Van Huyck, John B., Raymond C. Battalio, and Richard O. Biel, "Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure." American Economic Review, 1990, 80(1), pp. 234-48.

The Stag Hunt with Many Players:

$$\begin{aligned} \pi_i(e_i, e_{-i}) &= a \min\{e_1, ..., e_n\} - be_i \\ 0 &< b < a. \\ e_i &\in \{1, 2, ..., \overline{e}\}, \text{ integers} \end{aligned}$$

- What is efficient?
- What are the equilibria?
- Which equilibria are most appealing and why?

$$\begin{aligned}\pi_i(e_i, e_{-i}) &= a \min\{e_1, ..., e_n\} - b e_i \\ 0 &< b < a. \\ e_i &\in \{1, 2, ..., \overline{e}\}, \text{ integers}\end{aligned}$$

- Efficiency $e_i = \overline{e}$ all i
- Equilibria: any $e_i = e_j$ all i, j.
- Two deductive equilibrium selection criteria
  - Payoff Dominance: The equilibrium in not Pareto dominated by another equilibrium, i.e. there is no other equilibrium that makes everyone at least as well off.
    - Predicts $e_i = \overline{e}$
  - Risk Dominance/Security: Chose the equilibrium that maximizes the worst that can happen to you. This is sometimes called the mini-max solution
    - Predicts $e_i = 1$

Experimental design

TABLE 1—EXPERIMENTAL DESIGN

Experiment No.	Date	Size	A Payoff A Fullsize	B Payoff B Fullsize	A′ Payoff A Fullsize	C Payoff A Size Two[a]
1	June	16	1^p,2,...,10	–	–	–
2	June	16	1^p,2,...,10^p	11,...,15	16^p,...,20	–
3	June	14	1^p,2,...,10^p	11,...,15	16^p,...,20	–
4	Sept	15	1^p,2^p,...,10^p	11^p,...,15	16,...,20	21,...,27
5	Sept	16	1^p,2^p,...,10^p	11^p,...,15	16,...,20	21,...,27
6	Sept	16	1^p,2^p,...,10^p	11^p,...,15	16,...,20	21,...,25
7	Sept	14	1^p,2^p,...,10^p	11^p,...,15	16,...,22	23,...,25

^{p} ~ Denotes a period in which subjects made predictions.
[a] ~ In experiment 4 and 5 pairings were fixed, while in experiments 6 and 7 pairings were random.

PAYOFF TABLE *A*

		Smallest Value of *X* Chosen						
		7	6	5	4	3	2	1
Your	7	1.30	1.10	0.90	0.70	0.50	0.30	0.10
Choice	6	–	1.20	1.00	0.80	0.60	0.40	0.20
of	5	–	–	1.10	0.90	0.70	0.50	0.30
X	4	–	–	–	1.00	0.80	0.60	0.40
	3	–	–	–	–	0.90	0.70	0.50
	2	–	–	–	–	–	0.80	0.60
	1	–	–	–	–	–	–	0.70

PAYOFF TABLE *B*

		Smallest Value of *X* Chosen						
		7	6	5	4	3	2	1
Your	7	1.30	1.20	1.10	1.00	0.90	0.80	0.70
Choice	6	–	1.20	1.10	1.00	0.90	0.80	0.70
of	5	–	–	1.10	1.00	0.90	0.80	0.70
X	4	–	–	–	1.00	0.90	0.80	0.70
X	3	–	–	–	–	0.90	0.80	0.70
	2	–	–	–	–	–	0.80	0.70
	1	–	–	–	–	–	–	0.70

Results

TABLE 2—EXPERIMENTAL RESULTS FOR TREATMENT *A*

	Period									
	1	2	3	4	5	6	7	8	9	10
Experiment 1										
No. of 7's	8	1	1	0	0	0	0	0	0	1
No. of 6's	3	2	1	0	0	0	0	0	0	0
No. of 5's	2	3	2	1	0	0	1	0	0	0
No. of 4's	1	6	5	4	1	1	1	0	0	0
No. of 3's	1	2	5	5	4	1	1	1	0	1
No. of 2's	1	2	2	4	8	7	8	6	4	1
No. of 1's	0	0	0	2	3	7	5	9	12	13
Minimum	2	2	2	1	1	1	1	1	1	1
Experiment 2										
No. of 7's	4	0	1	0	0	0	0	0	0	1
No. of 6's	1	0	1	0	0	1	0	0	0	0
No. of 5's	3	3	2	1	0	0	1	1	0	1
No. of 4's	4	6	2	3	3	0	0	0	0	0
No. of 3's	1	4	2	5	0	1	1	0	1	0
No. of 2's	3	2	6	5	5	9	3	4	3	1
No. of 1's	0	1	2	2	8	5	11	11	12	13
Minimum	2	1	1	1	1	1	1	1	1	1
Experiment 3										
No. of 7's	4	4	1	0	1	1	1	0	0	2
No. of 6's	2	0	2	0	0	0	0	0	0	0
No. of 5's	5	6	1	1	1	0	0	0	0	0
No. of 4's	3	3	2	1	2	1	0	0	0	1
No. of 3's	0	0	7	6	0	2	3	0	0	0
No. of 2's	0	1	1	4	5	3	6	3	2	2
No. of 1's	0	0	0	2	5	7	4	11	12	9
Minimum	4	2	2	1	1	1	1	1	1	1
Experiment 4										
No. of 7's	6	0	1	1	0	0	1	0	0	0
No. of 6's	0	6	2	0	0	1	0	0	0	0
No. of 5's	8	5	5	5	0	1	0	0	0	0
No. of 4's	1	1	4	6	7	1	2	1	1	0
No. of 3's	0	2	3	2	4	3	2	2	1	0
No. of 2's	0	1	0	0	2	3	7	4	2	2
No. of 1's	0	0	0	1	2	6	3	8	11	13
Minimum	4	2	3	1	1	1	1	1	1	1

