Information acquisition and self-selection in coordination games*

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Abstract

We study experimentally costly information acquisition in coordination games with incomplete information (global games). We identify a novel behavioral channel in the form of self-selection where individual information choices influence subsequent behavior in the game. Subjects who self-select by choosing a higher (lower) precision set lower (higher) thresholds than those predicted by the theory (threshold-level effect). Moreover, the behavior of higher precision types is more stable and predictable than lower precision types, both individually and within pairs (threshold-quality effect). We provide evidence in support of the self-selection mechanism in initial rounds of the experiment. This behavior has significant welfare effects.

Key words: information acquisition, global games, experiments, self-selection.

JEL codes: C72, C90, D82

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1 Introduction

In many economic situations decision makers spend a considerable amount of time and resources to acquire additional information in order to improve their decision-making. In coordination environments these actions, while desirable from an individual perspective, may have welfare-reducing consequences because individuals do not necessarily take into account that their information choices, by influencing their subsequent actions, also affect aggregate outcomes.[1] The importance of these informational externalities has increased in recent decades due to the growing abundance of information.[2] These considerations have motivated an extensive theoretical literature that analyzes private information acquisition and its impact on economic outcomes in coordination settings (e.g., Colombo et al. (2014), Myatt and Wallace (2015), Szkup and Trevino (2015), or Yang (2015)). This literature provides a detailed characterization of optimal information choices and efficiency in a large number of economic settings, with Gaussian-quadratic games (as in Morris and Shin (2002)) and global games (as in Morris and Shin (1998)) being the prime examples. However, there is little empirical understanding of the forces and predictions suggested by the theory. This is mainly due to the difficulty of observing agents' information sets in naturally occurring economic situations.

This paper attempts to fill this gap and provide empirical evidence by studying experimentally costly information acquisition in a coordination game. Economic experiments are a unique tool to study information processing questions as they provide a controlled environment where the researcher can observe the information sets of agents and estimate how actions and outcomes depend on them. For our theoretical benchmark we take the standard global game of regime change (see Morris and Shin (1998, 2003)) and extend it to feature an initial stage of costly information acquisition (as in Szkup and Trevino (2015)), but with a finite number of players in order to serve as a benchmark for the experiment. We characterize the equilibrium in this game and test its theoretical predictions in the laboratory.[3] Our empirical analysis reveals three features that are not captured by the theory. First, subjects who acquire more precise information tend to exhibit a higher quality of play (cleaner and more stable individual strategies). Second, more precise information leads to more risky coordination. Third, subjects who acquire more precise information than what is prescribed in equilibrium on average earn higher payoffs than in equilibrium. We argue that these departures from the theory are driven, at least partly, by self-selection via costly information acquisition. Subjects exhibit substantial heterogeneity in their information choices. In par-

[1] Examples of coordination settings with spillover effects include price-setting decisions in monopolistically competitive settings (Hellwig and Veldkamp (2009)), investment choices in the presence of positive demand spillovers (Angeletos and Pavan (2004)), financing and rollover decisions (Morris and Shin (2004)), or political revolts (Edmond (2013)). For additional examples, see Cooper (1998) and Veldkamp (2011).

[2] See Goldfarb and Tucker (2019) or Chung and Veldkamp (2019) for a discussion on the increasing abundance of information and its falling cost in economic settings.

[3] While our theoretical model fits into the class of global games, our solution method departs from the typical techniques used in the literature due to having a finite number of players with potentially asymmetric noise distributions. Our theoretical equilibrium characterization is therefore a novel contribution.

ticular, approximately one third of subjects over-acquire information, one third chooses the equilibrium information level, and one third under-acquire information, and they do so persistently. We argue that as a result of self-selection, subjects who choose the highest precision use more efficient, cleaner, and more stable strategies that lead to higher payoffs than the equilibrium predictions, while the opposite is true for subjects who choose the cheapest, least precise information.

In particular, the heterogeneity in information choices translates into heterogeneity in the quality of play. We find that all subjects who choose a high precision of information use threshold strategies, which are the optimal type of strategies in global games. Moreover, the thresholds in this group are clean and predictable with low dispersion. The use of threshold strategies for subjects who acquire the equilibrium level of information (medium precision) decreases slightly to 93.75%. We see a significant decrease for subjects who choose the lowest precision since 75% of them use threshold strategies, while the remaining 25% choose a fixed action that does not depend on the signal they observe. We also observe a large dispersion of thresholds in the low precision group and less predictability overall, opposite to what we find for the high precision subjects. For comparison, experiments on global games with exogenous information structures typically find that about 90% of subjects use threshold strategies and find no differences in dispersion of thresholds across different precisions (see Heinemann et al. (2004) or Szkup and Trevino (2020)). Finally, using the strategy method, we find that subjects who acquire the most precise information tend to adjust their individual thresholds minimally from round to round and they set very similar thresholds to their opponent, while the opposite is true for subjects who choose noisier information. We refer to this as the threshold-quality effect associated to selecting different levels of precision of information.

The heterogeneity in information choices also translates into heterogeneity in the propensity and success of coordination for pairs that converge in their information choices. Subjects who select a high precision seek to coordinate more often by setting thresholds that are lower than the equilibrium threshold and closer to the efficient threshold. In contrast, subjects who acquire low precision set higher, less efficient thresholds than those predicted by the theory. This is in stark contrast to the theoretical predictions of the model where thresholds should be increasing in precision. We refer to this departure from the theory as the threshold-level effect. The presence of the threshold-level effect is consistent with the findings of Szkup and Trevino (2020) of non-increasing thresholds for subjects who are exogenously endowed with either a high, medium, or low precision. However, our result is starker since we find a decreasing and more pronounced relationship between thresholds and precision choices.

