Information, Bayes Rule, Cascades

James Andreoni

1 Introduction

- There is a severe new sickness that's been discovered: College Lecture Acute Sleeping Sickness (CLASS).
- Sadly, this disease afflicts 1 in every 10,000 college students
- But there is hope....a new test that can detect this disease.
  - If you have CLASS, the test will surely be positive.
  - Moreover, the test is 99% accurate, that is, there is only a 1% chance of a false positive.
- Before we give you the test, what is the probability that you have CLASS?
  - Write the answer in your notebook.
- Suppose we give you the test and, alas, your test result is *positive*!!
- What it the probability that you have the dreaded CLASS?
  - Write your answer down in your notebook.

Answer: After a positive test result, your chances are only about 1%.

Why?

- Out of 10,000 people, about 100 will get false positive test results, and 1 will get a true positive result.
- So out of 10,000 people, 101 will test positive, but only 1 of those has the disease.
- Your chances are 1/101
- So ...

Answer: After a positive test result, your chances are still about 1%, that is, 1 in 100.

Why?

- Out of 10,000 people, about 100 will get false positive test results, and 1 will get a true positive result.
- So out of 10,000 people, 101 will test positive, but only 1 of those has the disease.
- Your chances are 1/101
- So ...

WAKE UP!

2 Bayes’ Rule

How, in general, do we update our beliefs when we get new information?
The method is called Bayes’ Rule.

Example: My cup has 3 black and white stones in it, 2 of one and 1 of the other. So it is one of these two urns:

B Cup	or	**W** Cup
B		W
B		W
W		B

- I flip a coin to determine which cup to use.
- Let $p =$ my belief that we are using the Black cup
- Call p my *prior.*
- Before I take any draws from the cup, my $prior$ is $p = 0.5$.

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- I flip a coin to determine which cup to use.
- Let $p =$ my belief that we are using the Black cup
- Call p my *prior.*
- Before I take any draws from the cup, my $prior$ is $p = 0.5$.
- Now I take a draw. Suppose the draw is W. Now what do I believe the probability is that I chose the **B** cup?
- This is called the *posterior.* $p = prob(\mathbf{B}|W). =$ probability of the **B** cup given a draw of W.

B Cup	or	**W** Cup
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- How do I calculate my *posterior*? What is the chance it is the **B** cup after a W draw?

B Cup	or	**W** Cup
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- How do I calculate my *posterior*? What is the chance it is the **B** cup after a B draw?
- Method 1: Counting: Count up all the possible ways you could have drawn a W stone, and how many of those would be drawn from the **B** cup:

$$\begin{aligned}\Pr(\mathbf{B}|W) &= \frac{\text{Number of } W \text{ stones in the } \mathbf{B} \text{ cup}}{\text{Number of } W \text{ stones in both cups}} \\ &= \frac{1}{3} < \frac{1}{2}\end{aligned}$$

- Note, you got evidence that does not favor cub **B,** so your posterior belief is lower than your prior. Makes sense.

$$\Pr(\mathbf{B}|W) = \frac{\text{Number of } W \text{ stones in the } \textbf{B} \text{ cup}}{\text{Number of } W \text{ stones in both cups}}$$

- Let N be the number of marbles in a cup, i.e. 3 here. then

$$\begin{aligned} \Pr(\mathbf{B}|W) &= \frac{\Pr(W|\mathbf{B})N}{\Pr(W|\mathbf{B})N + \Pr(W|\mathbf{W})N} \\ &= \frac{\Pr(W|\mathbf{B})(1/2)}{\Pr(W|\mathbf{B})(1/2) + \Pr(W|\mathbf{W})(1/2)} \end{aligned}$$

- But suppose the likelihood of each urn is different, i.e. not 1/2 each. Then

$$\Pr(\mathbf{B}|W) = \frac{\Pr(W|\mathbf{B})\Pr(\mathbf{B})}{\Pr(W|\mathbf{B})\Pr(\mathbf{B}) + \Pr(W|\mathbf{W})\Pr(\mathbf{W})} \qquad \text{(Bayes' Rule)}$$

So this is Bayes' Rule:

$$\Pr(\mathbf{B}|W) = \frac{\Pr(W|\mathbf{B})\Pr(\mathbf{B})}{\Pr(W|\mathbf{B})\Pr(\mathbf{B}) + \Pr(W|\mathbf{W})\Pr(\mathbf{W})} \qquad \text{(Bayes' Rule)}$$

Apply it here:

$$\Pr(\mathbf{B}|W) = \frac{\frac{1}{3}\frac{1}{2}}{\frac{1}{3}\frac{1}{2} + \frac{2}{3}\frac{1}{2}} = \frac{1}{3}$$

Apply it again:
Prior $p = 1/3$.
Suppose we draw a $\mathbf{B}$:

$$\begin{aligned}\Pr(\mathbf{B}|B) &= \frac{\text{Pr}(B|\mathbf{B})\Pr(\mathbf{B})}{\text{Pr}(B|\mathbf{B})\Pr(\mathbf{B}) + \text{Pr}(B|\mathbf{W})\Pr(W)} \\ &= \frac{\frac{2}{3}\frac{1}{3}}{\frac{2}{3}\frac{1}{3} + \frac{1}{3}\frac{2}{3}} = \frac{1}{2}\end{aligned}$$

Can people successfully apply Bayes' Rule?

