3 Theory

- Basic question: how do people evaluate and compare outcomes in the money-time space? It is better to consume $100 today or $200 in five years?
- In general, consumption in each time period could be treated as a different good, and a general theory of preferences could be invoked
- But this is too complicated for a tractable analysis of decision-making over time
- Simplification 1: express utility over a stream of consumption in different periods as a weighted sum of a period utility of consumption, where the period utility function is the same in every period, and the weight on period t relative to the present period τ, or the discount function, is given by $D(\tau, t)$ (that is, $D(\tau, \tau)$ is normalized to 1)
- Simplification 2: assume that the discount function does not depend on calendar time, only on how far in advance one evaluates the utility; that is, $D(\cdot)$ only depends on $t - \tau$;
- Question: what discount function should be used?

Standard Model

Samuelson, Paul, “A Note on Measurement of Utility", Review of Economic Studies, 4(2), 1937, 155-161.

- **Exponential discounting**: $D_t = \delta^t$, where $\delta \in (0, 1]$

$$U = \sum_{s=t}^{T} \delta^{s-t} u(c_s)$$

where $\delta = 1/(1+\rho)$, $\delta =$ discount *factor,* and $\rho =$ discount *rate*.

Alternative Models

- These all have the same objective of explaining excessive impatience for good things and excessive procrastination of bad things.
- These have various names:
  - Present Bias
  - Hyperbolic Discounting
  - Quasi-hyperbolic Discounting
  - β-δ Preferences

Strotz, R.H., "Myopia and Inconsistency in Dynamic Utility Maximization, *Review of Economic Studies,* **23(3), 1956, 165-180.**

- Seriously considers the possibility of non-exponential discounting
- Idea: $D(t-\tau)$ drops dramatically between 0 and 1, but decreases more slowly afterwards
- Results in time inconsistency and present bias
  - From the point of view of today, period τ, the MRS between consumption in periods $\tau+1$ and $\tau+2$ is given by $u'(c_{\tau+1})/u'(c_{\tau+2}) = D(2)/D(1)$
  - However, come period $\tau+1$, this MRS will be given by $D(1)/D(0) < D(2)/D(1)$
  - As a result, the DM will want to overconsume in period $\tau+1$ relative to the plan he made in period τ

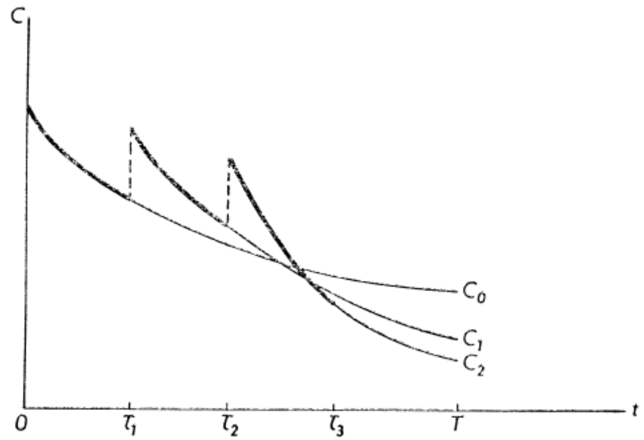


Figure 2

- This is the behavioral outcome when the DM is not self-aware of the present bias in his preferences, i.e., if he assumes that the preferences of his current self over future allocations do not coincide with preferences of his future selves over these allocations
- This is what subsequent literature labels as a *naive* DM
- But it is quite possible that at least with experience over time, a DM will become aware of this bias; that is, he will recognize that his behavior over time is given by an outcome of a *game among different temporal selves*
- This is what subsequent literature labels as a *sophisticated* DM

Loewenstein, George and Drazen Prelec, "Anomalies in Intertemporal Choice: Evidence and an Interpretation," *Quarterly Journal of Economics,* **107(2), 1992, 573-597.**

- Suppose that a DM is indifferent between receiving x today (time 0) and receiving $y > x$ in s periods from today; that is, $u(x) = u(y)D(s)$
- Under the present bias, if both outcomes are postponed t periods, then the later outcome will be preferred: $u(x)D(t) < u(y)D(s + t)$
- Question: how long does the later outcome need to be postponed to make the two outcome-time pairs indifferent?
- Hypothesis: This time is a linear function of t, namely kt, with $k > 0$; that is, for any $t > 0$, $u(x)D(t) = u(y)D(s + kt)$; $k = 1$ implies no present bias, $k > 1$ implies present bias, k depends on x and y
- Interpretation: "clocks" for the two outcomes run at different speeds
- L&P show that in order for this to be true for any x, $y > x$, and $t > 0$, $D(\cdot)$ must be a hyperbola:

$$D(t) = (1 + \alpha t)^{-\delta/\alpha},\ \alpha, \delta > 0$$

- Therefore the name *hyperbolic discounting*
- The parameter α determines how much $D(\cdot)$ differs from exponential discounting; the two become closer as $\alpha \to 0$; one can use the l'Hopital rule to show that the for the limiting case when