The starker reversal of comparative statics and clear heterogeneity in subjects' behavior that is related to their information choices (and illustrated by the threshold-level and threshold-quality effects) suggests a previously unexplored self-selection mechanism that takes place through the choice of information. Subjects that are more inclined to take the risky action in order to extract a higher payoff whenever possible might be more "invested" in the game and value information more than those subjects who might prefer to take a more conservative approach in the game or to not engage in the game as much. As a consequence, these subjects acquire better information and invest more often using cleaner, more predictable strategies than those subjects who are willing to forgo investment opportunities and save money on information. Since purchasing a high precision makes their signals very accurate, subjects who acquire precise signals are less likely to make the mistake of investing when coordinated investment is not profitable. As a consequence, they earn a higher payoff.

The self-selection mechanism is also manifested in the stability of individual thresholds and on the coordination of thresholds within pairs. Subjects in pairs that choose a high precision set extremely stable thresholds over time and exhibit a high degree of convergence in thresholds with their pair member. The individual stability of thresholds and convergence of thresholds within pairs is substantially reduced as subjects choose lower precisions. We find support for the self-selection mechanism from the early rounds of the experiment by showing that precision choices are heterogeneous and individually stable from the very beginning, meaning that subjects who start the experiment choosing high, medium, or low precisions continue to do so throughout the experiment. This implies a lack of experimentation and learning with precision choices and is consistent with the notion of subjects self-selecting early on as high, medium, or low precision types. When studying how different precision types play the game in the early rounds of the experiment we find further evidence for self-selection since high precision types show a stronger response to their signal than medium and low types. Moreover, individual thresholds for high types stabilize within the first 10 rounds, in contrast to the continued variation that we observe for medium and, in particular, low precision types.

Finally, we show that self-selection via information choices has significant welfare effects. In particular, the subjects who self-select into choosing better information see welfare gains with respect to equilibrium play, a result of setting low thresholds and exhibiting very predictable behavior that facilitates successful coordination. That is, their more aggressive and accurate behavior in the game leads to welfare gains that offset the higher cost of information. On the other hand, subjects who self-select into low precisions see large welfare losses with respect to equilibrium, a result of setting high thresholds and exhibiting more erratic behavior. As a consequence, these subjects experience welfare losses despite saving on information costs. These empirical observations further underscore the importance of self-selection mechanisms that is not captured by the theory.

Related Literature — Our paper contributes to different strands of the literature on global games and, more broadly, on coordination games. Heinemann, Nagel, and Ockenfels (2004) were the first to test the predictions of the global games model of Morris and Shin (1998) experimentally. Other early work in this literature includes Cabrales, Nagel, and Armanter (2007), who study whether subjects play risk-dominant strategies in global games and Duffy and Ochs (2012), who investigate experimentally how static and dynamic entry games differ from each other and from global games. Darai, Kogan, Kwasnica, and Weber (2017) and Avoyan (2019) study how different types of public signals and different types of communication, respectively, affect outcomes in experimental global games and Trevino (2020) studies informational contagion in a global games setting. The closest paper to ours is Szkup and Trevino (2020) who analyze how exogenous changes in information precision affect behavior in global games. They show that comparative statics are reversed compared to the theoretical predictions. In this paper, we consider endogenous information structures where subjects can chose the precision of their signal, at a cost. Thus, we are able to investigate how self-selection via information acquisition affects subjects' play. The results of Szkup and Trevino (2020) can be viewed as a natural control treatment for this paper. By comparing our results to theirs we can better understand the effects of endogenizing information, for different levels of precision.

Our paper is also related to the experimental literature on beauty contest models with incomplete information. This literature initially focused on understanding how exogenous changes in the information structure affect welfare (see, e.g., Cornand and Heinemann (2014, 2015), or Baeriswyl and Cornand (2014)). More recently Baeriswyl and Cornand (2021) and Szkup and Trevino (2022) analyzed experimentally information acquisition in that setting and found that subjects generally pay less attention to most common and least private signals and they tend to overacquire and overuse private information, respectively. Finally, our paper contributes to the broader literature that studies how strategic uncertainty affects subjects' play in coordination games (see Van Huyck, Battalio, and Beil (1990, 1991), Cooper, DeJong, Forsythe, and Ross (1990, 1992), or Straub (1995), among many others).

Clearly, our paper is closely related to the theoretical literature on global games. Global games, first introduced by Carlsson and van Damme (1993) and popularized by Morris and Shin (1998, 2003), have been widely studied theoretically. Exogenous changes in the information structure were studied theoretically by Iachan and Nenov (2014), Metz (2002), and Shadmehr and Bernhardt (2011) among others. Szkup and Trevino (2015) study costly information acquisition in a global game with a continuum of players and our theoretical model is a two-player version of their setting. Also related is Yang (2015) who considers flexible information acquisition in global games.

2 The Model

In this section we describe and solve the theoretical model that serves as a benchmark for our experiment. The model consists of two stages. In the first stage, which we call the information acquisition stage, players choose the precision of the private signal they receive at the beginning of the second stage. In the second stage, players observe their signals and play a standard global game (see Carlsson and van Damme (1993), Morris and Shin (2003), or Szkup and Trevino (2020)), with endogenously chosen precisions. Below, we follow Morris and Shin (2003) and interpret our model as an investment game where players have the opportunity to acquire additional information about the potential returns to investment before making the binary decision of investing in a project or not.[4] The return to investment depends on fundamentals and on aggregate investment, so the second stage of the game is modeled as a coordination game. In what follows, we refer to the second stage as the investment stage.