- It depends on the experiment.
- People are often biased by context and prejudice.
  - Even if you tell them that the white 45 year old librarian is equally likely to like Fifty Cent as to like Bach, people use their stereotypes and violate Bayes Rule.
  - This is called the *base rate bias:* The base rate (prior) is so strong that they fail to update it with new information.
  - This is why you might have overestimated your chance of having CLASS
- People are also subject to recognize patterns.
  - Suppose you make three draws above and get $W, B, B.$
  - Since this looks just like the **B** cup, people over estimate the likelihood that it is the **B** cup**.**
  - This is called the *representativeness bias.*
  - What is the likelihood of the **B** cup?

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  - This is called the *representativeness bias.*
  - What is the likelihood of the **B** cup? Answer: 2/3

Gary Charness and Dan Levin, "When Optimal Choices Feel Wrong: A Laboratory Study of Bayesian Updating, Complexity, and Affect" ***American Economic Review,*** **September 2005.**

- Study the "Win-stay, lose-switch" heuristic.
- This is the tendency to repeat the same move after a success and change after a failure.
- Example from basketball: Just before the tip-off your starting guard breaks a shoestring. You need to start the second string guard while the first-string guard changes his laces. This take 3 minutes. In that time the second string guard is the top scorer on the team. Do you leave him in, or substitute the first-stringer at the first opportunity?

- Consider this information structure:

Treatment 1

	Left urn	Right urn
Up ($p = 0.5$)	●●●●○○	●●●●●●
Down ($p = 0.5$)	●●●○○○	○○○○○○

- The true state of the world is either *Up* or *Down,* but you don't know which. Each state is equally likely.
- You get 2 draws, and you get paid $1 for each **black** ball you draw.
- Your first draw must be from the **left**
- The second ball you draw can be from either **left** or **right**

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- You get 2 draws, and you get paid $1 for each **black** ball you draw.
- Your first draw must be from the **left**
- The second ball you draw can be from either **left** or **right.**
- Suppose your first draw is black, do you stay or switch?
- Suppose your first draw is white, do you stay or switch?**.**

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	Left urn	Right urn
Up ($p = 0.5$)	●●●●○○	●●●●●●
Down ($p = 0.5$)	●●●○○○	○○○○○○

- Suppose your first draw is black, do you stay or switch? Answer: Switch
- Suppose your first draw is white, do you stay or switch?.Answer: Stay
- So, you *switch* after success and *stay* after failure, which is the opposite of the heuristic

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	Left urn	Right urn
Up ($p = 0.5$)	●●●●○○	●●●●●●
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- Suppose instead that you were forced to choose your first ball from **right.**
- Suppose your first draw is black, do you stay or switch?
- Suppose your first draw is white, do you stay or switch?

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	Left urn	Right urn
Up ($p = 0.5$)	●●●●○○	●●●●●●
Down ($p = 0.5$)	●●●○○○	○○○○○○

- Suppose instead that you were forced to choose your first ball from **right.**
- Suppose your first draw is black, do you stay or switch?Answer: Stay
- Suppose your first draw is white, do you stay or switch? Answer: Switch
- So, you *stay* after success and *switch* after failure, which is the same as the heuristic

Results

RESULT 1: Switching-error rates are very low when Bayesian updating and a reinforcement heuristic are aligned, but are quite large when these are opposed.

- When Bayes Rule and the heuristic are *aligned* (right draws) error rate is only 5%
- When Bayes Rule and the heuristic are in *conflict* (left draws), error rates are 36.8%
- For almost all subjects the error rates are higher when they first draw from the left.

The rationale for the bias is that people get emotional after a success, and this flood of good feelings clouds their judgment.
So, now let's do a new treatment where we don't pay them for the first draw, but only if they get a black on the second draw. Now they should have no emotional reaction to the first draw so can think clearly for the second one.

RESULT 2: Removing affect from the initial draw (by not paying for its outcome and not associating it with success or failure) reduces the error rate, particularly in the case of positive affect.

- The drop is particularly dramatic after black (successful) draws from Left, dropping to 13.5 percent from 36.8 percent.
- smaller drop when the first draw is white (failure), from 56.4 percent to 42.4 percent.

RESULT 5: The cost of an error has a strong influence on the frequency of the error. However, the presence of affect for the first draw, the transparency of the updating (Left versus Right), and the gender of the participant also appear to play important roles.

