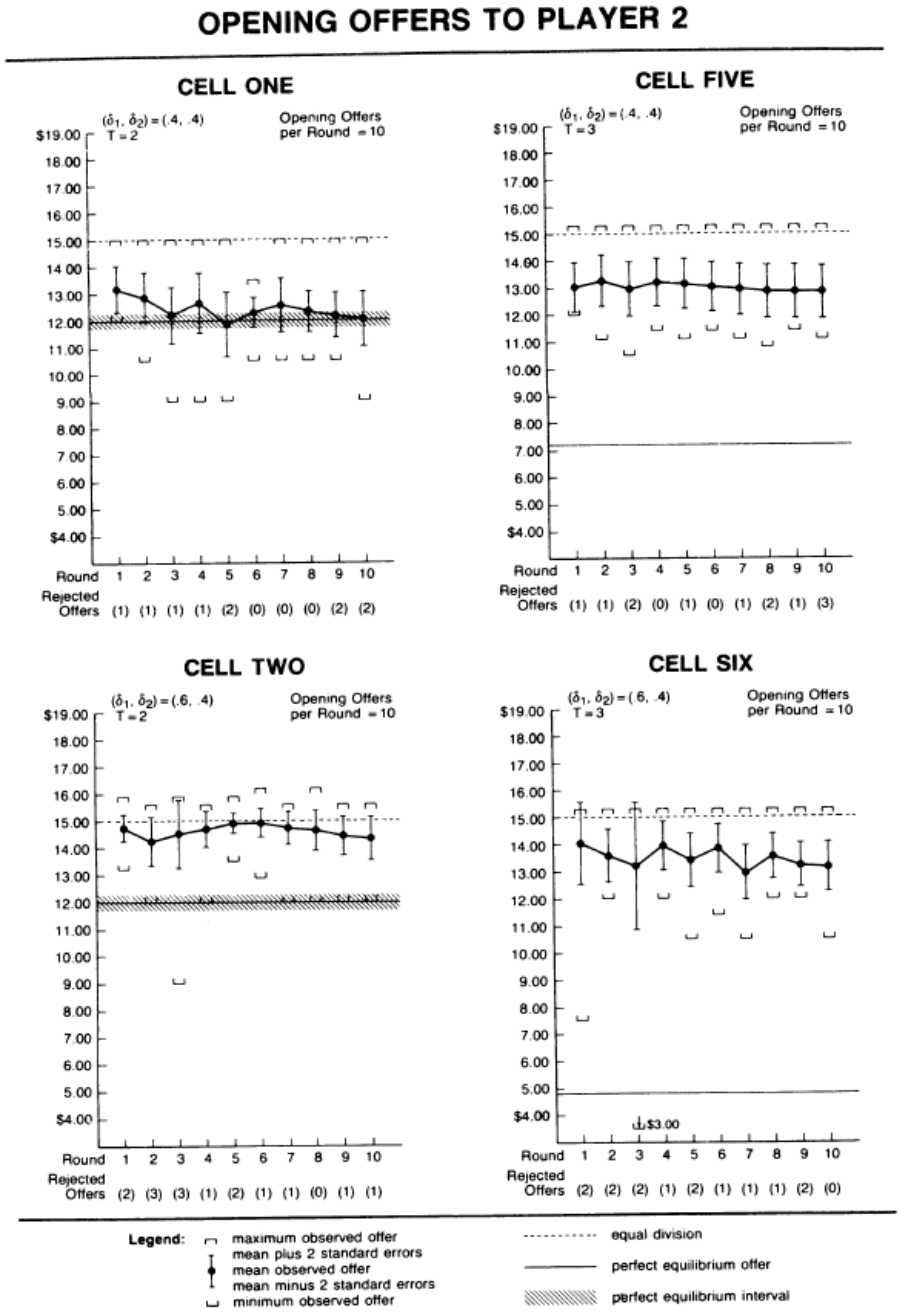


FIGURE 1A. OPENING OFFERS TO PLAYER 2 FOR CELLS ONE, TWO, FIVE,

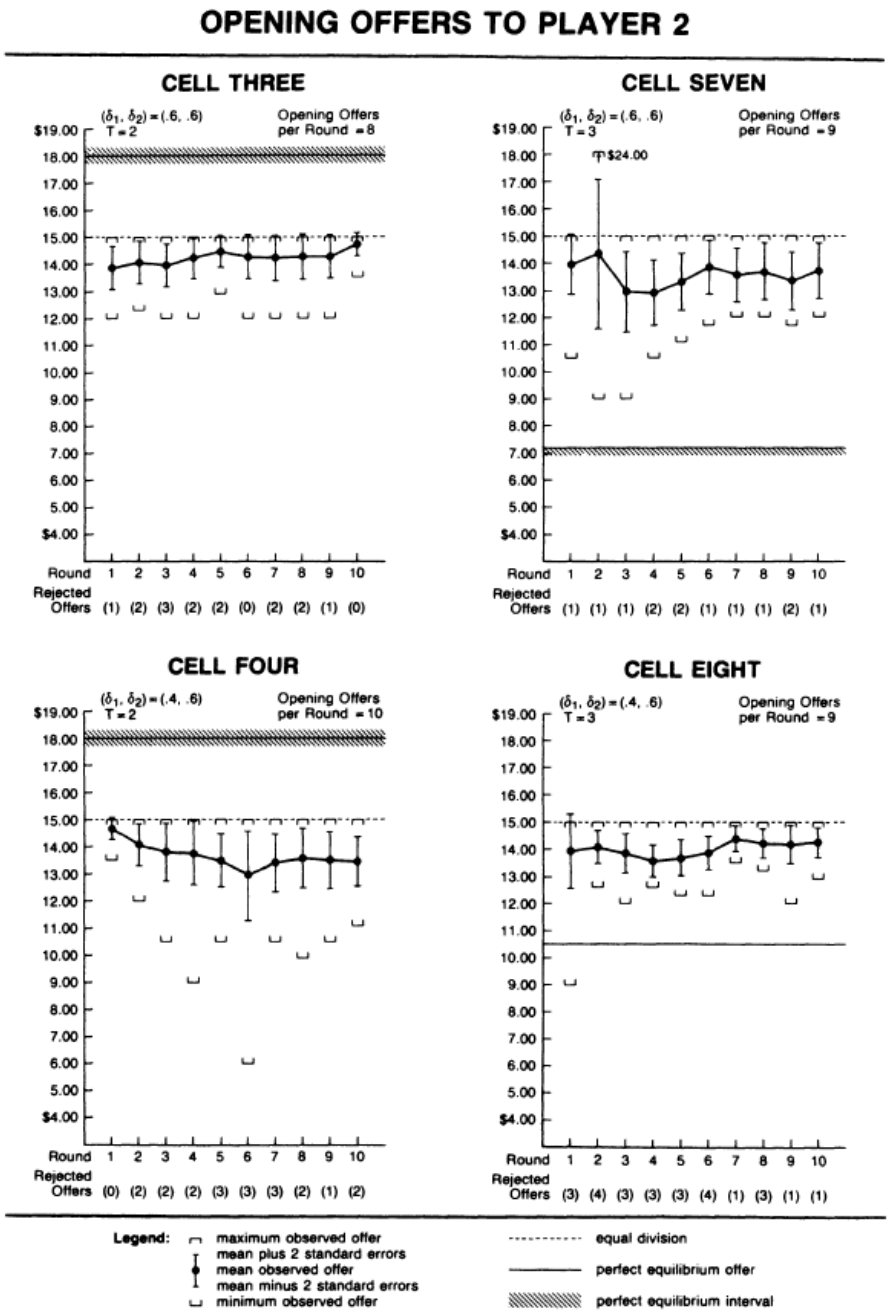


FIGURE 1B. OPENING OFFERS TO PLAYER 2 FOR CELLS THREE, FOUR, SEVEN, AND EIGHT

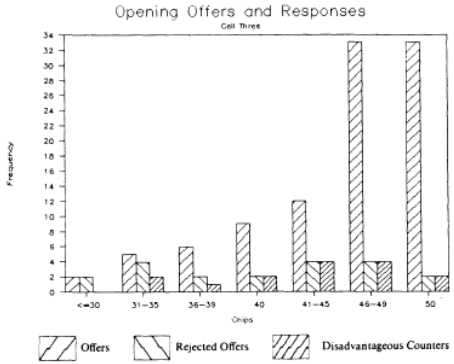


FIGURE 2D. OPENING OFFERS AND RESPONSES FOR CELL THREE

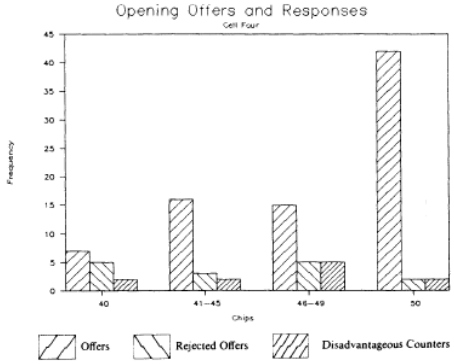


FIGURE 2C. OPENING OFFERS AND RESPONSES FOR CELL FOUR

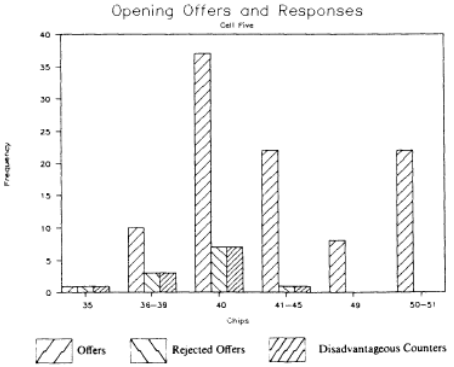


FIGURE 2E. OPENING OFFERS AND RESPONSES FOR CELL FIVE

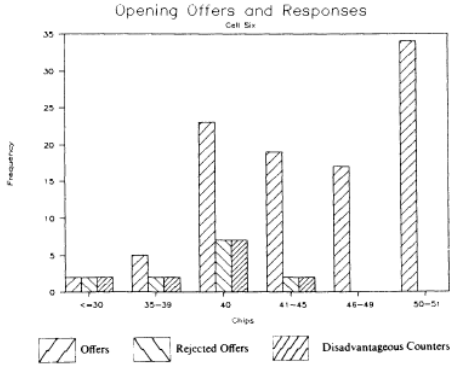


FIGURE 2F. OPENING OFFERS AND RESPONSES FOR CELL SIX

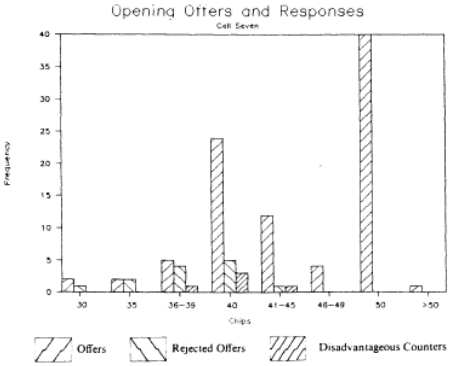


FIGURE 2G. OPENING OFFERS AND RESPONSES FOR CELL SEVEN

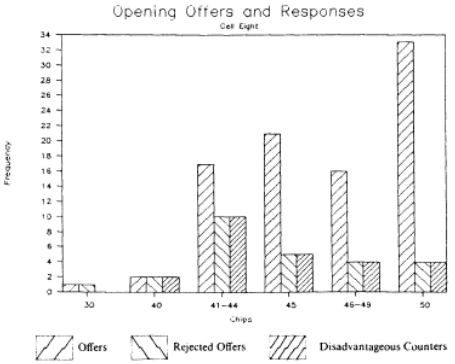


FIGURE 2H. OPENING OFFERS AND RESPONSES FOR CELL EIGHT