2.1 The setup

There are two identical investors, $i \in \{1, 2\}$, who choose privately the quality of their information (information acquisition stage) and then decide whether to invest in the project or not (investment stage). Investment is risky and if a player chooses to invest he has to pay a cost T and gets payoff θ if the investment is successful. We refer to θ as the return from investment. The investment is successful if either $\theta \geq \underline{\theta}$ and the other player also invests, or if $\theta \geq \overline{\theta}$, regardless of the action of the other player. Otherwise, the investment fails and the investor earns no return.[5] The payoff from not investing is certain and normalized to 0. Thus, players face the following payoffs:

	Success	Failure
Invest	$\theta - T$	$-T$
Not Invest	0	0

Note that for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right)$ investors would like to coordinate their actions. We further assume that $0 \leq \underline{\theta} < 2T < \overline{\theta}$, which implies that the risk dominant equilibrium of the complete information game (as defined by Harsanyi and Selten (1988)) belongs to the coordination region $\left[\underline{\theta}, \overline{\theta}\right)$.

The return θ is a random variable and follows a normal distribution with mean μ_θ and variance σ_θ^2, that is $\theta \sim N\left(\mu_\theta, \sigma_\theta^2\right)$. Investors do not observe θ. Instead, investor i observes a noisy private signal x_i about the realization of θ given by

$$x_i = \theta + \sigma_i \varepsilon_i,$$

where $\sigma_i > 0$ and $\varepsilon_i \sim N(0, 1)$ is an idiosyncratic noise, *i.i.d.* across investors, and independent of the realization of θ. The precision of the private signal, determined by σ_i, is player-specific and it is the vehicle by which investors choose the quality of their information.[6] After observing their respective private signals, investors simultaneously make their investment decisions. Once investment decisions are made, payoffs are realized and the game

[4] As in the global games literature, the second stage of our model admits other interpretations, such as speculative attacks against currencies (Morris and Shin (1998)), political revolts (Edmond (2013)), debt roll over decisions (Morris and Shin (2004)), or technology adoption (Frankel and Pauzner (2000)).

[5] That the success of investment depends on the action of the other player captures complementarities that are often present in investment decisions. See, e.g., Dasgupta (2007) for a detailed discussion.

[6] Recall that the precision of a normally distributed random variable is defined as the inverse of its variance. In our theoretical analysis, we use the following terms to refer to the same situation: higher precision, lower variance, or lower standard deviation.

concludes.

In the first stage of the game, each investor decides how much information about θ to acquire by choosing the precision of their private signal. This means that, starting from an initial common standard deviation $\underline{\sigma}$, investors can either keep this level of noise or improve their precision by "buying" a lower σ_i at a cost $C(\sigma_i)$. We assume feasible choices of $\sigma_i \in [0, \underline{\sigma}]$ and that the cost function $C(\cdot)$ is continuous, with $C(\underline{\sigma}) = 0$, $C'(\underline{\sigma}) = 0$, $C'(\sigma) < 0$, $C''(\sigma) > 0$, for all $\sigma \in (0, \underline{\sigma})$, and $\lim_{\sigma \to 0} C'(\sigma) = \infty$, which are standard assumptions in the literature; see, for example, Colombo et al. (2014). Precision choices are private and hence not observed by the other player.

We solve the model by backward induction and focus on pure strategy Perfect Bayesian Equilibria. The restriction to pure strategies implies that agents have non-degenerate beliefs about each others' precision choices. We first consider the investment stage game assuming an exogenous profile of information choice and then study incentives for unilateral deviation. Precision choice vector $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2\}$ corresponds to equilibrium precision choices if there are no incentives to deviate from the prescribed profile of information choice. This approach is standard (including the focus on pure-strategy equilibria) and follows Hauk and Hurkens (2001) and Amir and Lazzati (2016).

2.2 Equilibrium: Investment Stage

At the beginning of the second stage, each player observes his own signal $x_i = \theta + \sigma_i \varepsilon_i$, where σ_i corresponds to player i's choice of precision in the first stage, which we take as given. Note that the heterogeneity of signal precision differentiates our investment stage from standard global games. That is, the second stage of our game can be thought of as a standard 2×2 global game with the notable difference that players are potentially heterogeneous with respect to the precision of their signals.[7]

Let $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2\}$ be the vector of investors' precision choices. As is standard in global games, we focus on threshold strategies that determine each player's action choice (denoted by $a(x_i; \boldsymbol{\sigma})$), given observed signal x_i and precision choices $\boldsymbol{\sigma}$:

$$a(x_i; \boldsymbol{\sigma}) = \begin{cases} 1 \text{ (invest)} & \text{iff } \; x_i \geq x_i^*(\boldsymbol{\sigma}) \\ 0 \text{ (not invest)} & \text{iff } \; x_i < x_i^*(\boldsymbol{\sigma}) \end{cases}$$

That is, player i chooses to invest if and only if his signal is greater than a threshold x_i^* and does not invest otherwise. The optimal threshold x_i^* is the value of the signal for which investor i is indifferent between investing and not investing. That is, x_i^* satisfies the following equation

$$\mathbb{E}\Big[\theta \Pr\left(x_j \geq x_j^* | \theta\right) \Big| x_i^*, \theta \in \big[\underline{\theta}, \overline{\theta}\big]\Big] + \mathbb{E}\Big[\theta \Big| x_i^*, \theta > \overline{\theta}\Big] = T \tag{1}$$

[7]Most of the global games literature focuses on models with a continuum of players that receive signals with the same precision (see Morris and Shin (2003) and Angeletos and Lian (2018) for reviews of this literature). The notable exception is Yang (2015) who considers flexible information choices in a closely related coordination game with finitely many players.

The RHS of Equation (1) captures the cost of investing. The LHS captures the expected benefit from investing, conditional on observing signal x_i^*, taking into account that investment is successful if either $\theta > \overline{\theta}$ or if $\theta \geq \underline{\theta}$ and the other player invests (which happens when $x_j \geq x_j^*$). The next proposition shows that for any feasible choice of precisions, the investment stage of our game has a unique, dominance solvable equilibrium.

