Choices over Time

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1 Introduction

1. Which do you prefer: **A)** $25 today **B)** $26 in a week.
2. Which do you prefer: **A)** $25 in 30 weeks **B)** $26 in 31 weeks.
3. Fill in the blank: I am indifferent between $100 today and $_____ in one month.
4. Fill in the blank: I am indifferent between $100 today and $_____ in one year.
5. Suppose you get a spacious, luxury apartment for only one year of college, and a shared room on campus the other years. When do you prefer to have the luxury apartment?
   A) Luxury apartment freshman year only.

   B) Luxury apartment senior year only

2 Choice over time.

Let c_t be consumption at some time t.
Which is worth more, \$100 today or \$100 in a year?
What part of human nature may cause us to “discount” the future?

Model of utility

- – Begin with $U = U(c_0, c_1, c_2, ..., c_T)$. What properties or restrictions can we add to this to give us additional predictive powers?
- Discounting, or Impatience:
  – If $c_t = c_{t+k}$ for some $k > 0$,then $\partial U/\partial c_t > \partial U/\partial c_{t+k}$.

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- Consistency of optimal plans.
  - If future income is known for sure, then any plan I make today I will carry out in the future.
  - That is, I know today what my feelings will be in the future.
  - Let $U_t = U_t(c_t, c_{t+1}, ..., c_T)$
  - Then $\partial U_t/\partial c_{t+k}$ is the same for all t.
  - So if I plan today on a best consumption stream, $c_1^*, c_2^*, ...c_T^*,$,then as long as my income stays the same, I will never want to deviate from that plan in the future.

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- Stable preferences
  - My (instantaneous) tastes don't change over time.
  - That is, I like the same things in the future that I like today, and I like them in just the same way.

Model of utility

If utility follows these basic properties, then it can be shown that we can write the intertemporal utility function this way:

Exponential Discounting:

$$
\begin{aligned}
u(c_0, c_1, c_2, ..., c_T) &= u(c_0) + \delta u(c_1) + \delta^2 u(c_T) + ... + \delta^T u(c_T) \\
&= \sum_{t=0}^{T} \delta^t u(c_t) \\
\text{where } 0 &< \delta < 1
\end{aligned}
$$

This is a widely used model.

- We call δ the *discount factor.*
- In a comparison to interest rates and present value calculations we sometimes think of

  $$\delta = \frac{1}{1+\rho}$$

  where ρ is called the *discount rate.*
- So utility is like a present discounted value of future utility, but the discount rate is your own personal rate, not the market interest rate.

- (see Samuelson, Paul, “A Note on Measurement of Utility"**,** Review of Economic Studies, 4(2), 1937, 155-161).

Examples. Suppose: $u(x) = \ln(x)$

- Suppose you have a chocolate bar that will last 2 days. How do you allocate consumption of it over the two days?

$$\max \ln(c_0) + \delta \ln(c_1), \text{ s.t. } c_0 + c_1 = 1$$

or

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What is the solution?

$$c_0 = \frac{1}{1+\delta}, \; c_1 = \frac{\delta}{1+\delta}$$

which implies

$$c_1 = \delta c_0$$

so consumptions declines at the individual's discount factor.

- What if it lasts 4 days?

$$\max \ln(c_0) + \delta \ln(c_1) + \delta^2 \ln(c_2) + \delta^3 \ln(c_3)$$
$$\text{s.t. } c_0 + c_1 + c_2 + c_3 = 1$$

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$$\text{s.t.} \quad c_0 + c_1 + c_2 + c_3 = 1$$

What is the solution?

$$\begin{aligned} c_1 &= \delta c_0 \\ c_2 &= \delta c_1 = \delta^2 c_0 \\ c_3 &= \delta c_2 = \delta^3 c_0 \end{aligned}$$

substituting these into the budget constraint, we find

$$c_0(1 + \delta + \delta^2 + \delta^3) = 1$$

- Suppose you have to do some work that will take exactly W hours, and you have 2 periods to do it. That is, $w_1 + w_2 = W$. So let $24 - w_1$ and $24 - w_2$ be the leisure time available to you. Then your problem is

$$\max_{w_1, w_2} \ln(24 - w_1) + \delta \ln(24 - w_2)$$
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What is the solution?
Rewrite this as $\ell_1 = 24 - w_1$ and $\ell_2 = 24 - w_2$, and $\ell_1 + \ell_2 = 48 - W$, and the problem is

$$\max_{\ell_1, \ell_2} \ln(\ell_1) + \delta \ln(\ell_2)$$
$$\text{s.t. } \ell_1 + \ell_2 = 48 - W$$

and as above we get

$$\ell_1 = \frac{48 - W}{1 + \delta}, \ell_2 = \frac{\delta(48 - W)}{1 + \delta}, \text{ so } \ell_2 = \delta \ell_1$$

so you work more in the second period than the first.

General result:
Imagine being endowed with m_t money each period and think of moving some of it between period k and $k+1$. What transfer would maximize utility?
Note any transfer would make this true:

$$u'(c_k) = \delta u'(c_{k+1}).$$

So when we are optimizing, our marginal utilities must be rising at the rate δ.
This means when we are consuming good things, we should tend to consume more of it early and when we are consuming bad things, we should be consuming more of it later

Your answers

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Model of utility

What are the weak points of our model of utility?

- Discounting, or Impatience.
- Consistency of optimal plans.
- Stable preferences

Model of utility

What are the weak points of our model of utility?

- Discounting, or Impatience.
- Consistency of optimal plans.
  - We often think we have greater will power than we do.
    - Sticking with a diet, or exercise plan, or study plan.
    - "This year I'll do my taxes right away, not waiting until April 15th"
  - Often tend to move good things up too much ("I wish I hadn't eaten all that")
  - Often tend to postpone bad things too long (e.g. cramming for exams).
- Stable preferences
  - Can what we consume today influence our tastes tomorrow?
    - Hard to go back to a small apartment after living with luxury. (Habit formation).
    - Cigarettes are an extreme form of this by causing addiction.
  - There is also pleasure in anticipation of good stuff and pain in the dread of bad stuff.