Disadvantagous Counter Proposal: *Player 2 rejects on offer that would earn him X if he had accepted it, then makes a counter offer that earns him Y if Player 1 accepts it, but $Y < X$. That is, Player 2 would have earned more by accepting the original offer.*

Summary of the most interesting findings:

- Play is somewhat sensitive to bargaining game parameters, but fairness is clearly dominant.
- The appearance of disadvantageous counter-proposals is an important observation that must be addressed.

12 Changing Cultures

Roth, Alvin E., V. Prasnikar, M. Okunofujiwara and S. Zamir. "Bargaining and Market Behavior in Jerusalem, Ljubljana, Pittsburgh, and Tokyo: An Experimental Study." *American Economic Review*, December 1991, 81(5), pp. 1068-95.

These authors ask the question of how cultural differences influence bargaining outcomes. Went to different countries.

Special Considerations:

1. Experimenter Effects
   a. Problem: Need to speak the native language, so need different experimenters in each country
   b. Solution: Have the same people come to Pittsburgh and run studies there too.
2. Language Effects
   a. Problem: Can all terms and ideas be translated?
   b. Solution: Translate to language X, then have a third party translate back to English. Compare.
3. Payoff/Currency Exchange
   a. Problem: Making the stakes comparable across countries

b. Solution: Normalize amounts to a common commodity, e.g. a movie ticket or an hour's wage for college students.

The objective her is to compare the outcome for "Market Games" to ultimatum games, both within and across countries.

- In the market game, 9 sellers make a TIOLI offer to buy an indivisible object from the seller. All buyers have the same value if they buy the object. If the seller rejects the highest offer, all players get noting.
  - The analogy to UGs is that buyers can reject inequality by making low bids. Will competition overwhelm this?
- Equilibrium in the Market Game is also unequal, as in UG.