Proposition 1 *There exists $\overline{\sigma}_\theta > 0$ such that for all $\sigma_\theta > \overline{\sigma}_\theta$ and all $\sigma_i \leq \underline{\sigma}$, $i \in \{1,2\}$, the investment stage has a unique, dominance solvable equilibrium in which players use threshold strategies $\{x_1^*(\boldsymbol{\sigma}), x_2^*(\boldsymbol{\sigma})\}$. Moreover, as $\sigma_i, \sigma_j \searrow 0$, with $\frac{\sigma_i}{\sigma_j} \to c \in \mathbb{R}_+$, this equilibrium converges to the risk-dominant equilibrium of the complete information game (that is, $x_i^*(\sigma) \to 2T$, $i \in \{1,2\}$).*

The first part of Proposition 1 establishes that if the standard deviation of the prior is sufficiently large then the investment stage has a unique dominance solvable equilibrium, regardless of the investors' precision choices. That equilibrium is unique only if the public information is noisy enough is a standard result in the global game literature (see Hellwig (2002) and Morris and Shin (2003, 2004) for a detailed discussion). However, because our game features two asymmetric players who have unbounded utilities, to establish this result we take a different approach from the literature and rely instead on tools of monotone supermodular games.[8]

The second part of Proposition 1 shows that as the noise in private signals vanishes, the equilibrium converges to the risk-dominant equilibrium of the complete information game, regardless of the way this limit is approached (i.e., for any $\frac{\sigma_i}{\sigma_j} \to c$, $c \in \mathbb{R}_+$). This extends the result of Carlsson and van Damme (1993) to games with asymmetric noise structures.

2.3 Equilibrium: Information acquisition

Recall that in the information acquisition stage investors privately choose the noisiness of their signal, $\sigma_i \in [0, \underline{\sigma}]$. If an investor chooses not to acquire information he will observe a signal with a default precision $\underline{\sigma}$. The expected utility of investor i, whose signal has standard deviation σ_i, and who believes that his opponent's signal has standard deviation σ_j and that his opponent holds correct beliefs about his own choice of σ_i, is given by:

$$U_i(\sigma_i, \sigma_j) = \mathbb{E}\left[\mathbf{1}_{\{x_i \geq x_i^*(\boldsymbol{\sigma})\}} v_i(x_i, x_j^*(\boldsymbol{\sigma}); \boldsymbol{\sigma})\right] f(x_i; \sigma)\, dx_i - C(\sigma_i) \tag{2}$$

where $v_i(\cdot)$ is the expected payoff from investing when agent i observes signal x_i and agent j uses threshold $x_j^*(\boldsymbol{\sigma})$, $f(x_i; \sigma)$ is the PDF of x_i given agent i's precision choice, $C(\sigma_i)$ is the

[8] In particular, in the appendix we extend the result of Vives and van Zandt (2007) to prove the existence of a least and a greatest Bayesian Nash Equilibrium in monotone strategies in games with unbounded but integrable utility functions. We then show that the equilibrium conditions that determine thresholds define a univalent mapping, implying uniqueness of a monotone equilibrium and dominance solvability of the investment game.

cost associated with investor i's choice of precision, and $\mathbf{1}_{\{x_i \geq x_i^*(\boldsymbol{\sigma})\}}$ is an indicator function that takes value 1 if $x_i \geq x_i^*(\boldsymbol{\sigma})$.

Definition 1 (Equilibrium) *A pure strategy Bayesian Nash Equilibrium is a pair of information choices (σ_i^*, σ_j^*), optimal decision rules for the investment stage $a_i(x_i; \boldsymbol{\sigma})$, and belief functions $\mu_i : [0, \underline{\sigma}] \to [0, 1]$ such that for each $i = 1, 2$ we have:*

- $U_i(\sigma_i^*, \sigma_j^*) \geq U_i(\sigma_i, \sigma_j^*) \quad \forall \sigma_i \in [0, \underline{\sigma}]$,
- *The belief function μ_i satisfies $\mu_i(\sigma_j^*) = 1$ and $\mu_i(\sigma_j) = 0$ for $\sigma_j \neq \sigma_j^*$,*
- *For $i = 1, 2$, given the belief function μ_i, player i's action rule is given by*

$$a_i(x_i; \boldsymbol{\sigma}) = \begin{cases} 1 & \text{if } x_i \geq x_i^*(\sigma_i, \sigma_j^*) \\ 0 & \text{if } x_i < x_i^*(\sigma_i, \sigma_j^*) \end{cases}$$

where $x_i^*(\boldsymbol{\sigma})$ solves

$$v(x_i^*(\sigma_i, \sigma_j^*), x_j^*(\sigma_i^*, \sigma_j^*); \boldsymbol{\sigma}) = 0$$

The first condition is an optimality condition that requires player i to have no incentives to deviate from his equilibrium precision choice. Note, however, that since each player's precision choice is private, when considering deviations by one of them we keep the strategy and beliefs of the other one constant. The second condition is a restriction on the belief function, namely, that a player assigns positive probability only to the actual equilibrium choice of his opponent. Finally, the last condition requires each player to act optimally in the investment stage, even after unilateral deviations in the information acquisition stage.

Proposition 2 *There exists a pure-strategy Bayesian Nash Equilibrium of the investment game with information acquisition. In any equilibrium, both investors choose to acquire information (i.e., $\sigma_i < \underline{\sigma}$, $i \in \{1, 2\}$).*

Proposition 2 establishes existence of an equilibrium where players acquire information. Unfortunately, in contrast to the coordination games with a continuum of players that feature information acquisition (e.g., Colombo et al. (2014) or Szkup and Trevino (2015)), the fact that investors are non-atomic and take into consideration the impact of their own actions on their opponent's behavior hinders the analytical characterization of equilibria and comparative statics for a wide range of parameters.

Below we derive theoretical predictions for the game we implement in the experiment.