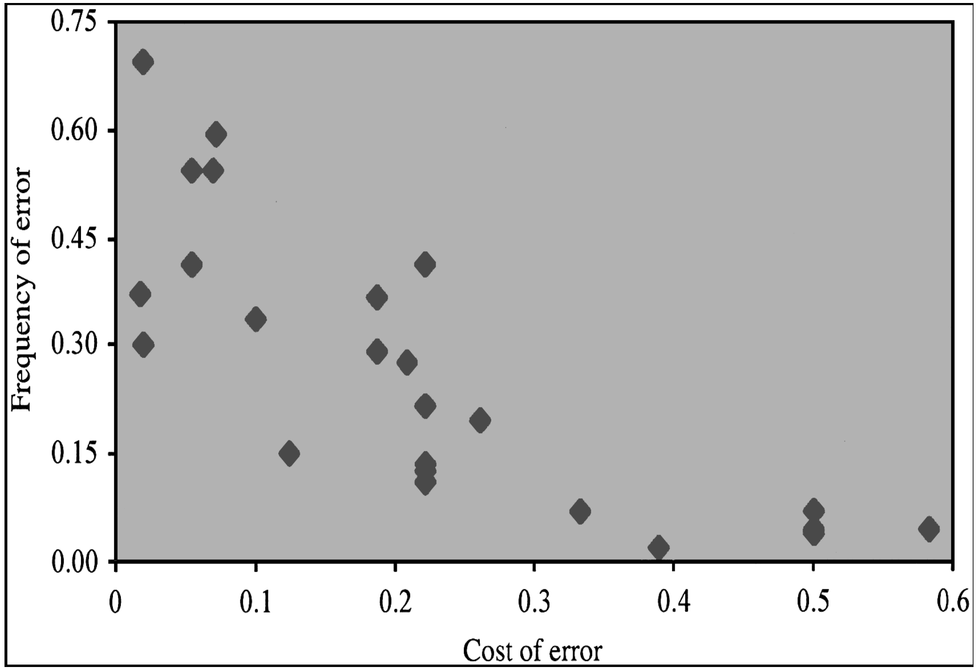

FIGURE 2. COST AND FREQUENCY OF ERRORS

$$\mathit{Frequency} = \underset{(0.045)}{0.317} - \underset{(0.203)}{1.49}*\text{Cost}$$

$$+ \underset{(0.334)}{1.31}*\text{Cost}^2 + \underset{(0.024)}{.097}*\text{Left}$$

$$+ \underset{(0.031)}{.099}*\text{Affect} + \underset{(0.025)}{.081}*\text{Female}$$

3 Cascade Demonstration

- I'll start by picking one of these at random:

B Cup	or	**W** Cup
B		W
B		W
W		B

- I will put you in groups of 5 or 6 (I hope)
- The first person pick a stone in private then announce a guess of which cup was used.
- I'll write all guesses on the board.
- You get \$2 if you guess right.
- Put the stone back and I'll do this again with the second person.
- When all 5 are done, we'll see which cup it was.

Question: You are in a new city. You are hungry and see two restaurants that look just the same to you—same cuisine, similar menu and prices. One restaurant has only one empty table, while the other is almost empty
Where do you go for lunch?

Suppose this happens:

- Person 1 bets **B**
  - your posterior?

Suppose this happens:

- Person 1 bets **B**
  - your posterior? $p_B = 2/3 = 67\%$
- Person 2 bets **B**
  - your prior is 80%
  - your posterior?

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  - your posterior? $p_B = \frac{\frac{2}{3}\frac{2}{3}}{\frac{2}{3}\frac{2}{3}+\frac{1}{3}\frac{1}{3}} = \frac{4}{5} = 80\%$
- You are person 3. You draw W
  - your prior is 80%
  - your posterior? $p_B = 2/3$
  - So you should bet **B** even though your signal is W.

Suppose this happens:

- Person 1 bets **B**
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- You are person 4. You draw W.
  - What's your prior?

Suspect this happens:

- Person 1 bets **B**
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- You are person 3. You draw W
  - your prior is 80%
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  - So you should bet **B** even though your signal is W.
- You are person 4. You draw W.
  - What's your prior? 80% – the last bet told you nothing, since the bet would be **B** either way.
    - $*$ remember, all person 4 sees is the bet, not the draw.
  - What's your posterior?

Suppose this happens:

- Person 1 bets **B**
  - your posterior? $p_B = 2/3 = 67\%$
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- You are person 4. You draw W.
  - What's your prior? 80% – the last bet told you nothing, since the bet would be **B** either way
    - $*$ remember, all person 4 sees is the bet, not the draw.
  - What's your posterior? $p_B = 2/3$ again. Since $2/3 > 1/2$, bet **B.**
- Now we are stuck...everyone will bet **B** no matter what they draw!!

Think again about the restaurant. Suppose both are empty and the first person makes a choice by flipping a coin. The second person shows up and guesses that the first people might have known something, so they make the same choice. And so on. Eventually one restaurant has a wait while the other is empty. Yet, in truth no one knows it is the better restaurant. Finally, someone comes by with a restaurant guide that says the empty restaurant is better...but they discard this information on the belief that all these people can't be wrong. This is called herding behavior, or cascades—people ignore their personal information and follow the crowd, and it makes sense to do so.

Where are herding and cascades seen? Examples could be bank runs, "hot stocks", and the housing bubble