$\alpha = 0$, $D(t) = e^{-\delta t}$, or exponential discounting

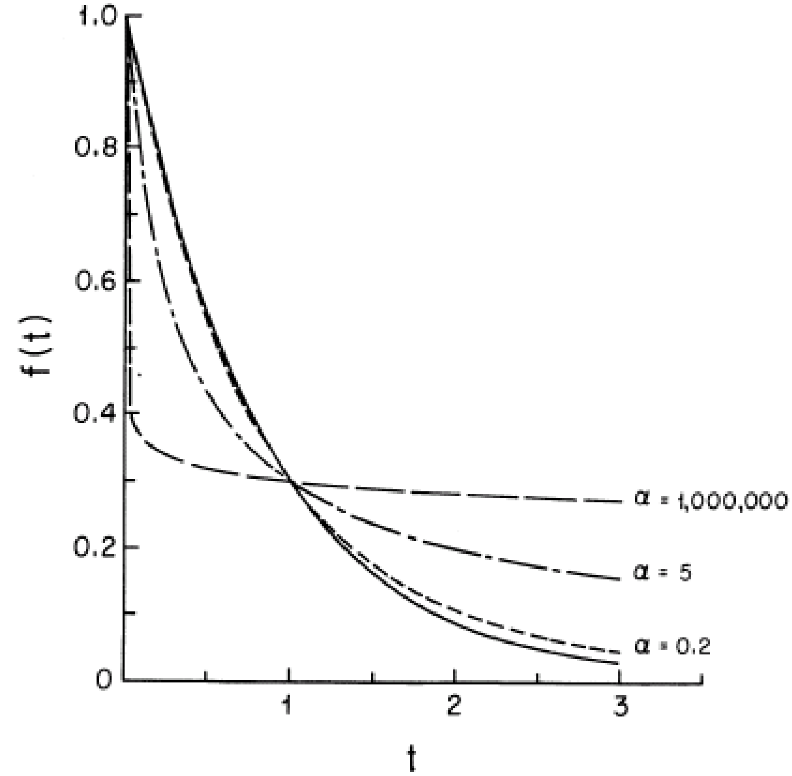


FIGURE I
The Hyperbolic Discount Function $\phi(t) = (1 + \alpha t)^{-\beta/\alpha}$ for Three Different Levels of α. All βs are adjusted so that curves cross at $\phi(1) = 0.3$. The most steeply sloped curve represents conventional exponential discounting.

Phelps, Edmund S. and R.A. Pollack, "On Second Best National Saving and Game-Equilibrium Growth", *Review of Economic Studies,* **35(2), (1968), 185-199.**
Laibson, David, "Golden Eggs and Hyperbolic Discounting", *Quarterly Journal of Economics,* **112(2), (1997), 443-478.**

- Hyberbolic discount function is not very tractable
- Phelps and Pollak (1968) propose a different discount function which Laibson (1997) later calls *quasi-hyperbolic discounting* (or sometimes called β-δ preferences), since it approximates a hyperbola by mimicking one key property of the present bias: large discounting between the present (time 0) and a future time period s relative to the discounting between $t > 0$ and $t + s$

$$D(t) = \begin{cases} 1 & \text{if } t = 0 \\ \beta\delta^t,\ \beta, \delta \in (0,1) & \text{if } t > 0 \end{cases}$$

- With $\beta = 1$, this becomes a standard exponential discount function
- Terminology: $\beta =$ short-term discount factor, $\delta =$ long-term discount factor

- So, whereas the standar utility would be

$$U = u(x_0) + \sum_{t=1}^{T} \delta^t u(x_t),$$

we instead have

$$U = u(x_0) + \beta \sum_{t=1}^{T} \delta^t u(x_t)$$

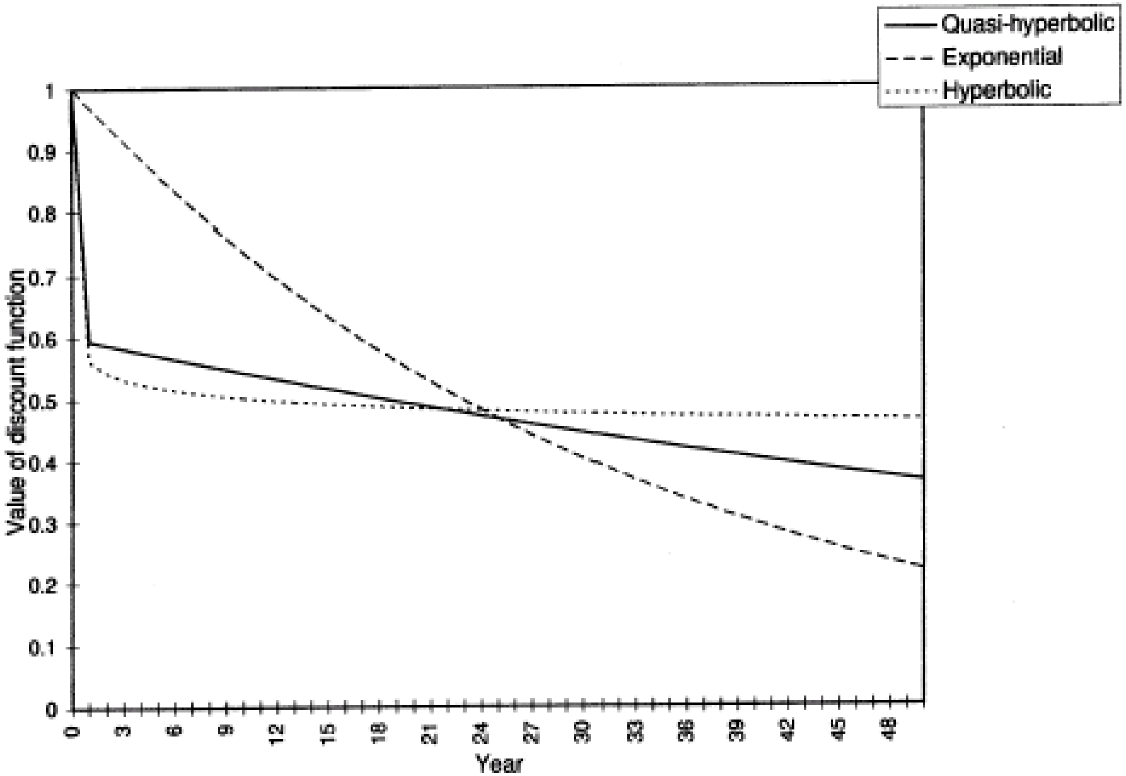


FIGURE I
Discount Functions

- Model explains why you may constrain your future self with actions today.
  - Take on debt for college so you work hard in your job.
  - 401k savings with penalty for early withdrawl

– Promise to teach a course to force yourself to learn stuff you wish you knew but never took the time for.