2.4 Theoretical predictions for the experiment

The theoretical model is governed by a set of parameters $\Theta = \left\{\mu_\theta, \sigma_\theta, (\underline{\theta}, \overline{\theta}), T, \{\sigma_i\}, \{C(\sigma_i)\}\right\}$. We choose the following parameters for our experiment:

$$\Theta = \{50, 50, (0, 100), 18, \{1, 3, 6, 10, 16, 20\}, \{6, 5, 4, 2, 1.5, 1\}\}$$

In particular:

- The fundamental θ is randomly drawn from a normal distribution with mean 50 and standard deviation of 50.
- The coordination region is for values of $\theta \in [0, 100)$.
- The cost of taking the risky action is $T = 18$.
- A discrete choice set of precisions and the cost associated with each precision is presented in the form of a menu of six precision levels, standard deviations, and costs:[9]

Precision level	Standard deviation	Cost
1	1	6
2	3	5
3	6	4
4	10	2
5	16	1.5
6	20	1

Table 1: Precision choices

We decided not to have a default precision chosen for subjects in order to avoid status quo biases. The reason to introduce a discrete choice set for precisions was to simplify the choice for subjects and the data analysis. We believe six is a reasonable number of options to observe dynamics in the level of informativeness that subjects choose, without losing statistical power.

Given these parametric assumptions we can characterize the predictions of the model for our experiment. The model predicts that subjects use threshold strategies in the second stage of the game, for any precision choices in a pair. The unique equilibrium prediction is that subjects coordinate on a precision level of 4 and set a symmetric threshold of 28.31. For non-equilibrium precision choices, the model predicts that thresholds are increasing in individual precision choices. We have chosen parameters such that the mean of the prior is high with respect to the cost of investing to ensure that the effect of precisions on thresholds does not depend on the precision of the prior (see Szkup and Trevino (2015)). Implicit in this prediction is that precision choices are strategic complements, which leads players to coordinate on both precisions and actions. As an additional feature of the model, note that the equilibrium prediction in the limit, as signal noise vanishes, is equal to $2T = 36$, which corresponds to the risk dominant equilibrium of the underlying complete information game.

[9] In the experimental part of the paper we refer to information choices as precision choices to be consistent with the language used in the implementation of the experiment. We use the term precision as a qualitative measure of informativeness of the signals, i.e., we compare levels of precision, and not magnitudes of standard deviations.

Previous experimental evidence shows that subjects coordinate on this limiting threshold when playing a global game without information acquisition (see Heinemann et al. (2004)).[10]

3 The experiment

We present results of a series of laboratory experiments designed to test the implications of the model from Section 2. The experiment was conducted at the Center for Experimental Social Science at New York University using the usual computerized recruiting procedures. Each session lasted 90 minutes and subjects earned on average $25. All subjects were undergraduate students from New York University.[11]

Our experimental design is related to the work of Heinemann et al. (2004), who test the predictions of the model of Morris and Shin (1998) in the laboratory and find that, on average, 92% of the strategies observed are consistent with the use of undominated thresholds that coincide with the theoretical prediction of equilibrium in the limit, as noise vanishes. Our experiment is closely related to Szkup and Trevino (2020) who used the same coordination game to study how subjects behave in an environment with different levels of exogenously given precisions. There are no studies, that we are aware of, that introduce costly information acquisition into a global games model and test the predictions experimentally.

3.1 Experimental design

We implement a between subjects design that allows us to directly compare the behavior of subjects across treatments. Each session consists of 50 independent rounds.

There are two main treatments. In the first treatment subjects play the investment game with costly information acquisition as in Section 2, where subjects choose from a set of discrete precisions with no default option, as described in Table 1, then receive a signal, and choose an action. We refer to this treatment as the treatment with Direct Action choice (DA), for which we have 60 subjects. Consistent with previous literature we find widespread use of threshold strategies in our DA treatment, so we design the second treatment with the purpose of studying the evolution of thresholds across rounds. In this treatment subjects play the game with information acquisition as in the DA treatment but we use the strategy method to elicit thresholds in the investment stage. That is, before observing their signal, subjects have to choose a cutoff value such that they would take the risky investment action if their signal was higher than this cutoff and choose the safe action of not investing if their signal was lower than the cutoff they report. Hence, this treatment allows us to observe thresholds directly, rather than infer them as in the DA treatment.[12] We refer to this as

[10] Note that in our case with 2 players the risk dominant equilibrium coincides with the prediction of global games in the limit, as the noise vanishes. Moreover, there is experimental evidence that shows that the risk dominant equilibrium is often selected in 2×2 coordination games (see Cabrales et al. (2000)).

[11] Instructions for all treatments can be found at http://econweb.ucsd.edu/~itrevino/pdfs/instructions_st_endogenous.pdf.

[12] This feature of our treatment is related to the study of Duffy and Ochs (2011) who use the strategy method to elicit thresholds in coordination games. See Brandts and Charness (2011) for a survey comparing

the Strategy Method treatment (SM), for which we had 44 subjects. We had in total 104 subjects.

Our design follows closely the design and parametrization of Szkup and Trevino (2020). In their treatments subjects play only the second stage of the game with a symmetric and exogenously set precision for their private signals, therefore they are natural control treatments for our experiment.

Subjects were randomly matched in pairs at the beginning of the session and play with the same partner in all rounds. We chose fixed pairs because the theoretical predictions assume that players' belief about information choices are correct in equilibrium. Fixing pairs, as opposed to randomizing pairs each round, helps subjects' to form correct beliefs about their partner's play. This is particularly important given the complexity of our environment. In addition, fixed pairs allows us to compare our results against Szkup and Trevino (2020), who also used fixed pairings.[13] It is important to note that our theoretical benchmark is still relevant even when subjects face this repeated interaction. Under the parametrization chosen for the experiment, the game that subjects play in each round (stage game) has a unique equilibrium. Since there are finitely many rounds and the stage game is identical across rounds, a standard backward induction argument implies that the finitely repeated game has a unique subgame perfect equilibrium where players play the unique stage-game equilibrium in every round. Therefore, we can still use the theoretical predictions of the one shot game.[14] Another possible concern is that fixed pairs might facilitate subjects' learning of their partner strategies. As we discuss in Section 3.3 below, we do not find evidence of such learning. Instead, we find that subjects pay only limited attention to each other's precision choices in earlier rounds, with their own early play being the best predictor of their subsequent play.