TABLE 2—EXPERIMENTAL RESULTS FOR TREATMENT *A*, Continued

	Period									
	1	2	3	4	5	6	7	8	9	10
Experiment 5										
No. of 7's	2	2	3	1	1	1	1	0	0	0
No. of 6's	1	3	1	0	0	0	0	0	0	0
No. of 5's	9	3	0	4	1	0	2	0	0	0
No. of 4's	3	4	6	2	1	2	0	2	1	1
No. of 3's	1	2	2	4	6	0	0	0	0	1
No. of 2's	0	2	2	3	4	6	5	2	5	3
No. of 1's	0	0	2	2	3	7	8	12	10	11
Minimum	3	2	1	1	1	1	1	1	1	1
Experiment 6										
No. of 7's	5	3	1	1	1	1	2	2	2	3
No. of 6's	2	0	0	0	1	0	0	0	0	0
No. of 5's	5	1	0	0	0	1	0	0	0	0
No. of 4's	2	3	4	0	0	0	0	0	0	0
No. of 3's	1	5	4	2	2	2	1	0	2	0
No. of 2's	0	2	4	5	3	3	6	4	5	5
No. of 1's	1	2	3	8	9	9	7	10	7	8
Minimum	1	1	1	1	1	1	1	1	1	1
Experiment 7										
No. of 7's	4	3	1	1	1	1	1	1	1	1
No. of 6's	1	0	0	0	0	0	0	0	0	0
No. of 5's	2	3	0	0	0	0	0	0	0	0
No. of 4's	4	0	1	2	1	0	0	0	0	0
No. of 3's	1	3	2	1	1	0	0	0	0	0
No. of 2's	1	3	2	2	4	4	4	4	5	3
No. of 1's	1	2	8	8	7	9	9	9	8	10
Minimum	1	1	1	1	1	1	1	1	1	1

TABLE 3—EXPERIMENTAL RESULTS FOR TREATMENT B AND TREATMENT A'

	Treatment B					Treatment A'				
	11	12	13	14	15	16	17	18	19	20
Experiment 2										
No. of 7's	13	15	16	16	16	8	2	0	0	0
No. of 6's	1	0	0	0	0	0	0	0	0	0
No. of 5's	0	1	0	0	0	1	0	0	0	0
No. of 4's	1	0	0	0	0	1	2	0	0	0
No. of 3's	1	0	0	0	0	1	1	1	1	0
No. of 2's	0	0	0	0	0	3	3	4	2	0
No. of 1's	0	0	0	0	0	2	8	11	13	16
Minimum	3	5	7*	7*	7*	1	1	1	1	1*
Experiment 3										
No. of 7's	13	13	12	13	14	6	2	2	1	1
No. of 6's	0	0	1	1	0	1	0	0	0	0
No. of 5's	0	0	1	0	0	0	2	1	0	0
No. of 4's	1	0	0	0	0	1	0	0	0	1
No. of 3's	0	1	0	0	0	0	0	0	0	0
No. of 2's	0	0	0	0	0	2	4	2	3	0
No. of 1's	0	0	0	0	0	4	6	9	10	12
Minimum	4	3	5	6	7*	1	1	1	1	1
Experiment 4										
No. of 7's	12	13	14	14	15	3	1	0	0	0
No. of 6's	0	0	0	0	0	0	0	0	0	0
No. of 5's	1	0	0	1	0	0	0	0	0	0
No. of 4's	0	1	1	0	0	2	0	0	0	0
No. of 3's	0	1	0	0	0	2	0	0	0	0
No. of 2's	0	0	0	0	0	2	1	2	0	0
No. of 1's	2	0	0	0	0	6	13	13	15	15
Minimum	1	3	4	5	7*	1	1	1	1*	1*
Experiment 5										
No. of 7's	13	13	15	15	15	1	0	0	0	0
No. of 6's	0	0	0	0	0	0	0	0	0	0
No. of 5's	1	1	0	0	0	0	0	0	0	0
No. of 4's	1	1	0	0	0	0	0	0	0	0
No. of 3's	0	0	0	0	0	1	1	0	0	0
No. of 2's	0	0	0	0	0	3	4	2	2	3
No. of 1's	1	1	1	1	1	11	11	14	14	13
Minimum	1	1	1	1	1	1	1	1	1	1
Experiment 6										
No. of 7's	13	13	12	12	13	2	2	2	2	2
No. of 6's	0	1	1	1	0	0	0	0	0	0
No. of 5's	0	1	1	0	1	0	0	0	0	0

What about smaller groups?