O'Donoghue, Ted and Matthew Rabin, "Doing It Now or Later", *American Economic Review,* March 1999..

- Consider the implication of the present bias on the timing of a one-time activity
- They distinguish between activities with
  - immediate costs: the cost is realized when the activity is performed, while a benefit comes later (writing a paper, working out)
  - immediate benefits: the benefit is realized when the activity is performed, while a cost comes later (seeing a movie, eating a chocolate ice-cream)
- As we discussed in the case of the Strotz paper, they distinguish between present-biased DMs that are
  - naive (*naifs*, N): do not realize the time inconsistency; think that future selves will implement the plan of the current self
  - sophisticated (*sophisticates*, S): do realize the time inconsistency; solution computed by subgame-perfect equilibrium
- They compare the timing of an action of both types to the timing of time consistent (TC) agents for whom $\beta = 1$ and time inconsistent, with $\beta < 1$.

- Setup: the activity must be performed exactly once in one of the time periods $1, ..., T$
- For simplicity let $\delta = 1$ (it's just easier that way—all intuitions follow).
- Preferences in any period $t \in \{1, .., T\}$:

$$U_t = u_t + \beta[u_{t+1} + ... + u_T]$$

- If performed in period t, an activity has a benefit of v_t and a cost of c_t
- When evaluating timing of the activity in period t, if the person completes the activity in period $\tau \geq t$,
  - for activities with immediate costs

$$U_t = \begin{cases} \beta v_\tau - c_\tau & \text{if } \tau = t \\ \beta v_\tau - \beta c_\tau & \text{if } \tau > t \end{cases}$$

  - for activities with immediate rewards

$$U_t = \begin{cases} v_\tau - \beta c_\tau & \text{if } \tau = t \\ \beta v_\tau - \beta c_\tau & \text{if } \tau > t \end{cases}$$

- **Proposition 1:** If costs are immediate, then naifs procrastinate (complete the activity later than TCs). If rewards are immediate, then naifs preproterate (complete the activity sooner than TCs).
- **Proposition 2:** For all cases, sophisticates complete the activity sooner than naifs. As a result,

sophistication mitigates procrastination, but exacerbates preproperation.

- Example 1: immediate costs, delayed benefit: $T = 4$, $\beta = 1/2$, $v_i = v$, $c = (3, 5, 8, 13)$.
  - $*$ TCs: (Y, Y, Y, Y), best to do it immediately
  - $*$ Naifs: (N, N, N, Y)
  - $*$ Sophisticates: (N, Y, N, Y)
    - · Know at time 2 they won't do it at time 3, so do it at time 2.
    - · Know at time 1 they will do it at time 2, so better to wait.

- Example 2: immediate rewards, delayed cost: $T = 4$, $\beta = 1/2$, $v = (3, 5, 8, 13)$, $c = (0, 0, 0, 0)$.
  - TCs: (N, N, N, Y), best to wait until the end.
  - Naifs: (N, N, Y, Y)
    - In period 1 and 2 think they'll wait until the end, so wait, but suffer impatience in period 3.
  - Sophisticates: (Y, Y, Y, Y)
    - Know they'll have a self control problem in period 2 and won't wait to the end, so doing it now is better.

- It turns out that the sophistication effect may completely undo, even overundo, the procrastination effect (preemptive overcontrol):
  - Example 3: immediate costs, but delayed benefits $T = 3$, $\beta = 1/2$, $v = (12, 18, 18)$, $c = (3, 8, 13)$.
    - ∗ TCs: (N, Y, Y)
      - · Time 2 maximizes $v - c$
    - ∗ Naifs: (N, N, Y)
    - ∗ Sophisticates: (Y, N, Y)
      - · So moves things to the opposite pole that procrastination does.

- It is important to distinguish in applications whether a particular result is driven by the present bias itself, or the present bias in conjunction with sophistication.

4 Empirical Evidence on Exponential and Hyperbolic Discounting

Frederick, Shane, George Loewenstein, and Ted O'Donoghue, "Time Discounting and Time Preference: A Critical Review", *Journal of Economic Literature,* **40(2), (2002), 351-401.**

- Survey theoretical and empirical literature on discounting and its applications
- Criticize the standard discounted utility (DU) model of Samuelson

$$U = \sum_{s=t}^{T} \delta^{s-t} u(c_s)$$

where $\delta = 1/(1+\rho)$, $\delta =$ discount *factor,* and $\rho =$ discount *rate*.

Key properties of the standard DU model

1. **New alternatives are integrated** into existing plans: if you are offered to give up $100 now for $200 next year, you incorporate this into your consumption planning and potentially adjust consumption in all future periods. That, utility is over consumption, not changes in consumption.
2. **Additive separability over time**; the time path of utility is irrelevant as long as its discounted sum is the same; rules out, for example, preference for a non-decreasing path of utility over time
3. **Consumption independence**: utility in period s depends only on consumption in period s, but not on consumption in any other period; rules out habit formation, for example
4. **Stationarity** of the period utility function over time; rules out a possibility of changing tastes
5. **Independence** of discounting from consumption; rules out simultaneous patience in some aspects of consumption (career development) and impatience in others (smoking)
6. **Exponential discounting**; rules out present bias

- Experimental measurements of the discount factor δ:

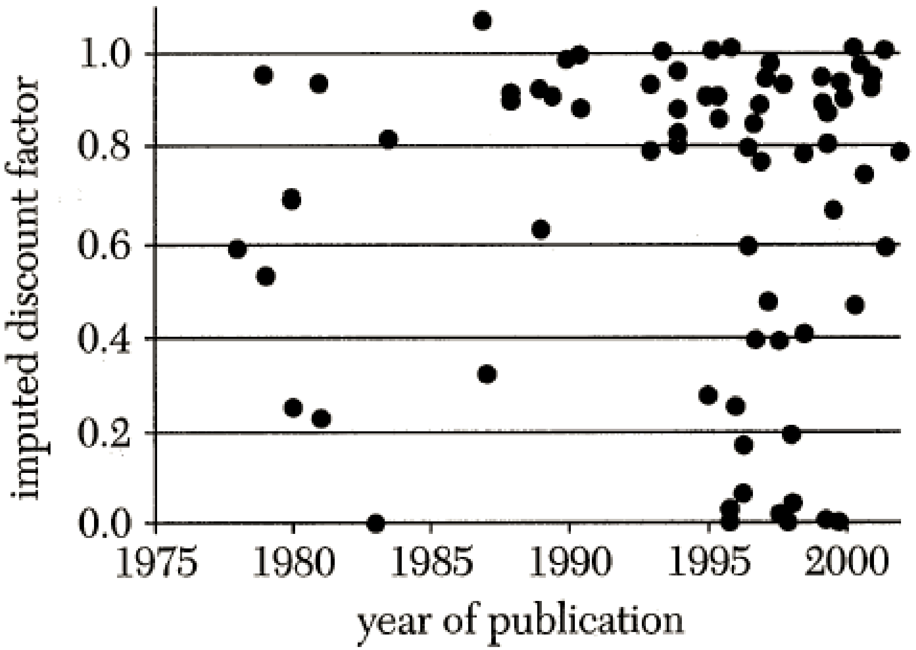


Figure 2. Discount Factor by Year of Study Publication

- Note: a consensus is **not** forming....should lead us to question the measurements as well as the model of preferences.

- Methods for measuring δ:
  - **Choice task:** “Do you prefer 100 units today or 120 units in a year?”
    - ∗ subjects are usually asked a series of questions of this sort
    - ∗ each of them provides an upper or a lower bound for the discount factor
    - ∗ problem: presentation effects:
      - · *anchoring effect*: people will be more likely to choose 120 next year if this choice task is preceded by a task of choosing between 100 today and 103 in a year

- **Matching task:** "100 units now is the same as ____ units in one year"
  - Fill in the blank.
  - Are these choices incentivized? Any reason to reveal true value?
- **Pricing task:** respondents specify a willingness to pay to obtain (avoid) some outcome occurring at a particular time
  - Note WTP is typically less than WTA (see the *Endowment Effect* elsewhere).
- **Rating task:** respondents evaluate an outcome occurring at a particular time by rating its attractiveness or aversiveness.
  - Psychology

- Typical way of measuring δ :
  - If indifferent between x_t and x_{t+k}, find the δ such that $x_t = \delta^k x_{t+k}$.
  - If indifferent between X now and a stream x_t for k periods, then find δ such that $X = \sum_{i=1}^{k} \delta^i x_{t+i}$.

- Issues:
  - **Real vs. hypothetical rewards**: Kirby and Marakovic (1995) show that discount rates are lower/discount factors are higher for hypothetical rewards)
  - **Intertemporal arbitrage:** If people have access to capital markets, present value is all that should matter. Then the key will be knowing what rate of interest subjects have access to.
  - **Concave utility:** Measuring δ assumes utility is approximately linear. For example, someone indifferent between 16 at t and 25 at $t+1$ would have a per period discount factor of $\delta = 16/25 = 0.64$. But if utility is $u(x) = \sqrt{x}$, then the true discount factor is $\delta = u'(x_t)/u'(x_{t+1}) = 5/4$
  - **Uncertainty or Ambiguity about getting payment:** Will the experimenter be there with cash? What if I need to be out of town on that day? Will I hit a borrowing constraint before I get paid?
  - **Transaction costs (not discussed in FLO):** Will there be added costs with getting paid later, e.g. check *vs.* cash, change of address, coming to an office on campus, remembering to pick it up, remembering the amount I am owed, other costs of worry and memory.
  - **Inflation:** Can we observe their beliefs about inflation?
  - **Change in utility function** or the baseline consumption
- NOTE: These all suggest that finding a read discount rate will be difficult, especially using money on students without access to capital markets, since their present bias may be rational. But

using money with older subjects who can access capital markets means you only test net-present-value maximization.

- The best commodity to use may be time rather than money.
  - Reactions?

- Field studies measurement: Infer from choices made in real markets
  - Riskier job with higher salary (Viscusi and Moore, 1989) $\rho \in [0.02, 0.14]$
  - Auto safety choices (Dreyfus and Viscusi, 1995): $\rho \in [0.11, 0.17]$
  - Life-cycle savings in macro model (Lawrence 1991),: $\rho \in [0.04, 0.13]$
  - Life-cycle savings, with some borrowing constraints (Carroll and Samwick, 1997), $\rho \in [0.05, 0.14]$.
  - Natural Experiment of pension buyout, get $22,283 now or $3,714/year for 18 years. 90%chose now, implying $\rho = 0.175$.
    - Is this high? It is about what credit cards charge, so maybe not.

Experimental Evidence on Hyperbolic Discounting

Thaler, Richard, "Some Empirical Evidence on Dynamic Inconsistency", *Economics Letters,* **8, (1981), 201-207.**

- Uses experiment with hypothetical payoffs to measure discount functions
- Based on a matching task
- Looking separately at gains and losses
  - Gains: Subjects were told that they had won some money in a lottery held by their bank. They could take the money right away or wait until later. They were asked how much they would require to make waiting just as attractive as getting the money now
  - Losses: Traffic ticket that could be paid now or later.
- Each subject received a 3×3 table to fill in, with dimensions being the amount of money and the time of delay

The sizes of the prizes (fines) and the length of time to be waited was varied among the forms. The figures used are given in table 1.

Table 1

Form	Amounts	Time to wait
(1)	\$15, \$250, \$3000	3 mo. 1 yr. 3 yrs.
(2)	75, 250, 1200	6 mo. 1 yr. 5 yrs.
(3)	15, 250, 3000	1 mo. 1 yr. 10 yrs.
(4)	−15, −100, −250	3 mo. 1 yr. 3 yrs.