To avoid framing effects, the experiment and instructions use a neutral terminology. Subjects are told to choose between two actions, A or B, avoiding terms such as "investment" or "coordination." To avoid bankruptcies, subjects enter each round with an endowment of 24 tokens from which they subtract their precision cost. From Table 1 we can see that even if subjects choose the most precise information, the lowest payoff they can get in a round is zero (in case of an unsuccessful investment). To be consistent with the neutral framing of the experiment, in the analysis of the data we refer to the action "invest" as the risky action and

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the strategy method and direct action choices in experiments.

[13]For robustness Szkup and Trevino (2020) ran an additional session of the game with random matching and found no evidence that matching protocol affected their results.

[14]It has been shown that in experiments cooperation tends to emerge even in games with a finite horizon (see, for example, Selten and Stoecker (1986), Andreoni and Miller (1993), or Cooper et al. (1996)). However, these studies also report the "endgame effect," where subjects revert to playing the unique stage-game equilibrium in the last rounds. Such endgame effect also emerges in theoretical models that attempt to rationalize cooperation in play in finitely repeated games (Kreps et al. (1992)). In our experiment, we do not observe endgame effects, in fact, subjects' behavior in the last round is the same as their play in earlier rounds. Moreover, we do not observe any evidence of cooperation, likely due to the complex nature of our 2-stage game and the incomplete information structure. In such a complex setup, it is difficult to cooperate and to effectively detect or punish deviations.

to “not invest” as the safe action. We also refer to investment stage as coordination stage.

Before starting the first paying round, subjects had access to a practice screen where they could generate signals for the different available precisions, and they were given an explanation of the payoffs associated with each possible action, given their signal and the underlying state θ. They could spend up to 5 minutes on the practice screen.

Each round of the experiment consisted of two decision stages:

1. Subjects choose from a menu of precisions and associated costs (see Table 1).
2. Subjects observe a signal $x_i \sim N(\theta, \sigma_i^2)$ and simultaneously decide whether to invest (A) or not invest (B).

As stated above, in treatment SM subjects had to choose a cutoff value for their signal, before observing the actual signal realization, in order to choose an action in the second step.

After each round, subjects received feedback about their own choice of precision, their own private signal, their choice of action, the realization of θ, how many people in their pair chose A, whether A was successful or not, and their individual payoff for the round. In addition, each subject could have accessed the history of precision choices of both pair members made over the previous rounds by pressing a button. At the end of the experiment, the computer randomly selected five of the rounds played and subjects were paid the average of the payoffs obtained in those rounds, using the exchange rate of 3 tokens per 1 US dollar.

The experiment was programmed and conducted with the software z-Tree (Fischbacher (2007)).

3.2 Experimental results

We are interested in understanding how information choices affect subjects' subsequent actions and how overall behavior compares to the theoretical predictions. In order to do so we start by focusing on the first stage of the game, presenting results about individual precision choices and about precision choices within pairs. We then analyze how these precision choices affect behavior in the second stage of the game. We study the effect of precision choices on individual actions and how the use and level of thresholds depends on precision choices. We then study the evolution of thresholds over time to better understand our results and we discuss welfare implications of our findings.

The analysis that follows is based on behavior in the last 25 rounds of the experiment, once behavior has stabilized. This is done mainly to allow subjects to stabilize in their individual precision choices in order to be able to estimate individual strategies in the coordination game. We analyze behavior in the first 25 rounds of the experiment in Section 3.3 where we explore self-selection in early rounds.

3.2.1 Information choices

Table 2 shows the frequencies of individual precision choices for the last 25 rounds of the experiment. In order to better understand the evolution of precision choices, we also show choices in the first 5 rounds and the last 5 rounds of the experiment.[15] As we can see, the most popular precision choice is level 4, the equilibrium precision, which seems to be the result of choices shifting from other precisions as subjects gained more experience in the experiment. However, we also see significant heterogeneity in information choices that prevails until the end of the experiment.

Precision level	Standard deviation	Cost	Precision choices in rounds: 26-50	1-5	45-50
1	1	6	14.5%	18.46%	14.04%
2	3	5	9.46%	12.88%	8.85%
3	6	4	17.77%	23.27%	17.88%
4	10	2	35.15%	22.31%	35.19%
5	16	1.5	3.77%	5%	4.42%
6	20	1	19.35%	18.08%	19.62%

Table 2: Frequencies of individual precision choices, DA and SM treatments

Since thresholds depend on precision choices, we first need to establish stability of individual precision choices over time in order to discuss the use of threshold strategies.[16] Once we establish stability of individual precision choices we analyze actions for the precision level that was most often chosen by each subject.

We find that, on average, in the last 25 rounds of the experiment an individual subject chooses the same level of precision for 22 out of the last 25 periods and that the most popular precision choice is the equilibrium level 4. To illustrate this result, Figure 1 contains the transition matrix of precision choices in the last 25 rounds. The entry a_{ij} of the matrix shows the probability of choosing precision level j in round $t+1$, given that a subject chose precision level i in round t, for $i, j \in [1, 6]$ and $t > 25$.