Start with fixed pairs. What do you think will happen? Why?

What about pairs that randomly change partners. What will happen?

Results with fixed pairs:

TABLE 4—EXPERIMENTAL RESULTS FOR TREATMENT *C*: FIXED PAIRINGS

	Period						
	21	22	23	24	25	26	27
Experiment 5							
Pair 1							
Subject 1	7	7	7	7	7	7	7
Subject 16	7	7	7	7	7	7	7
Minimum	7*	7*	7*	7*	7*	7*	7*
Pair 2							
Subject 2	7	2	7	7	7	7	7
Subject 15	1	7	3	6	7	7	7
Minimum	1	2	7	7	7	7	7
Pair 3							
Subject 3	1	1	1	1	1	1	1
Subject 14	1	1	7	1	1	1	7
Minimum	1*	1*	1	1*	1*	1*	1
Pair 4							
Subject 4	1	7	7	7	7	7	7
Subject 13	7	2	5	7	7	7	7
Minimum	1	2	5	7*	7*	7*	7*
Pair 5							
Subject 5	1	7	4	7	7	7	7
Subject 12	1	4	7	7	7	7	7
Minimum	1	4	4	7*	7*	7*	7*
Pair 6							
Subject 6	5	7	7	7	7	7	7
Subject 11	7	7	7	7	7	7	7
Minimum	5	7*	7*	7*	7*	7*	7*
Pair 7							
Subject 7	1	7	6	7	7	7	7
Subject 10	5	3	6	7	7	7	7
Minimum	1	3	6*	7*	7*	7*	7*
Pair 8							
Subject 8	7	6	6	7	7	7	7
Subject 9	3	5	7	7	7	7	7
Minimum	3	5	6	7*	7*	7*	7*
Experiment 6							
Pair 1							
Subject 2	7	7	4	5	6	6	7
Subject 15	2	3	6	6	7	7	7
Minimum	2	3	4	5	6	6	7*
Pair 2							
Subject 3	5	7	7	7	7	7	7
Subject 14	7	7	7	7	7	7	7
Minimum	5	7*	7*	7*	7*	7*	7*
Pair 3							
Subject 4	1	1	1	1	4	4	1
Subject 13	7	1	1	3	1	1	2
Minimum	1	1*	1*	1	1	1	1
Pair 4							

Random pairing in the small groups:

TABLE 5—DISTRIBUTION OF ACTIONS FOR TREATMENT C: RANDOM PAIRINGS

	Period				
	21	22	23	24	25
Experiment 6					
No. of 7's	5	5	4	10	8
No. of 6's	0	1	3	0	0
No. of 5's	2	5	3	3	4
No. of 4's	3	1	1	1	1
No. of 3's	1	1	1	0	0
No. of 2's	1	1	2	2	2
No. of 1's	4	2	2	0	1
Experiment 7					
No. of 7's	–	–	6	5	5
No. of 6's	–	–	1	0	1
No. of 5's	–	–	0	3	0
No. of 4's	–	–	2	1	4
No. of 3's	–	–	2	0	0
No. of 2's	–	–	0	0	1
No. of 1's	–	–	3	5	3

Conclusion:

- Risk dominance predicts equilibrium better in the larger groups
- Payoff dominance predicts well in small groups, with reputations
- In games of $n = 2$ with random matching, risk dominance yields some to payoff dominance with experience.

Roberto Weber, “Managing Growth to Achieve Efficient Coordination in Large Groups,” *American Economic Review,* 2006, 96 (1), March, 114-126.

- Observation:
  - Van Huyck, et al., get efficient outcomes with small groups, but not large groups.
  - In the real world there are many examples of successful coordination in large groups
    - When does a small group become large?
    - Shouldn’t path dependence matter?
    - Norms?
- Idea:
  - Start people out in small groups, using the VanHuyck, et al., Stag-Hunt game, and build to large groups.
  - Can we “manage growth” to build coordination in large groups?