- about 20 responses for every form
- Results:

Table 2
Median responses and (continuously compounded discount rates in percent).

	Amount of early prize	Later prize paid in		
		3 mo.	1 yr.	3 yrs.
(A)	$15	$ 30 (277)	$ 60 (139)	$ 100 (63)
	$250	$ 300 (73)	$ 350 (34)	$ 500 (23)
	$3000	$3500 (62)	$4000 (29)	$ 6000 (23)
		6 mo.	1 yr.	5 yr.
(B)	$75	$ 100 (58)	$ 200 (98)	$ 500 (38)
	$250	$ 300 (36)	$ 500 (69)	$ 1000 (28)
	$1200	$1500 (45)	$2400 (69)	$ 5000 (29)
		1 mo.	1 yr.	10 yrs.
(C)	$15	$ 20 (345)	$ 50 (120)	$ 100 (19)
	$250	$ 300 (219)	$ 400 (120)	$ 1000 (19)
	$3000	$3100 (39)	$ 400 (29)	$10000 (12)

	Amount of early fine	Later fine due in		
		3 mo.	1 yr.	3 yrs.
(D)	$15	$ 16 (26)	$ 20 (29)	$ 28 (20)
	$100	$ 102 (6)	$ 118 (16)	$ 155 (15)
		$ 251 (1)	$ 270 (8)	$ 310 (7)

- Average discount rates drop sharply with
  a. the length of delay – suggests hyperbolic discounting
  b. size of the prize – suggests fixed costs to waiting
- Average discount rates are smaller for losses than for gains

Benzion, Uri, Amnon Rapoport, and Joseph Yagil, "Discount Rates Inferred from Decisions: An Experimental Study", *Management Science,* **35(3), (1989), 270-284.**

- Similar idea
- 204 Israeli subjects with at least two-year background in econ or finance
- Four designs:
  a. **A** (postpone receipt): earned $\$y$ in a "financially solid" public institute, but the institute is "temporarily short of funds"; but the money will be available t periods in the future; state indifferent amount $\$x$ at that time

  b. **B** (postpone payment): debt $\$y$ is due to the institute; state indifferent amount $\$x$ t periods in the future, t given

  c. **C** (expedite a receipt): institute must pay $\$y$ t periods from now; state indifferent amount $\$x$ to receive now

  d. **D** (expedite a payment): debt $\$y$ is due to the institute in t periods; state indifferent amount $\$x$ to pay now
- For each design, a 4×4 factorial design is used with $t = 0.5, 1, 2, 4$ years and $y = \$40, \$200, \$1,000, \$5,000$
- Together in 64 questions (plus 16 filler questions); order of questions within a block randomized, order of 16 blocks randomized too

Results:

TABLE 1
Means and Standard Deviations of Inferred Discount Rates

	Scenario A t				Scenario B t			
Sum	0.5	1	2	4	0.5	1	2	4
40	0.598 (0.613)	0.393 (0.341)	0.263 (0.196)	0.219 (0.189)	0.334 (0.378)	0.219 (0.218)	0.193 (0.157)	0.141 (0.102)
200	0.428 (0.408)	0.255 (0.251)	0.230 (0.202)	0.195 (0.142)	0.260 (0.263)	0.167 (0.195)	0.158 (0.149)	0.128 (0.104)
1000	0.407 (0.462)	0.275 (0.246)	0.200 (0.176)	0.185 (0.151)	0.217 (0.242)	0.155 (0.193)	0.152 (0.200)	0.121 (0.116)
5000	0.184 (0.192)	0.162 (0.203)	0.151 (0.172)	0.116 (0.115)	0.153 (0.176)	0.105 (0.150)	0.088 (0.126)	0.075 (0.095)
	Scenario C t				Scenario D t			
Sum	0.5	1	2	4	0.5	1	2	4
40	0.379 (0.503)	0.244 (0.259)	0.189 (0.223)	0.137 (0.126)	0.535 (0.660)	0.330 (0.455)	0.265 (0.261)	0.206 (0.182)
200	0.288 (0.430)	0.174 (0.212)	0.134 (0.120)	0.131 (0.118)	0.321 (0.433)	0.236 (0.264)	0.210 (0.218)	0.157 (0.127)
1000	0.217 (0.278)	0.157 (0.190)	0.122 (0.151)	0.123 (0.123)	0.310 (0.385)	0.219 (0.241)	0.166 (0.170)	0.163 (0.190)
5000	0.171 (0.246)	0.139 (0.175)	0.105 (0.168)	0.100 (0.107)	0.261 (0.419)	0.192 (0.226)	0.149 (0.197)	0.136 (0.133)