	Prec 1	Prec 2	Prec 3	Prec 4	Prec 5	Prec 6
Prec 1	**0.94**	0.04	0.002	0.006	0.002	0.01
Prec 2	0.06	**0.83**	0.06	0.025	0	0.025
Prec 3	0.006	0.03	**0.84**	0.09	0.004	0.03
Prec 4	0	0.01	0.04	**0.91**	0.02	0.02
Prec 5	0.01	0.01	0.08	0.14	**0.63**	0.13
Prec 6	0.01	0.01	0.02	0.04	0.02	**0.9**

Figure 1: Transition matrix of precision choices in the last 25 rounds, DA and SM treatments

[15] This includes choices in treatments DA and SM. We aggregate the data because the distributions of precision choices are not statistically different between these two treatments. This was expected since the treatment effect, if present, would be in the second stage of the game.

[16] If individual precision choices were changing a lot over the last 25 periods, it would be difficult to estimate thresholds by conditioning on a precision choice, since it would be constantly changing.

By looking at the diagonal entries of the transition matrix, we can see that most precision levels seem to be absorbent states, i.e., if a subject chooses a precision in one period, it is very likely he will make the same choice in the next period.[17] This effectively means that individual precision choices are stable over the last 25 rounds. Given this stability result, we characterize subjects by their preferred precision choice.

Table 3 shows the frequency of subjects that choose each precision level as their preferred precision, confirming a substantial heterogeneity of subjects' preferences for information. Notice that the frequency of individual preferred precisions is similar to the frequency of precision choices in the last 25 rounds, implying that the heterogeneity of choices reported in Table 2 is driven by the between-subject heterogeneity reported in Table 3. We will revisit this heterogeneity when we study how these different groups of subjects play the coordination game, suggesting that the first stage of the game might serve as a self-selection device in the two-stage game.

Precision level	Standard deviation	Cost	Preferred precision
1	1	6	14.42%
2	3	5	8.65%
3	6	4	17.31%
4	10	2	36.54%
5	16	1.5	4.81%
6	20	1	18.27%

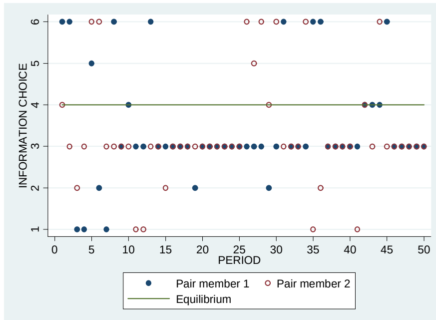
Table 3: Preferred individual precision choices in the last 25 rounds, DA and SM treatments

We have established stability of subjects' individual precision choices. However, recall from Section 2.2 that thresholds depend on the precision choices of both pair members. Thus, before we can estimate thresholds we have to establish a notion of convergence in precision choices within a pair.

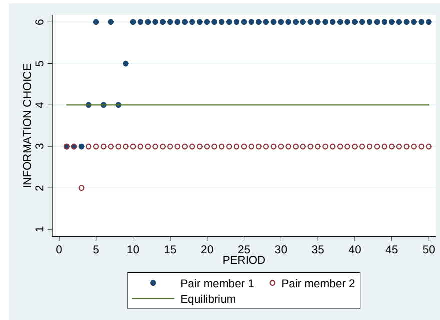
Our categorization is illustrated in Figure 2. We define individual convergence in precision as a situation when a subject chooses the same precision level for the last 25 rounds, with at most three deviations, and we say that a pair exhibits non-stable behavior if at least one of its members does not converge individually in his precision choice (panel (a) of Figure 2). A pair that exhibits stability but not convergence is a pair in which both members converge individually in their own precision choices, but the levels at which they converge are more than one level apart (panel (b) of Figure 2). We define weak convergence as pairs in which both members converge individually to a level of precision and these two precision levels are at most one level away from each other (panel (c) of Figure 2). We say that a pair exhibits full convergence if both members converge individually to the same level of precision for the last 25 rounds of play (panel (d) of Figure 2).[18]

[17] Precision level 5 is the least absorbent state. However, this is the least popular precision choice.

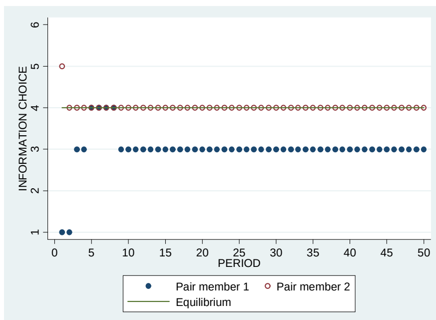
[18] Table A.1 in the appendix shows all the combinations of precision choices made by the different pairs in our experiment (for both the DA and SM treatments). The diagonal entries correspond to the pairs that



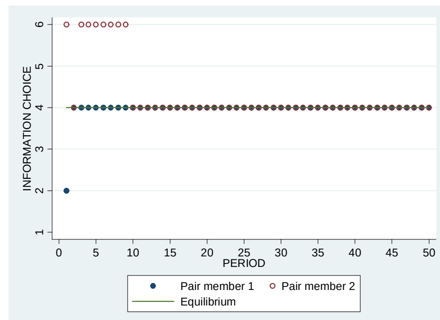
(a) Non-stable behavior



(b) Stability, no convergence



(c) Weak convergence



(d) Full convergence

Figure 2: Examples of types of convergence in precision within a pair

In the analysis of thresholds of Section 3.2.3 we use the notion of weak convergence and restrict our attention to pairs that coordinate on high precision (levels 1 and 2), medium precision (levels 3 and 4), and low precision (levels 5 and 6). Note that weak convergence includes full convergence. Table 4 summarizes the combinations of precision choices across pairs according to this notion of weak convergence. If we use this qualitative characterization we find that approximately two thirds of the pairs in our experiment exhibit weak convergence in precision choices. Moreover, among these pairs, the majority converge to medium precisions, which corresponds to the theoretical prediction.