RESULTS:

1. Market game reaches equilibrium in all countries, and fairly rapidly. All countries go to inequality fast. There are no interesting differences across countries.
2. With UGs their are differences

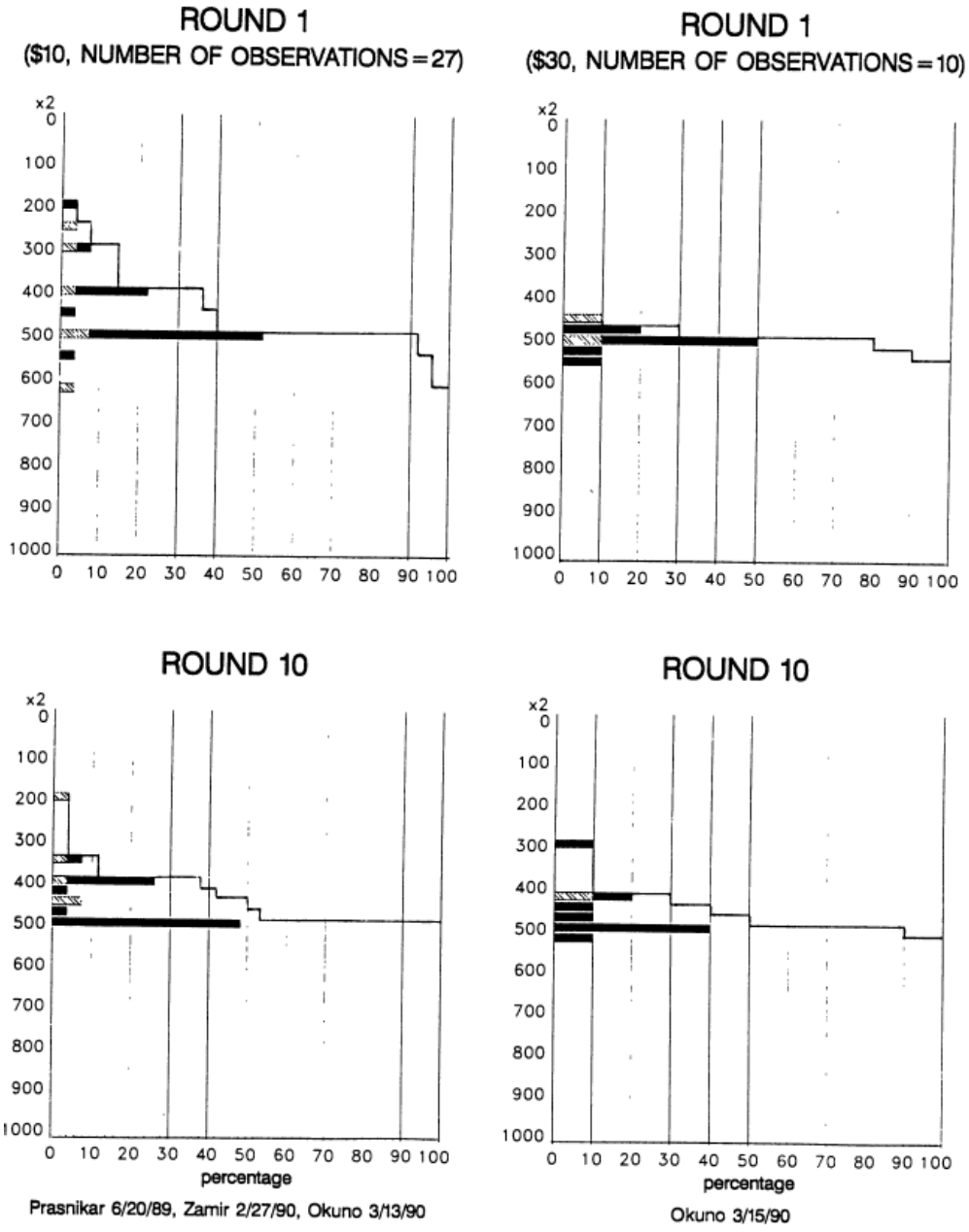


FIGURE 3. DISTRIBUTION OF BARGAINING OFFERS IN THE UNITED STATES (SOLID BARS = ACCEPTED OFFERS; STRIPED BARS = REJECTED OFFERS)