Background

- Use VanH's game:

TABLE 1—PAYOFFS (IN DOLLARS) FOR MINIMUM-EFFORT GAME

		Minimum choice of all players						
		7	6	5	4	3	2	1
Player's choice	7	0.90	0.70	0.50	0.30	0.10	−0.10	−0.30
	6		0.80	0.60	0.40	0.20	0.00	−0.20
	5			0.70	0.50	0.30	0.10	−0.10
	4				0.60	0.40	0.20	0.00
	3					0.50	0.30	0.10
	2						0.40	0.20
	1							0.30

- Summary of results from other experiments:

TABLE 2—DISTRIBUTIONS OF FIFTH-PERIOD GROUP MINIMA IN VARIOUS 7-ACTION MINIMUM-EFFORT STUDIES
(1 = inefficient; 7 = efficient)

Minimum choice in fifth period							Group	Number of	
7	6	5	4	3	2	1	size	groups	Source
86%	3%	3%	3%	0%	0%	5%	2	37	VHBB, CK
18%	4%	0%	11%	15%	15%	37%	3	27	KC, CK
0%	0%	0%	0%	10%	10%	80%	6	10	KC
0%	0%	0%	0%	0%	0%	100%	8	5	CSS
0%	0%	0%	0%	0%	0%	100%	9	2	CC
0%	0%	0%	0%	0%	0%	100%	14–16	7	VHBB

Sources: Van Huyck et al., 1990 (VHBB); Camerer and Knez, 2000 (CK); Knez and Camerer, 1994 (KC); Gerard P. Cachon and Camerer, 1996 (CC); Chaudhuri et al., 2001 (CSS).

Theory of Growing Groups

- Adaptive Dynamics, three types of agents
  - *incumbents*
  - *informed entrants*, who observe prior sequence of outcomes
  - *uninformed entrants* who known nothing about past moves by incumbents
- Model can show: Compare two groups of size n after T periods. The Fixed Group was always of size n, but the Grown Group reached size n over a number of periods, starting with two players and adding *informed* entrants one at a time. The Grown Group will always be at least as successful as the Fixed Group.
  - This result does not hold if Grown Groups used *uninformed* entrants.

Experiment

- 12 subjects per session
- Play 22 periods
- Three conditions
  - Control: All 12 play every period
  - Growth: group grows from size 2 to 12
    - History: Told the history
    - No-History: told nada, zip, zilch
      - As subjects waited, they were told they would receive a “fixed fair amount” for each period they waited.
      - No-history subjects were kept in another room.
- Subjects wrote 1...7 on paper, and the min of these was written on the board.
  - Subjects could record this, then calculate own payoff.
  - number erased and next round begun.

Results

Baseline:

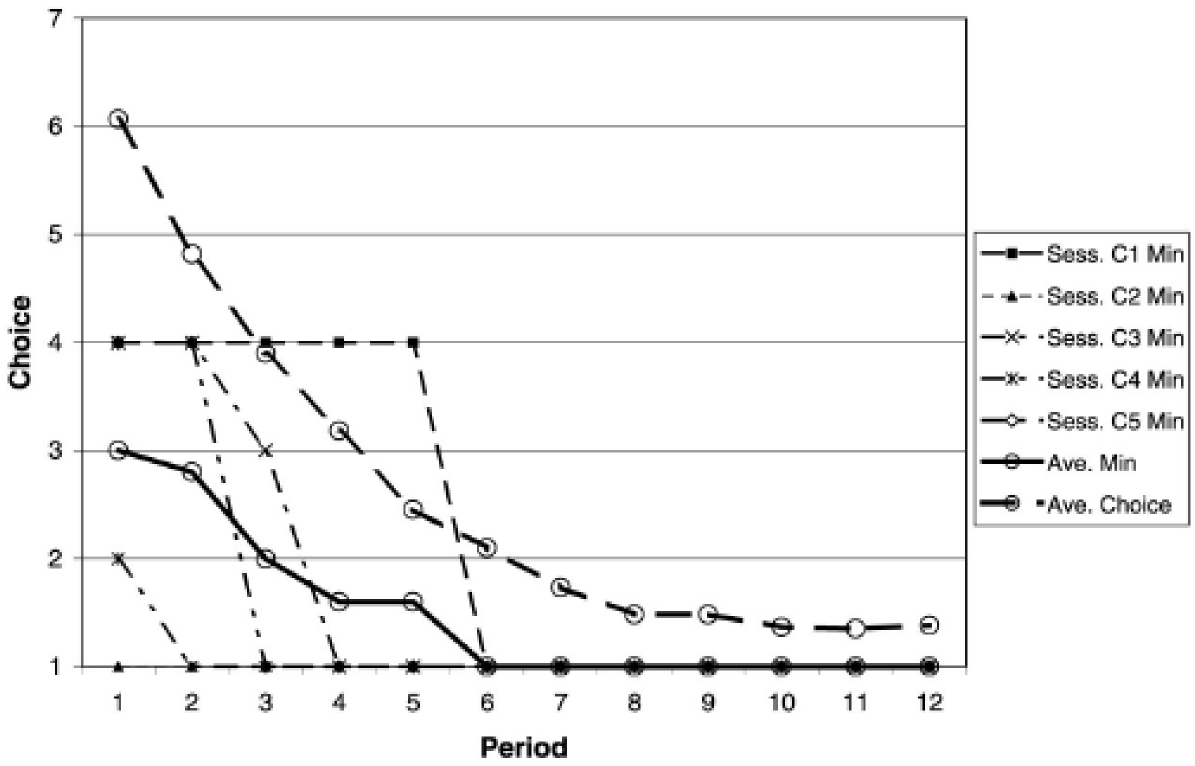

FIGURE 1. PERIOD MINIMA IN CONTROL SESSIONS

Growth Paths:

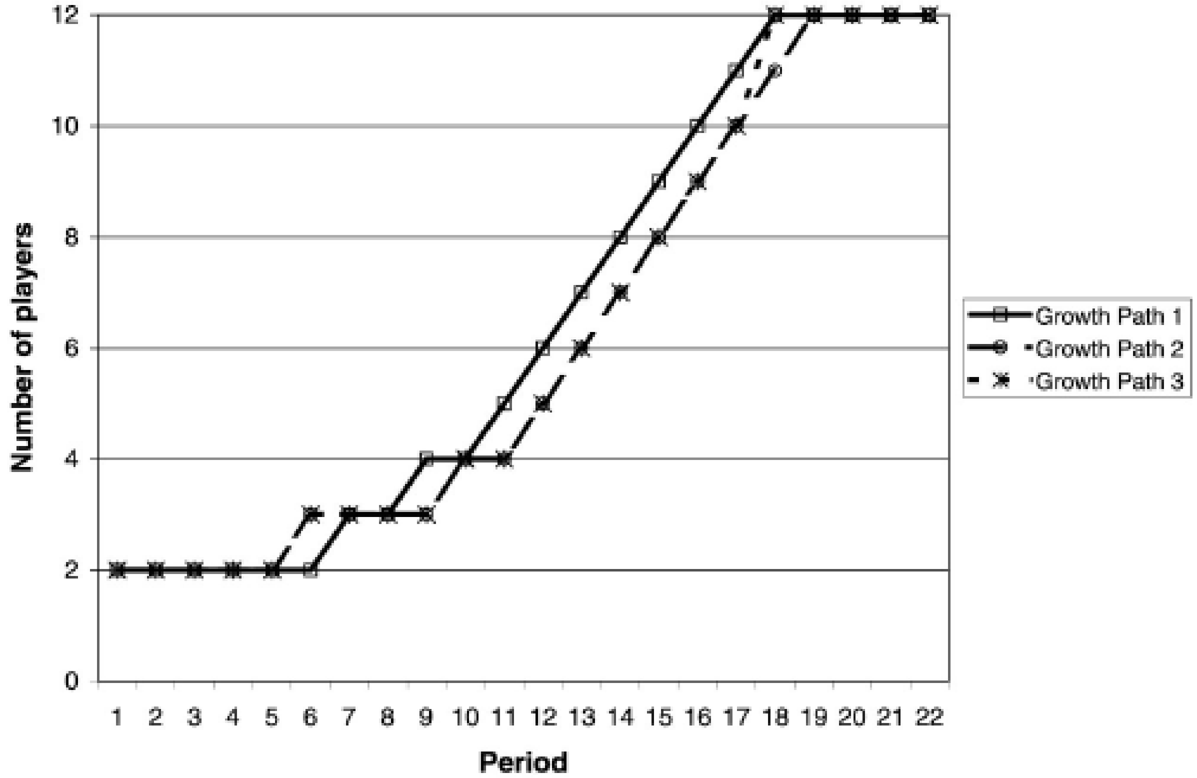

FIGURE 2. PREDETERMINED GROWTH PATHS

Growth Sessions:

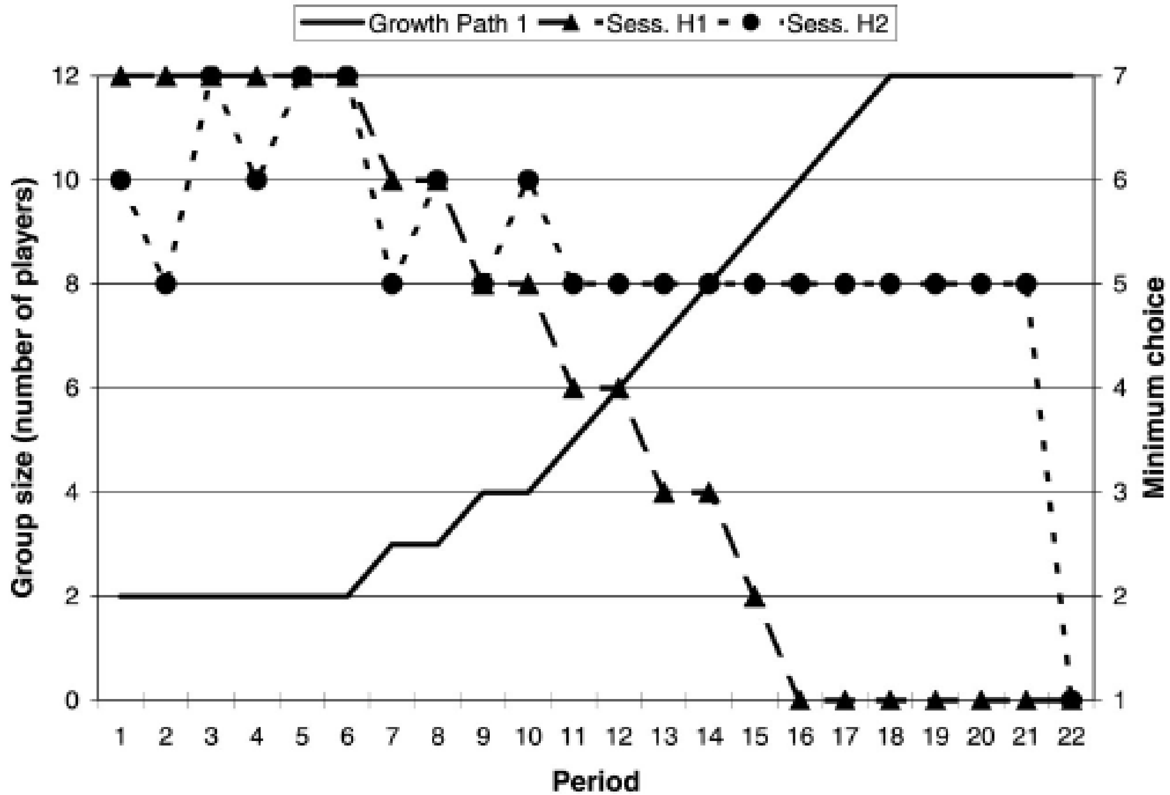

FIGURE 3A. GROWTH PATH 1 AND PERIOD MINIMA FOR SESSIONS 1 AND 2 (HISTORY)

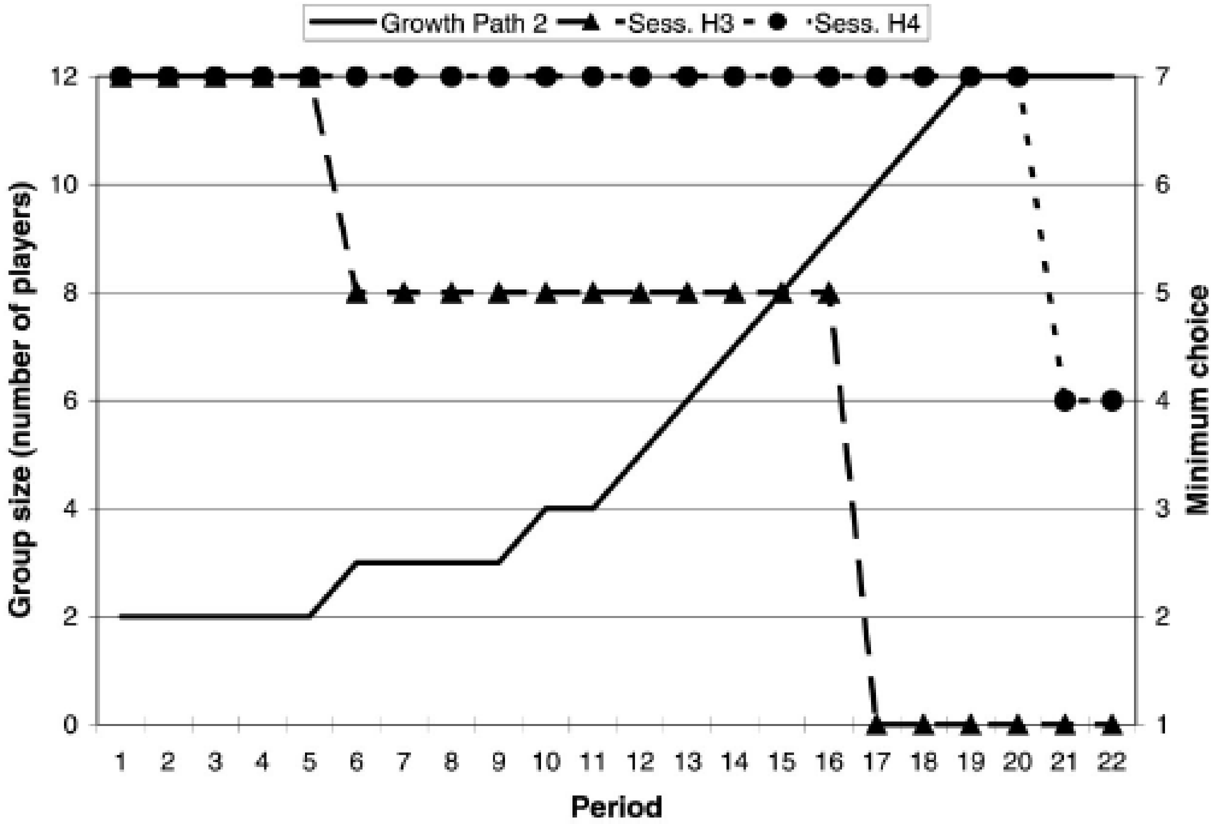

FIGURE 3B. GROWTH PATH 2 AND PERIOD MINIMA FOR SESSIONS 3 AND 4 (HISTORY)