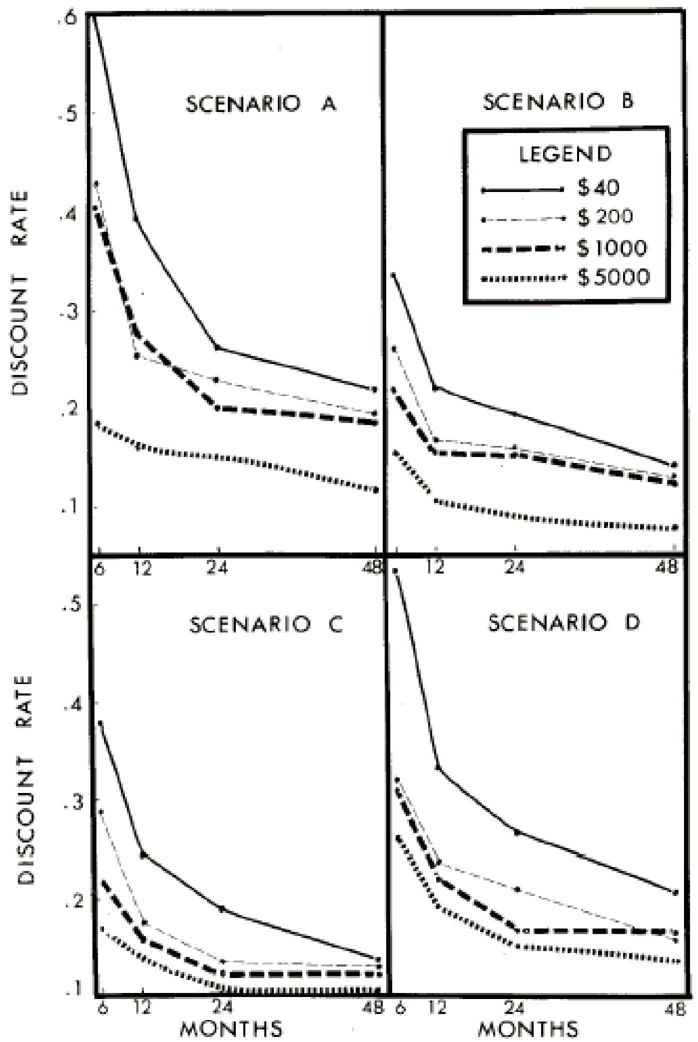


FIGURE 1. Mean Discount Rates for Different Time Delays by Sum of Cashflow and Scenario.

TABLE 2

Mean Discount Rates across Each of the Three Design Factors

TABLE 2.A. Mean Discount Rates across Sum by Time Period and Scenario

Scenario \ Time (years)	0.5	1	2	4	Mean Across Time
A	0.404	0.271	0.211	0.179	0.267
B	0.241	0.161	0.148	0.116	0.167
C	0.264	0.179	0.138	0.123	0.176
D	0.357	0.244	0.198	0.166	0.241
Means Across Scenario	0.317	0.214	0.174	0.146	0.213

TABLE 2.B. Mean Discount Rates across Time by Sum and Scenario

Sum ($) \ Scenario	A	B	C	D	Mean Across Scenario
40	0.368	0.222	0.237	0.334	0.290
200	0.277	0.178	0.182	0.231	0.217
1000	0.267	0.161	0.155	0.215	0.200
5000	0.153	0.105	0.129	0.185	0.143
Mean Across Sum	0.267	0.167	0.176	0.241	0.213

TABLE 2.C. Mean Discount Rates across Scenarios by Sum and Time Period

Sum ($) \ Time (years)	0.5	1	2	4	Mean Across Time
40	0.462	0.296	0.228	0.176	0.290
200	0.324	0.208	0.183	0.153	0.217
1000	0.288	0.201	0.160	0.148	0.200
5000	0.192	0.150	0.123	0.107	0.143
Mean Across Sum	0.317	0.214	0.174	0.146	0.213

- As in Thaler (1981), average discount rates drop with prize and time
- $\rho_A > \rho_B$ (difference 0.1) and $\rho_C < \rho_D$ (difference 0.065)

• Results for forward rates (discount rates between a consecutive pair of future time periods)

TABLE 3

*Means and Standard Deviations of Inferred Forward Rates**

	Scenario A				Scenario B			
	t				t			
Sum	0.5	1	2	4	0.5	1	2	4
40	0.598	0.300	0.169	0.199	0.334	0.182	0.179	0.100
	(0.613)	(0.553)	(0.234)	(0.379)	(0.378)	(0.662)	(0.202)	(0.139)
200	0.428	0.155	0.221	0.176	0.260	0.112	0.162	0.108
	(0.408)	(0.411)	(0.262)	(0.231)	(0.263)	(0.375)	(0.199)	(0.152)
1000	0.407	0.224	0.143	0.188	0.217	0.123	0.167	0.102
	(0.462)	(0.464)	(0.205)	(0.287)	(0.242)	(0.345)	(0.318)	(0.170)
5000	0.184	0.170	0.156	0.102	0.153	0.079	0.080	0.069
	(0.192)	(0.408)	(0.266)	(0.254)	(0.176)	(0.318)	(0.194)	(0.137)
	Scenario C				Scenario D			
	t				t			
Sum	0.5	1	2	4	0.5	1	2	4
40	0.379	0.180	0.161	0.105	0.535	0.242	0.256	0.186
	(0.503)	(0.410)	(0.332)	(0.194)	(0.660)	(0.648)	(0.420)	(0.367)
200	0.288	0.127	0.111	0.139	0.321	0.237	0.220	0.128
	(0.430)	(0.428)	(0.179)	(0.198)	(0.433)	(0.703)	(0.405)	(0.225)
1000	0.217	0.138	0.102	0.136	0.310	0.204	0.131	0.179
	(0.278)	(0.367)	(0.215)	(0.231)	(0.385)	(0.524)	(0.226)	(0.354)
5000	0.171	0.130	0.090	0.107	0.261	0.176	0.123	0.142
	(0.246)	(0.268)	(0.296)	(0.169)	(0.419)	(0.414)	(0.302)	(0.267)

* The first period rates are Spot rather than forward rates.