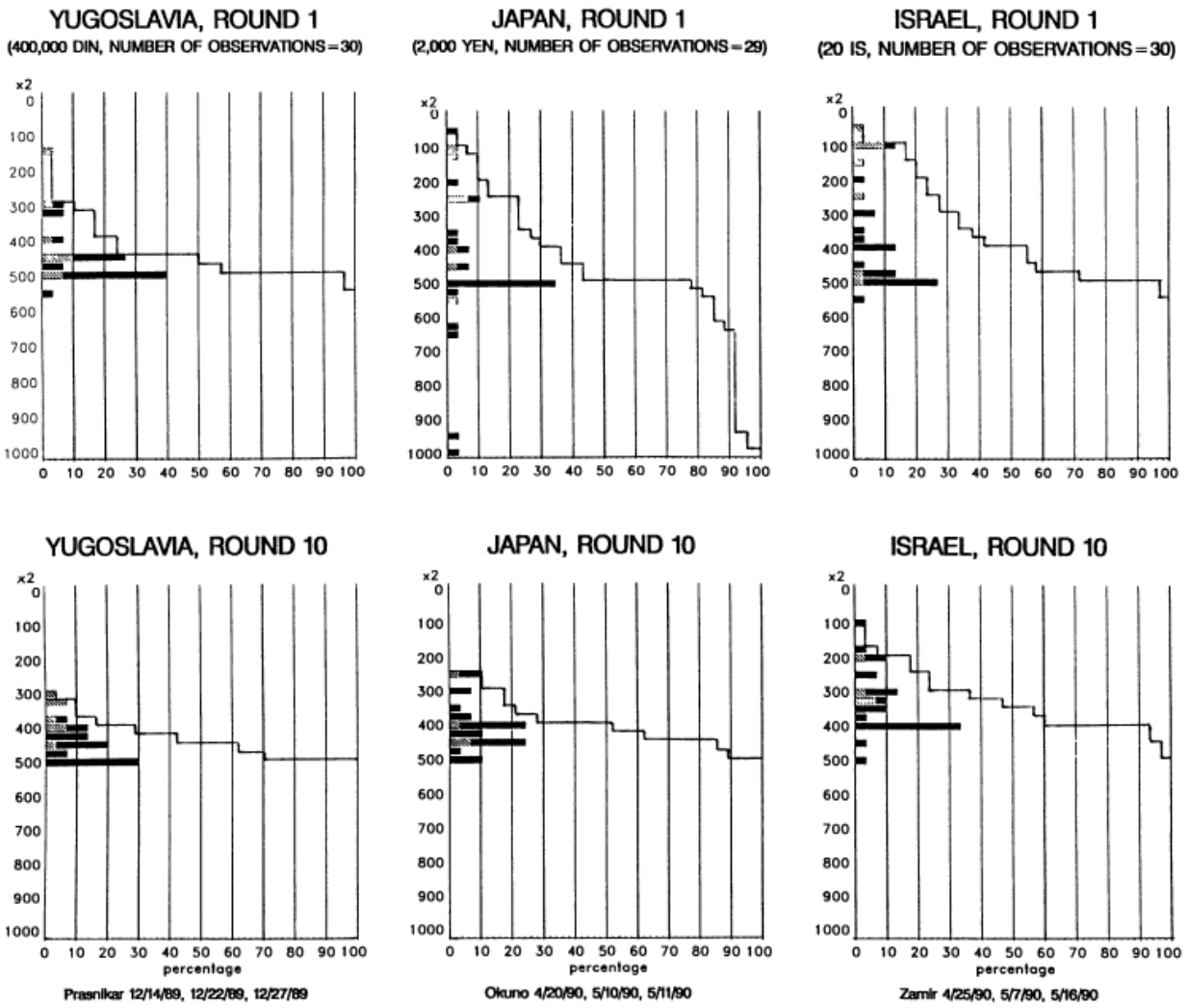


FIGURE 4. DISTRIBUTIONS OF BARGAINING OFFERS IN YUGOSLAVIA, JAPAN, AND ISRAEL (SOLID BARS = ACCEPTED OFFERS; STRIPED BARS = REJECTED OFFERS)

CONCLUSION

There are indeed significant and systematic differences across cultures in bargaining behavior. This indicates that "fairness" may be something that societies determine through socially sanctioned behaviors.

13 Stakes and Experience

Slonim, Robert and Alvin E. Roth, "Learning in High Stakes Ultimatum Games: An Experiment in the Slovak Republic." *Econometrica*, 66, 3, May 1988, 569-96.

This paper takes head on the concerns expressed about prior experiments.

- First, they are single shot and so people have not had a chance to learn about the game.
- Second, the stakes are relatively small. What if people played for a month's salary not simply an hour's wage?

