Choices under Risk.

James Andreoni

Demonstrations 1: Imagine the following gambles. Choose one from each row:

a1:	100% chance of $20	a2:	10% chance of $100 89% chance of $20 1% chance of $0
a3:	10% chance of $100 90% chance of $0	a4:	11% chance of $20 89% chance of $0

On your card, put your :

- ID
- a1 or a2.
- a3 or a4.

- What are people doing when they make these types of risky choices?
- Maximize Expected Value?

$$EV = pw_1 + (1-p)w_2$$

- The challenge to this approach started with the St. Petersburg Paradox. (Discovered by Nicholas Bernoulli 1720, resolved by his younger brother Daniel Bernoulli in 1738. Translated and reprinted in Econometrica, 1954)

I'll flip a coin until it comes up heads. This table lists the prizes you will get if the first head is on the nth flip.

Notice, the prize equals $\$2^n$.

How much would you be willing to pay for this gamble?

EV?

n	P(n)	Prize		
1	1/2	$2		
2	1/4	$4		
3	1/8	$8		
4	1/16	$16		
5	1/32	$32		
6	1/64	$64		
7	1/128	$128		
8	1/256	$256		
9	1/512	$512		
10	1/1024	$1024		
11	And so on…			

Find the EV of this gamble

The EV = sum of p(n)*prize(n)

EV=sum (1/2)^n * 2^n = infinity.

So will you pay an infinite amount for this gamble?

Why not?

This is called the
"St Petersburg Paradox"

n	P(n)	Prize	Expected value
1	1/2	$2	$1
2	1/4	$4	$1
3	1/8	$8	$1
4	1/16	$16	$1
5	1/32	$32	$1
6	1/64	$64	$1
7	1/128	$128	$1
8	1/256	$256	$1
9	1/512	$512	$1
10	1/1024	$1024	$1
11	And so on…		

Answer: Risk aversion.

- The series is infinite, so the expected payoff of the game is infinite. But people will only pay $12 or so to play. Bernoulli’s insight was that this could be explained by diminishing marginal utility.
- But how do we formalize a model of choice with risk and uncertainty?