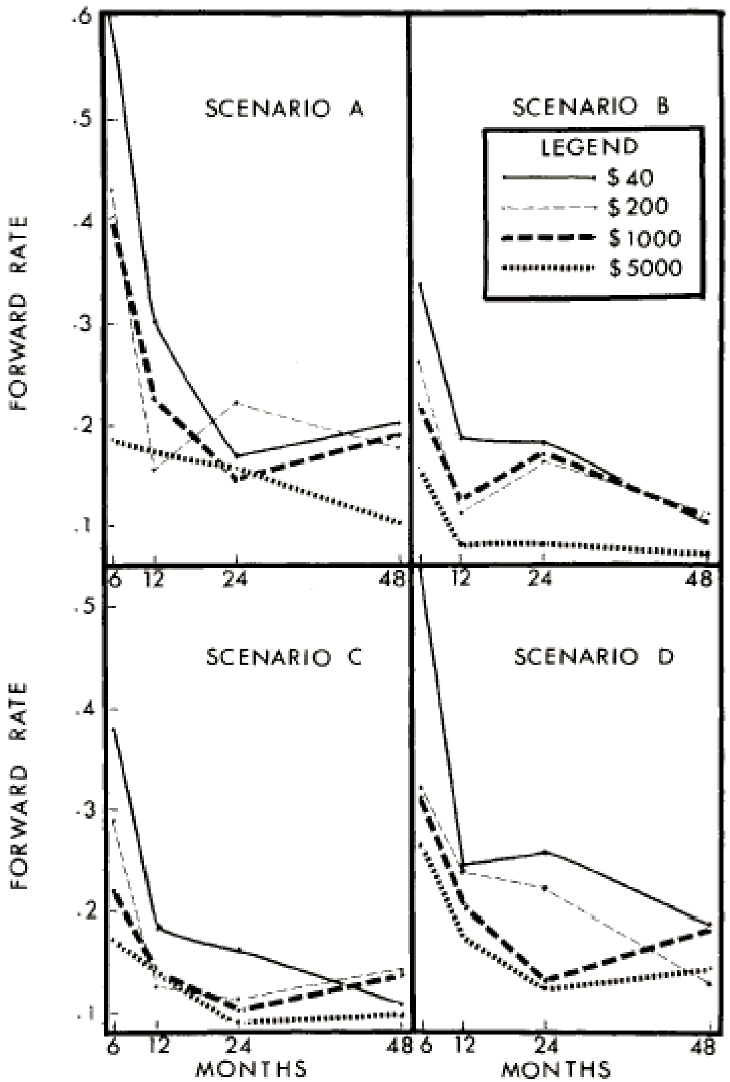


FIGURE 2. Mean Forward Rates for Different Time Delays by Sum of Cashflow and Scenario.

Benhahib, Jess, Alberto Bisin, and Andrew Schotter, “Present-Bias, Quasi-Hyperbolic Discounting, and Fixed Costs", unpublished manuscript, 2006.

- Observation from prior studies is that there may be some fixed costs of delay. These could come from the fear of not getting paid, or the inconvenience, or the memory storage costs, etc.
- The innovation here is try to get enough information from a particular individual to estimate this fixed cost, as well as nested versions of discounting theories.
- Using a matching task, both for future and present equivalent payoff to control for framing effects, try to distinguish among various theories of discounting.
- 27 subjects at two *two different times* (within subject design)

– Time 1, *Q-Present:* "What amount of money, \$$x$, if paid to you today, would make you indifferent to \$$y$ paid to you in t days?"

- report x
- $y \in \{\$10, \$20, \$30, \$50, \$100\}$,
- $t \in \{$3 days, 1 week, 2 weeks, 1 month, 3 months, 6 months$\}$, $5 \times 6 = 30$ matching tasks
- Incentivized choices with Becker-DeGroot-Marschak mechanism
  - pick one out of 30 tasks at random
  - draw a number from a uniform distribution $[0, y]$
  - if lower than x, subject has to wait, if higher, x is paid immediately.
  - true w.t.p. is dominant strategy.
- Subjects paid by check either at the end of the experiment, or the check mailed to their address so that it gets there with a proper time delay **(important, because the transaction costs of obtaining the money are the same)**

– Time 2, *Q-Future:* “What amount of money, \$$y$, would make you indifferent between \$$x$ today and \$$y$ t days from now?"

  * report y.
  * x’s generated from answers at time 1. equal to the minimum x reported across subjects.
  * y’s were censored to be no larger than the corresponding y’s on Time 1 task.
  * the rest is similar.
  * This allows a test of framing of the questions.

- Econometric specification:

$$u(y,t) = D(y,t;\theta,r,\alpha,b)y, \text{ where}$$
$$D(y,t;\theta,r,\alpha,b) = \varepsilon(y,t)\begin{cases} 1 & \text{if } t=0 \\ \alpha[1-(1-\theta)rt]^{1/(1-\theta)} - b/y & \text{if } t>0 \end{cases}$$

  – For $\theta \to 1$, $\alpha = 1$ and $b = 0$ this becomes e^{-rt}, or exponential discounting

  – For $\theta = 2$, $\alpha = 1$ and $b = 0$ this becomes $1/(1+rt)$, or hyperbolic discounting

  – For $\theta \to 1$, $\alpha < 1$ and $b = 0$ this becomes quasi-hyperbolic discounting