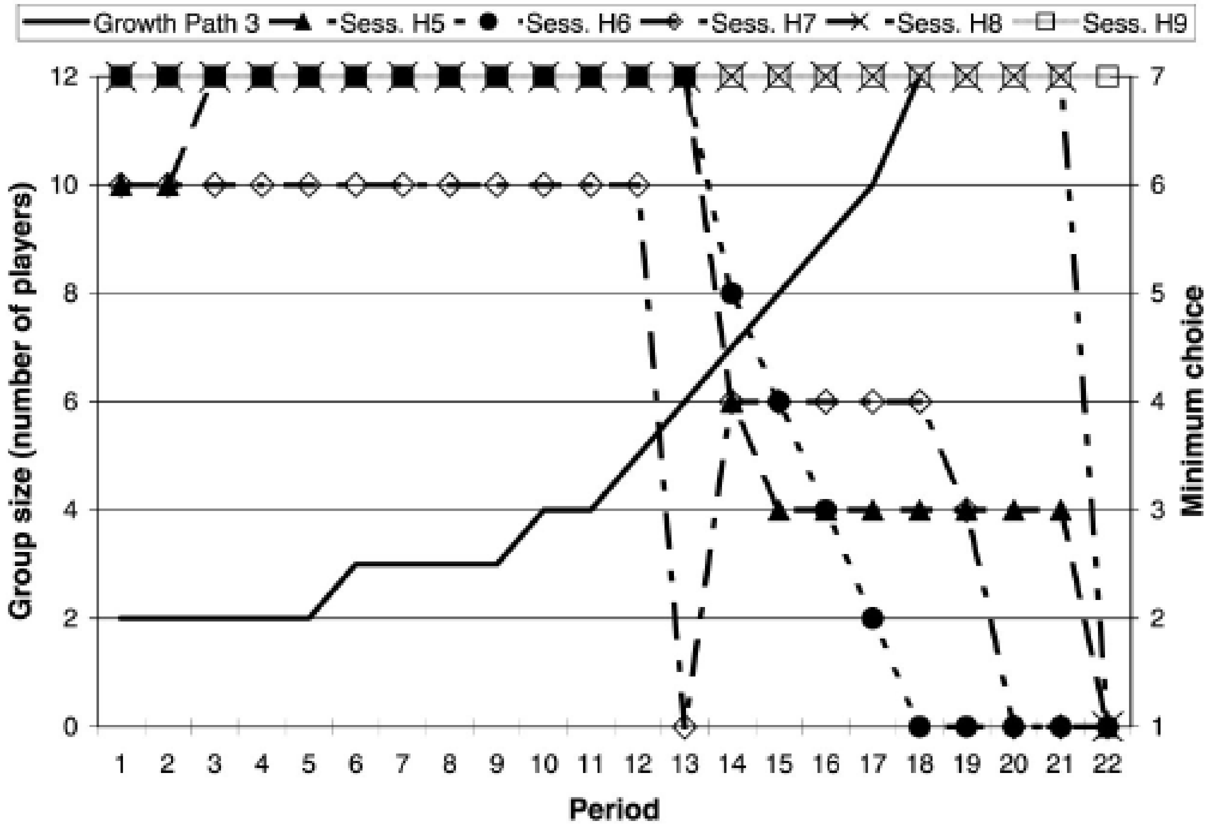

FIGURE 3C. GROWTH PATH 3 AND PERIOD MINIMA FOR SESSIONS 5 THROUGH 9 (HISTORY)