Problems in addressing these two issues:

1. Problem: Experience also changes the earnings of subjects, so there are income effects. People also learn about the types of people they play with, so there could be reputation effects.
2. Solution: Have subjects play against randomly changing partners.
3. Problem: A month's wage is a lot of money, at least in the US.
4. Solution: Go to a poor country, where $50 is a month's wage. Then compare an hour's wage to a month's wage at an affordable price.

EXPERIMENTAL DESIGN

2. EXPERIMENTAL DESIGN AND PERFECT EQUILIBRIUM PREDICTIONS

In the ultimatum game, subjects participated in a sequence of ten games against different anonymous opponents.[10] During the ten game session a subject learned only the results of his or her own negotiations. Each subject was randomly assigned to be a proposer or responder, and a subject played the same role throughout the ten game session. In all games the pie was 1000 points and proposed divisions could be made in units of 5 points (0, 5, 10,... 995, 1000). The exchange rate for 1000 points was 60, 300, or 1500 Slovak Crowns (Sk), depending on the session. Ten ultimatum sessions were conducted, three at 60 Sk, four at 300 Sk, and three at 1500 Sk.

The subgame perfect assumption (with the additional assumption that subjects only want to maximize their monetary payoffs) means the responder will accept any positive offer, since rejecting any positive offer is inconsistent with wanting to maximize monetary reward. Since the smallest positive amount a proposer can offer is 5 points, no proposer will offer more than 5 points because responder will surely accept that amount. Thus, two subgame perfect equilibria exist: in one, proposer offers responder 5 points and keeps 995 for himself, and responder accepts (but would have rejected an offer of 0 points). In the other, proposer offers responder 0 points and responder accepts.[11]

RESULTS

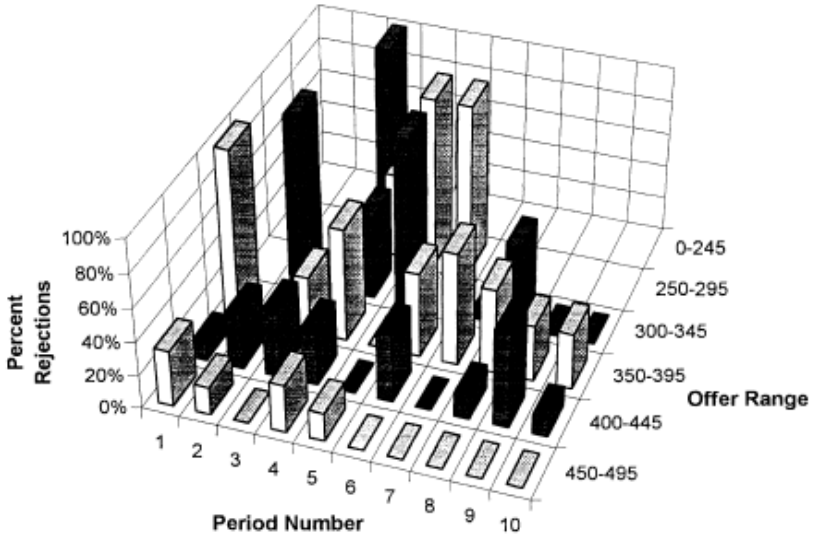
TABLE I

SUMMARY OF ULTIMATUM GAME

Offers and Rejections by Range of Offers and Basic Statistics

	60 Sk, $N = 24$		300 Sk, $N = 33$		1500 Sk, $N = 25$		
Offer Ranges	% Off	% Rej	% Off	% Rej	% Off	% Rej	#Offers
> 500	6.3	6.7	6.7	4.5	7.2	0.0	55
	(15)	(1)	(22)	(1)	(18)	(0)	
= 500	28.7	0.0	21.5	1.4	30.8	1.3	217
	(69)	(0)	(71)	(1)	(77)	(1)	
450–495	21.7	9.6	22.7	5.3	6.0	0.0	142
	(52)	(5)	(75)	(4)	(15)	(0)	
400–445	24.6	23.7	21.8	12.5	32.4	4.9	212
	(59)	(14)	(72)	(9)	(81)	(4)	
350–395	11.3	40.7	9.4	9.7	5.2	0.0	71
	(27)	(11)	(31)	(3)	(13)	(0)	
300–345	4.6	45.5	10.6	22.9	7.2	11.1	64
	(11)	(5)	(35)	(8)	(18)	(2)	
250–295	2.5	66.7	3.9	30.8	3.2	37.5	27
	(6)	(4)	(13)	(4)	(8)	(3)	
< 250	0.4	100.0	3.3	90.9	8.0	60.0	32
	(1)	(1)	(11)	(10)	(20)	(12)	
All Offers	100.0	17.1	100.0	12.1	100.0	8.8	820
	(240)	(41)	(330)	(40)	(250)	(22)	
Offers < 500	35.1	25.6	71.2	16.0	61.6	13.6	548
	(156)	(40)	(237)	(38)	(155)	(21)	
Average (all)	445		423		427		
Average (7 exclusions)	440		428		415		