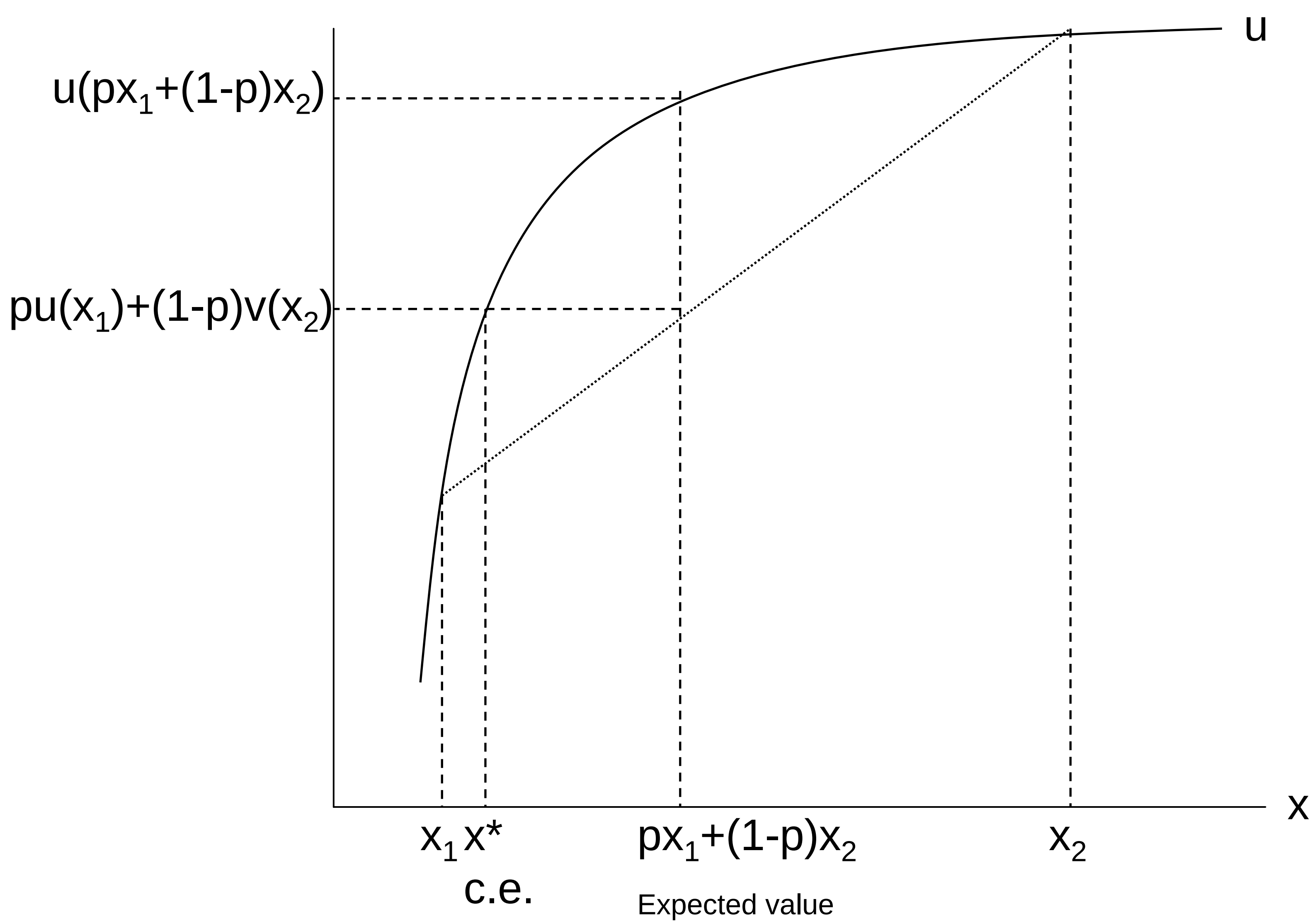
Model of Expected Utility

- Define:
  - p1, p2, etc, the *Probability Distribution*
  - x2, x2 etc. as the *Prizes* or *States of Nature*
- Utility should be a function of these characteristics:
  - U = U(p1,p2, x1, x2)
- Can we put some more structure on this to give us more predictive power?
- Add two assumption:
  - Preference are reparable over states of nature, that is what you care about one state doesn't depend on what could have been.
  - How you feel about a particular state of nature doesn't change if its probability of happening changes.
- If these are true, you can show that there is a function u(x) such that we can write the utility of a gamble "p of x1 and (1-p) of x2" as:

$$U = pu(x1)+(1- p)u(x2)$$

In this case we say preferences have the *Expected Utility Property*.

Easiest to think about a gamble with two outcomes: good and bad.



u
u(px1+(1-p)x2)
pu(x1)+(1-p)v(x2)
x
x1 x*
c.e.
px1+(1-p)x2
Expected value
x2

- Note: $U(EV) > EU$
  - This is what it means to be risk averse.
  - Prefer the same EV but without the added risk
- How much are you willing to pay for this gamble?
  - Define: *Certainty Equivalent:* The amount of money for sure that would make you indifferent between the sure thing or the gamble
    - $u(x^*)=pu(x1)+(1-p)u(x2)$
    - x^* is the certainty equivalent.
    - Note, $x^*<EV$
    - This can solve the St. Petersburgh paradox.

Suppose $u = x^{0.5}$. x is the amount of money you have, in dollars.

A friend offers you a gamble where you get $64 if you win, and you get $4 if you lose. The probability of winning is 0.5. Obviously this is a tempting gamble.

- Calculate the Expected Value of this gamble.
- Calculate the EU of the gamble.
- Calculate the CE

- What if $U(EV) < EU$?

- What if U(EV) < EU?
  - Risk loving.
  - CE > EV … prefer to have the added risk

- What if U(EV) = EU everywhere?
  - Risk neutral.
  - CE=EV   … indifferent to the risk.

How well does the EU model predict?

von Neuman and Morgenstern derived EU from axioms about behavior over lotteries.

Preferences over lotteries must be complete and transitive:

Either on the same indifference, or on others.

Indifference curves can't cross.

Continuity:

For any three outcomes X > Y > Z, there's a unique p s.t. people are indifferent between a lottery over x and z, and y for sure.

Independence:

utility of an outcome does not depend on the chance of getting it.

Marshak-Machina (MM) Triangle:

- Way of visualizing the restrictions that vNM's version of EU puts on choices under risk.
- Consider a "prospect" with 3 outcomes: $x_1 < x_2 < x_3$.

Suppose people maximized expected utility.

$$U^* = p_1\, u(x_1) + p_2\, u(x_2) + p_3\, u(x_3)$$

$$U^* = p_1\, u(x_1) + (1 - p_1 - p_3)\, u(x_2) + p_3\, u(x_3)$$

Find Iso-expected utility lines in p1 and p3 space.

$U^* = p_1\, u(x_1) + (1 - p_1 – p_3)\, u(x_2) + p_3\, u(x_3)$

or

$p_3 = p_1\, [u(x_1) – u(x_2)]/[u(x_2) - u(x_3)] – U^*/[u(x_2) - u(x_3)]$

or

$p_3 = p_1 A + U^* B$

Where $A = [u(x_1) – u(x_2)]/[u(x_2) - u(x_3)] > 0$

And $B = - 1/[u(x_2) - u(x_3)] > 0$

These follow because $x_1 < x_2 < x_3$.