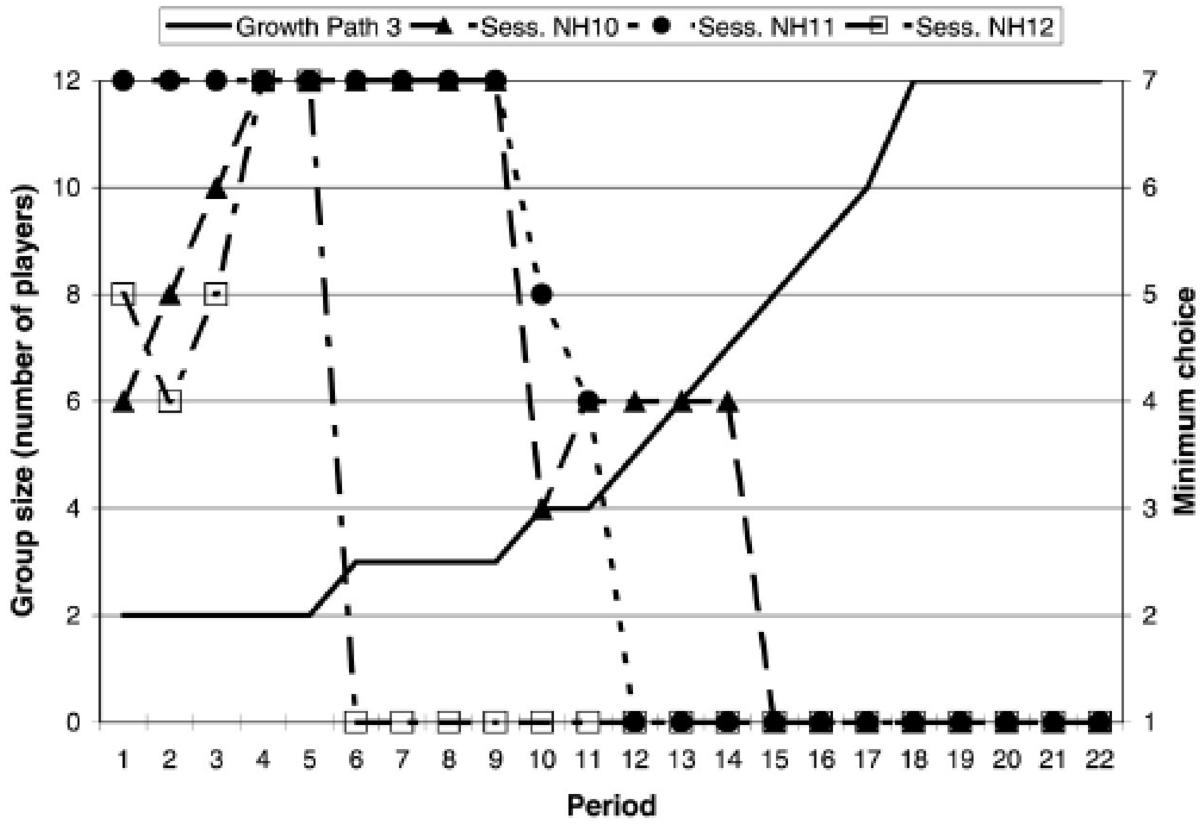

FIGURE 3D. GROWTH PATH 3 AND PERIOD MINIMA FOR SESSIONS 10 THROUGH 12 (NO HISTORY)

TABLE 3—AVERAGE MINIMA (MEDIANS) BY SESSION FOR RANGES OF GROUP SIZE

	Growth path 1: Group size (number of periods at that size)					First n = 12 minimum
	2 (6)	3 (2)	4–6 (4)	7–11 (5)	12 (5)	
Session 1 (h)	7.0 (7)	6.0 (6)	4.5 (4.5)	2.0 (2)	1.0 (1)	1
Session 2 (h)	6.3 (6.5)	5.5 (5.5)	5.3 (5)	5.0 (5)	4.2 (5)	5
	Growth path 2: Group size (number of periods at that size)					
	2 (5)	3 (4)	4–6 (4)	7–11 (5)	12 (4)	
Session 3 (h)	7.0 (7)	5.0 (5)	5.0 (5)	3.4 (5)	1.0 (1)	1
Session 4 (h)	7.0 (7)	7.0 (7)	7.0 (7)	7.0 (7)	5.5 (5.5)	7
	Growth path 3: Group size (number of periods at that size)					
	2 (5)	3 (4)	4–6 (4)	7–10 (4)	12 (5)	
Session 5 (h)	6.6 (7)	7.0 (7)	7.0 (7)	3.3 (3)	2.6 (3)	3
Session 6 (h)	7.0 (7)	7.0 (7)	7.0 (7)	3.5 (3.5)	1.0 (1)	1
Session 7 (h)	6.0 (6)	6.0 (6)	4.8 (6)	4.0 (4)	2.0 (1)	4
Session 8 (h)	7.0 (7)	7.0 (7)	7.0 (7)	7.0 (7)	5.8 (7)	7
Session 9 (h)	7.0 (7)	7.0 (7)	7.0 (7)	7.0 (7)	7.0 (7)	7
Session 10 (nh)	5.8 (6)	7.0 (7)	3.8 (4)	1.8 (1)	1.0 (1)	1
Session 11 (nh)	7.0 (7)	7.0 (7)	2.8 (2.5)	1.0 (1)	1.0 (1)	1
Session 12 (nh)	5.6 (6)	1.0 (1)	1.0 (1)	1.0 (1)	1.0 (1)	1

TABLE 4—DISTRIBUTIONS OF SUBJECT CHOICES IN FOURTH PERIOD AS 12-PERSON GROUPS

		Control	Growth and history	Growth and no history
Choice	7	3 (5%)	32 (30%)	0 (0%)
	6	0 (0%)	3 (3%)	0 (0%)
	5	6 (10%)	13 (12%)	0 (0%)
	4	22 (37%)	8 (7%)	0 (0%)
	3	7 (12%)	10 (9%)	0 (0%)
	2	9 (15%)	9 (8%)	1 (3%)
	1	13 (22%)	33 (31%)	35 (97%)
Total		60	108	36
Minima		1, 1, 1, 1, 4	1, 1, 1, 1, 3, 4, 5, 7, 7	1, 1, 1

Conclusion

- Growth can alleviate coordination problems.
  - But information is essential.
  - No prior experiments showed any min $>$ 1 after repeated play in large groups.
- Interesting “norm” developed in some groups:
  - when someone is added, the group min should fall by 1.
  - indicates higher order thinking in these games