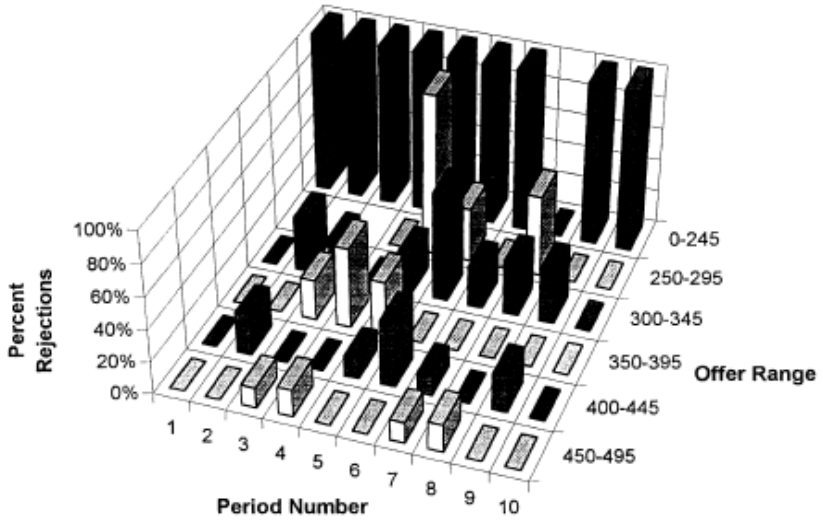
Notes: The number in parentheses below each percent offer is the number of offers made in the range and the number in parentheses below percent rejected is the number of offers rejected in the range. The average (7 exclusions) removes all offers of the six subjects that made more than four offers greater than 50% and also excludes the one subject that made the offer of .5% of the pie in every round.



60 Sk: Rejections / Offers

Period	Offer Ranges 450 -495	400 -445	350 -395	300 -345	250 -295	0 -245	ALL
1	1/3	1/10	1/1	0/0	0/0	0/0	3/14
2	1/6	2/6	0/3	1/1	0/0	0/0	4/16
3	0/3	2/5	1/3	0/1	0/1	1/1	4/14
4	2/7	1/3	2/3	1/2	1/2	0/0	7/17
5	1/6	0/3	0/2	2/2	2/2	0/0	5/15
6	0/6	2/5	2/4	0/0	1/1	0/0	5/16
7	0/4	0/4	2/3	0/1	0/0	0/0	2/12
8	0/7	1/7	1/2	1/2	0/0	0/0	2/18
9	0/5	4/7	1/3	0/1	0/0	0/0	5/16
10	0/5	1/9	1/3	0/1	0/0	0/0	2/18
1-10	5/52	14/59	11/27	5/11	4/6	1/1	40/156

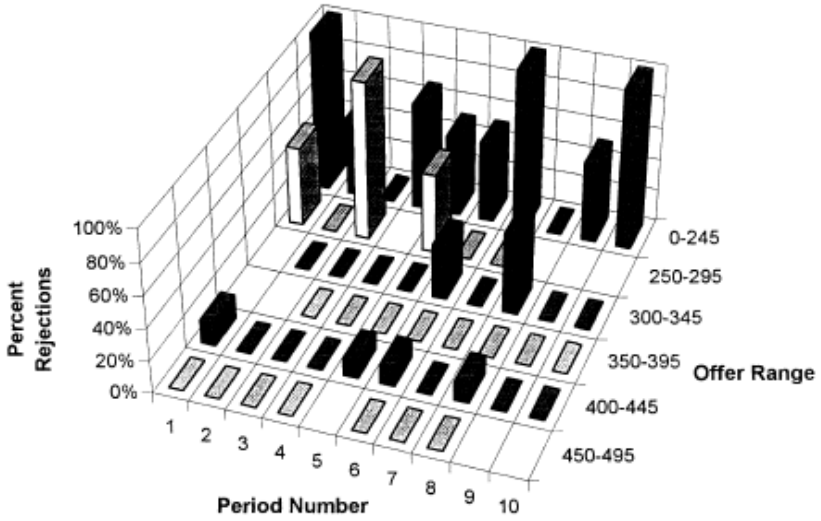
FIGURE 1a.—Low stakes (60 Sk).