So the Iso-utility curves are upward sloping lines.

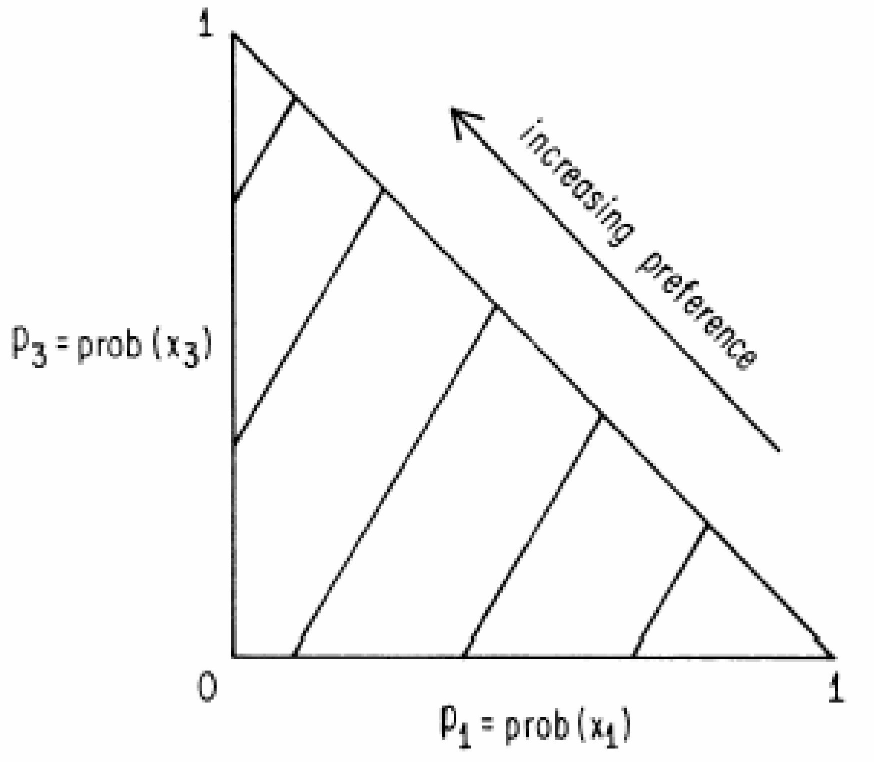


Fig. 2. Expected utility indifference curves in the triangle diagram

Since x_2 is the intermediate outcome, movement to the NE is a mean preserving increase in spread. (Same EV, more variance.)

So the more risk averse you are, more vertical the lines.

Note that the indifference curves must **still** be parallel.

a1:	100% chance of $20	a2:	10% chance of $100 89% chance of $20 1% chance of $0
a3:	10% chance of $100 90% chance of $0	a4:	11% chance of $20 89% chance of $0

So what did you choose?

Most people choose a1 and a3.

a1:	100% chance of \$20	a2:	10% chance of \$100 89% chance of \$20 1% chance of \$0
a3:	10% chance of \$100 90% chance of \$0	a4:	11% chance of \$20 89% chance of \$0

Most people choose a1 and a3. Let u(0)=0. Then

a1 > a2 means: U(20) > .1U(100)+.89u(20)

Rearrange to get: .11u(20) > .1u(100)

But this contradicts the finding that a3>a4.

a1:	100% chance of $20	a2:	10% chance of $100 89% chance of $20 1% chance of $0
a3:	10% chance of $100 90% chance of $0	a4:	11% chance of $20 89% chance of $0