  – If $b > 0$, there is a fixed cost of delay

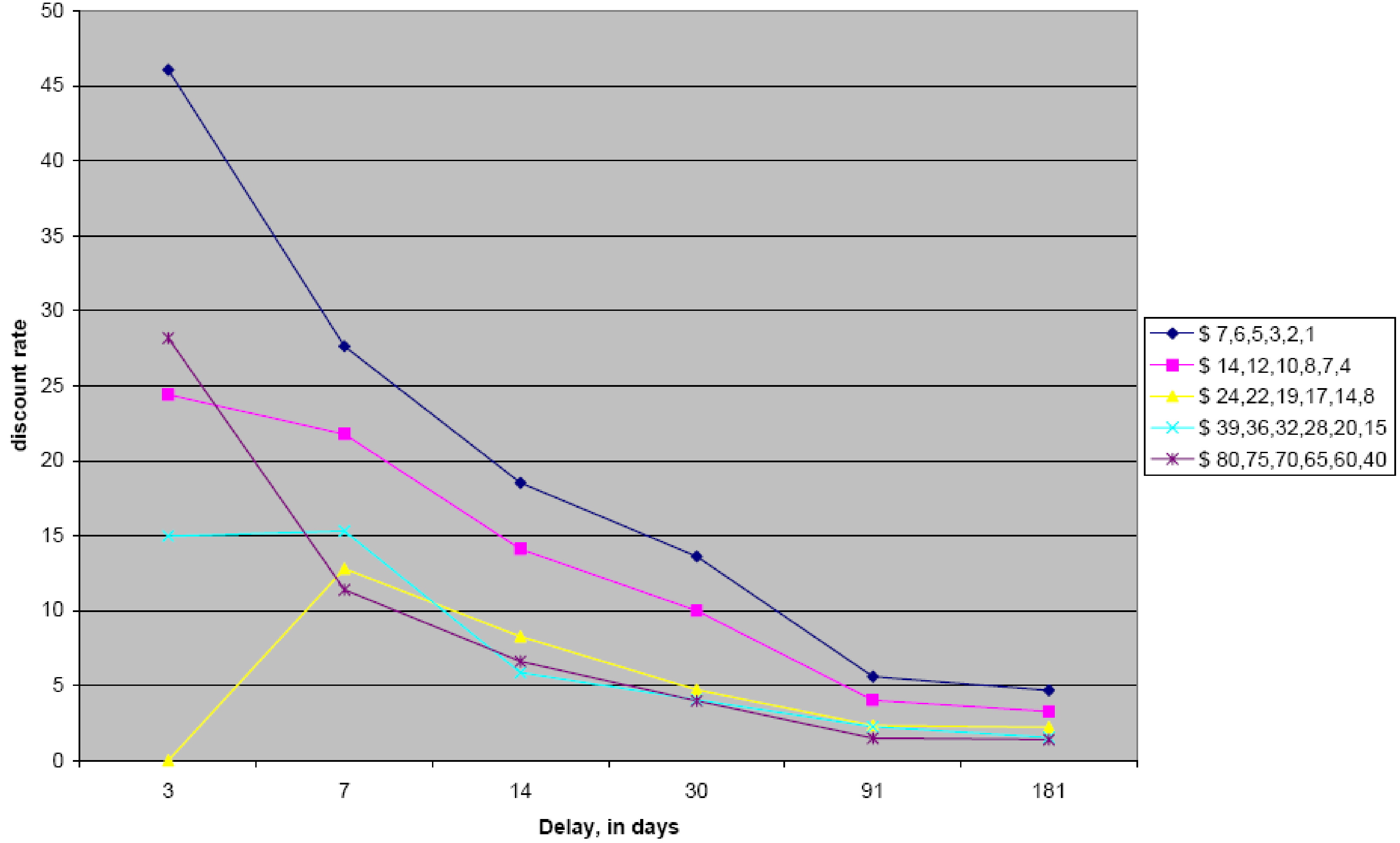
  – $\varepsilon(y,t)$ assumed log-normal

  – Nonlinear least squares used for estimation

Results:

- Raw results: Data have both the *present bias* and *magnitude effects* of earlier studies:

Figure 1: annual discount rate, subject 27 [Q-*future*]



- Estimate discount rates separately for every subject:

Table 3: Question $[Q - future]$; Specification with Fixed Cost and Quasi-Hyperbolic Component

person	θ	se(θ)	r	se(r)	b	se(b)	α	se(α)
1	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
2	-8.13	9.66	0.13	0.05	0.73	0.17	0.98	0.01
3	-0.92	2.19	0.48	0.15	3.51	0.47	0.94	0.01
4	-1.28	1.38	0.68	0.24	4.22	1.02	1.01	0.03
5	-0.62	1.31	0.66	0.18	2.62	0.58	0.98	0.02
6	1.70	0.64	1.97	0.52	3.42	0.80	1.01	0.03
7	21.74	24.80	0.41	0.48	1.68	0.28	0.98	0.01
8	-0.51	1.28	0.68	0.18	3.41	0.60	1.09	0.02
9	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
10	4.39	1.70	3.43	2.28	4.94	1.16	0.97	0.05
11	-3.21	2.39	0.42	0.17	1.27	0.69	1.06	0.03
12	5.60	1.52	3.15	1.55	4.53	0.66	1.00	0.03
13	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
14	6.21	0.78	8.90	4.12	3.87	0.52	0.92	0.04
15	-0.06	2.06	0.83	0.43	6.17	1.52	0.96	0.04
16	0.21	0.92	0.85	0.19	2.66	0.52	0.99	0.02
17	-2.66	5.84	0.27	0.13	0.72	0.39	0.99	0.01
18	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
19	-11.59	19.25	0.15	0.21	6.17	1.43	0.93	0.03
20	-48.75		0.04	0.00	3.04	1.28	1.20	0.04
21	1.21	0.80	1.35	0.35	4.62	0.77	1.21	0.03
22	4.57	0.75	1.74	0.50	3.54	0.85	1.02	0.03
23	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
24	41.33	23.02	17944.30		4.16	1.07	0.97	0.13
25	3.95	0.79	5.11	2.20	5.29	0.89	1.02	0.05
26	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
27	-0.02	1.44	0.74	0.23	3.79	0.70	1.16	0.03

Things to note:

- $\alpha \approx 1$ for most.
- $b > 0$ for most, and relatively similar across subjects.
- $b \approx 4$ on average, interpretation of a $4 fixed cost (or certainty equivalent adjustment in the case of risk).
- θ is not very precisely measured, or very stable across individuals.

Framing:

- Yes, parameters estimates are not the same across the two conditions.
- But this is hard to attribute to a true framing effect. For instance, what if we had some subjects make the *same* choices again? Would they differ too?

Conclusions:

- Clear experimental evidence *against* exponential discounting.
- The data favors a specification of discounting which contains a present bias in the form of a fixed cost, and no quasi-hyperbolic component.
- Curvature of discounting (exponential vs. hyperbolic), in the fixed cost specification, is not precisely estimated.
- This implies that present bias vanishes with large rewards and that the evidence for hyperbolic discounting is perhaps weaker than previously documented using more restrictive specifications.
- But....experiments with relatively large rewards are needed to confirm the fixed cost representation of present bias