300 Sk: Rejections / Offers

Period	Offer Ranges 450 -495	400 -445	350 -395	300 -345	250 -295	0 -245	ALL
1	0/11	0/3	0/2	0/3	0/0	1/1	1/20
2	0/11	1/4	0/3	1/3	0/0	1/1	3/22
3	1/8	0/6	1/4	1/4	0/0	1/1	4/23
4	1/6	0/8	1/2	0/4	0/1	1/1	3/22
5	0/8	1/8	1/3	1/4	2/2	1/1	6/26
6	0/7	4/10	0/1	2/3	1/3	1/1	8/25
7	1/8	1/8	0/3	1/4	0/1	2/2	5/26
8	1/6	0/8	0/4	1/3	1/2	0/1	3/24
9	0/5	2/9	0/6	1/3	0/2	1/1	4/26
10	0/5	0/8	0/3	0/4	0/2	1/1	1/23
ALL	4/75	9/72	3/31	8/35	4/13	10/11	38/237

FIGURE 1b.—Middle stakes (300 Sk).



1500 Sk: Rejections / Offers

Period	Offer Ranges: 450 -495	400 -445	350 -395	300 -345	250 -295	0 -245	ALL
1	0/4	1/7	0/0	0/0	1/2	2/2	4/15
2	0/4	0/6	0/0	0/1	0/1	1/2	1/14
3	0/2	0/8	0/2	0/1	1/1	0/1	1/15
4	0/1	0/7	0/2	0/1	0/0	2/3	2/14
5	0/0	1/10	0/1	0/1	1/2	1/2	3/16
6	0/1	1/9	0/1	1/3	0/1	1/2	3/17
7	0/2	0/8	0/2	0/1	0/1	3/3	3/17
8	0/1	1/9	0/1	1/2	0/0	0/2	2/15
9	0/0	0/8	0/3	0/4	0/0	1/2	1/17
10	0/0	0/9	0/1	0/4	0/0	1/1	1/15
ALL	0/15	4/81	0/13	2/18	3/8	12/20	21/155

FIGURE 1c.—High stakes (1500 Sk).

Logit Regressions:

$$(1) \qquad Reject = f(a + b_{off} * off),$$

$$(2) \qquad Reject = f(a + b_{off} * off + b_m * pieM + b_n * pieH),$$

.

$$(3) \qquad Reject = f(a + b_{off} * off + b_{avrej} * avrej_i),$$

$$(4) \qquad Reject = f(a + b_{off} * off + b_m * pieM + b_h * pieH + b_{avrej} * avrej_i),$$

Where
Reject $= 1$ if offer is rejected
off $=$ offer
PieM=1 if medium pie,
PieH=1if high pie
averej $=$ average number of rejections by person j, except current round.

TABLE II

LOGIT REGRESSION RESULTS: PROBABILITY OFFER IS REJECTED

	Round 1		All Rounds			
Parameter	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Intercept	4.22	7.08*	2.93***	4.29***	4.66***	4.39***
b_{off}	−15.7**	−20.3**	−15.8***	−17.6***	−17.5***	−17.7***
b_m		−4.61 ($p = .13$)		−0.73* ($p = .028$)	−0.69* ($p = .037$)	−0.78* ($p = .023$)
b_h		−1.17 ($p = .35$)		−1.30** ($p = .002$)	−1.29** ($p = .002$)	−1.39** ($p = .001$)
b_{avrej}			5.54***	5.29***	5.30***	5.49***
b_{round}					−0.07 ($p = .156$)	
$b_2, \ldots, b_{10}$						1^x
# Observations	49	49	548	548	548	548
−2 Log Likelihood	30.08	23.95	336.28	325.15	323.12	311.04
Model Comparisons:		vs. model 1 $\chi^2_{(2)} = 6.13$ ($p = .046$)		vs. model 3 $\chi^2_{(2)} = 11.13$ ($p = .0038$)	vs. model 4 $\chi^2_{(1)} = 2.03$ ($p = .154$)	vs. model 4 $\chi^2_{(2)} = 14.1$ ($p < .118$)

Notes: 1^x—parameter estimates for round dummy variables not shown. $^*p < .05$, $^{**}p < .01$, $^{***}p < .001$.

CONCLUSIONS

1. Stakes matter, but less than one might have guessed.
   - The larger the pie, the less likely it is that a given offer will be rejected.
   - That is, a 10% offer is more likely to be rejected if it comes from an hour's wage than from a month's wage.
2. Repetition matters, but less than one might have guessed.
   - Are people learning subgame perfection?
   - Or are they learning that fairness pays?
   - Or both?....more papers are needed to figure this out.

CONCLUSIONS

1. Stakes matter, but less than one might have guessed.
2. Repetition matters, but less than one might have guessed.