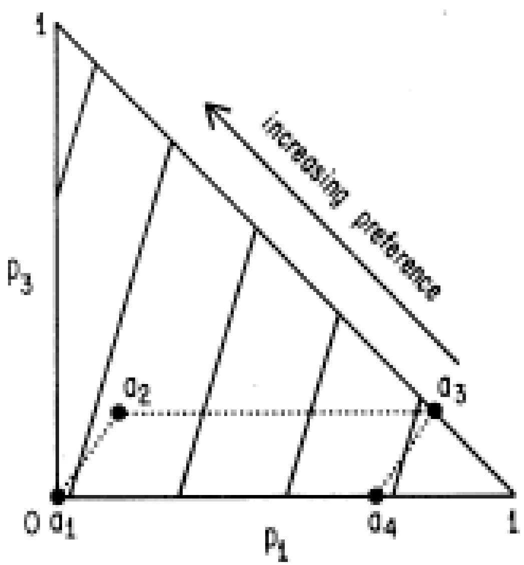


Fig. 4a. Expected utility indifference curves and the Allais Paradox

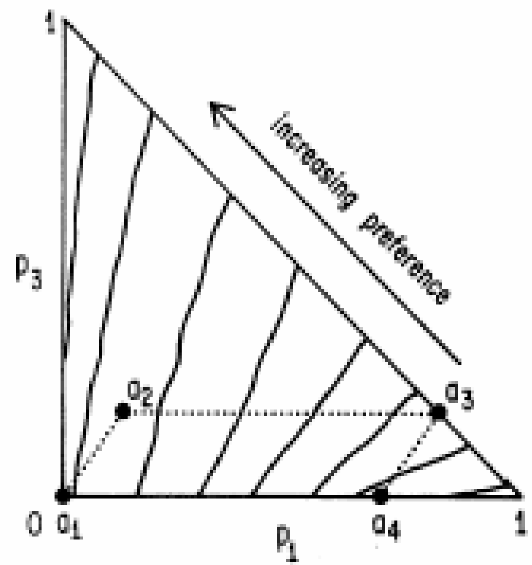


Fig. 4b. Indifference curves which 'fan out' and the Allais Paradox

Typically, this is fixed with

“Subjective Expected Utility”

So, from maximizing

$$EV = p(w_1) + (1\text{-}p)(w_2)$$

to

$$EU = p\ u(w_1) + (1\text{-}p)\ u(w_2)$$

now to

$$SEU = w(p)\ u(w_1) + (1\text{-}w(p))\ u(w_2)$$

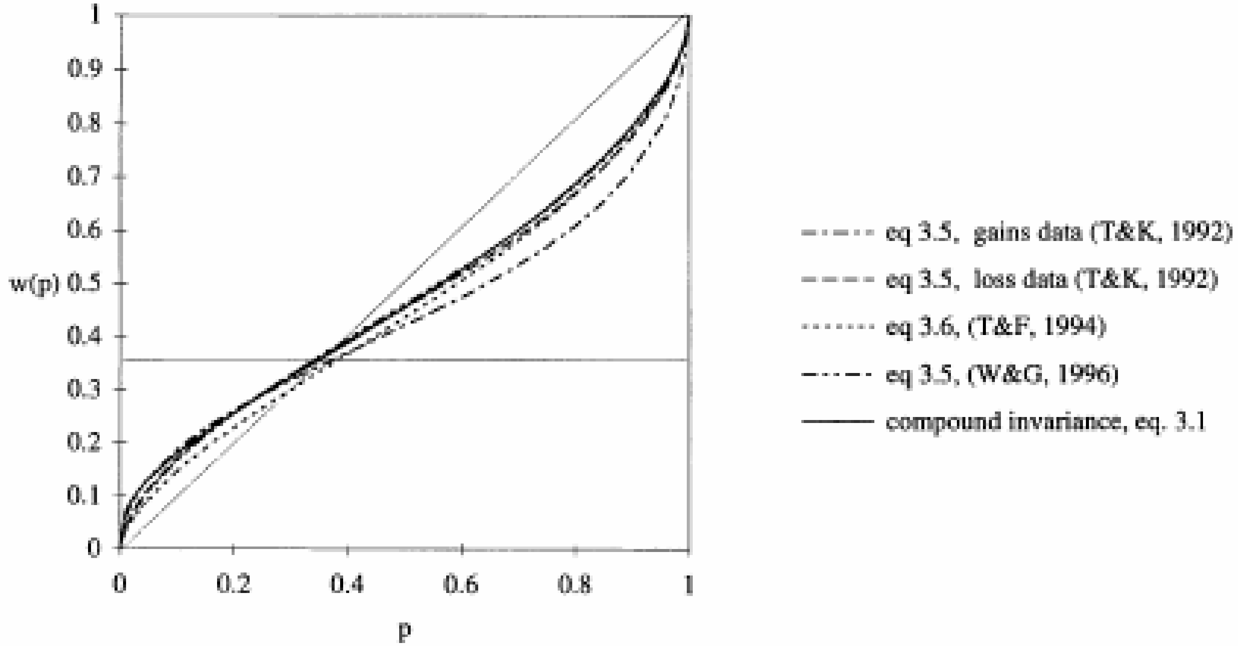


FIGURE 1.—The compound invariant form (solid line) and several empirical probability weighting functions. Estimates of the one-parameter equation (3.5) are taken from Tversky and Kahneman (1992) and Wu and Gonzalez (1996a); estimates of the two-parameter equation (3.6) are taken from Tversky and Fox (1994).