	High	Medium	Low
H	**13.46%**	19.23%	5.77%
M		**32.69%**	19.23%
L			**9.62%**

Table 4: Weak convergence of precision choices, DA and SM treatments

Tables 2-4 imply that subjects exhibit substantial and persistent heterogeneity in their precision choices. Below, we show that these differences in behavior at the information acquisition stage translate into systematic differences in behavior at the coordination stage that diverge from the theoretical predictions. These results suggest that subjects use precision choices to self-select in terms of how they intend to act at the coordination stage.

exhibit full convergence.

3.2.2 Information choices and actions

We first study how different precision choices affect individual actions and then how precision choices impact the use and estimation of thresholds.

For the DA treatment Figure 3 plots the cumulative distribution function (CDF) of the decision to take the risky action (invest) for each signal realization, by individual precision choice. The value of the signal for which subjects take the risky action with probability 0.5 determines the cutoff for their action rule. Looking at the intersection of the curves corresponding to the different precision levels with the 0.5 line, from left to right, we can see that those cutoff points tend to be larger for lower precisions. This suggests that a higher precision leads to a higher likelihood of taking the risky action.[19] While at odds with the theoretical predictions, this is in line with existing experimental evidence on global games with exogenously set precisions (see Szkup and Trevino (2020)). We explore this observation further in Section 3.2.3.

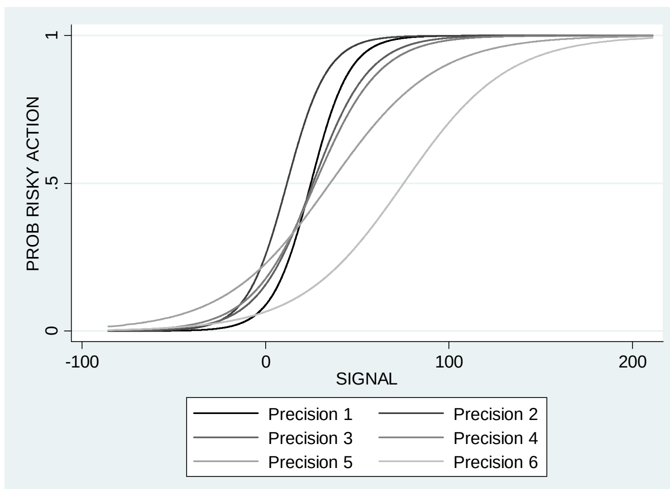


Figure 3: Probability of taking the risky action by precision choices, DA treatment

We also see that as we move towards lower precision levels the slopes of the CDFs decrease, indicating that the dispersion of actions within each precision group increases as the precision of information decreases. In particular, when subjects choose the lowest precision their choices exhibit a large dispersion, indicating a lower predictability of their actions based on their signals compared to subjects who choose higher precisions. This suggests that there is a relationship between subjects' information choices and the stability and consistency of their actions. Subjects who choose higher precisions (and are thus more "invested" in the game) tend to have a more predictable action pattern (lower dispersion) than those who choose lower precisions. Importantly, this relationship between predictability of actions and precision of information is not present when precision is exogenously set (see Figure A.1 in Appendix A).

[19]We find further evidence of this relationship by running two regressions (one for the DA and one for the SM treatment) to determine the statistical effect that each level of precision has on individual actions (for DA) and on reported thresholds (for SM). This analysis is reported in Tables A.2 and A.3 in the Appendix.

This is suggestive of subjects potentially using precision choices to self-select in the way they play the game, not only in terms of the likelihood of taking the risky action (intersection with the 0.5 line), but in the predictability or stability of their strategies (dispersion of precision CDFs). We further explore these two effects when we discuss the estimation of thresholds (Section 3.2.3) and their stability over time (Section 3.2.4).

3.2.3 Threshold estimation

For the DA treatment we identify three types of individual action rules in the coordination game: subjects who use thresholds, those who use a degenerate strategy, and those who act randomly. The use of thresholds can be either by a perfect or almost perfect threshold. We say that a subject uses a perfect threshold if he takes the safe action for low values of the signal and the risky action for high values of the signal, with exactly one switching point (i.e., the intersection of the sets of signals for which a subject chooses either action is null). For almost perfect thresholds we allow these two sets to overlap for at most three observations. This means that subjects take the safe action for low signal values and the risky action for high signal values, but these two sets can intersect for at most three observations. We say that a subject uses a degenerate strategy if his choice of action is constant and does not depend on the signal (i.e., always taking the risky or safe action). A subject exhibits random behavior when his choices are erratic and do not seem to follow a pattern based on the observed signal.

In total, 93.33% of the subjects in the DA treatment use threshold strategies for their preferred precision choice, which is similar to the use of thresholds in the literature with exogenous information structures (see, for example, Heinemann et al. (2004) and Szkup and Trevino (2020)). Therefore, we find support for the theoretical prediction of threshold use when information is endogenously determined. However, the frequency of threshold use depends on precision choices. As shown in Table 5, we find that 100% of subjects whose preferred precision is level 1, 2, or 3 use threshold strategies. For precision level 4, 95.45% of subjects use thresholds, the one subject who does not use a threshold uses the degenerate strategy of always taking the risky action. There are only 2 subjects choosing precision level 5, one uses a threshold and one behaves randomly. For precision level 6, 77.78% of the subjects use threshold strategies and the rest use the degenerate strategy of always choosing the safe action.

Precision	Thresholds	Degenerate strategies	Random behavior
1	100%	-	-
2	100%	-	-
3	100%	-	-
4	95.45%	4.55% (risky)	-
5	50%	-	50%
6	77.78%	22.22% (safe)	-

Table 5: Use of threshold strategies by preferred precision, DA treatment

These observations support our conjecture of subjects using precision choices to self-select in terms of how "invested" they are in the game, with subjects who choose a higher precision being more likely to have monotonic, signal-dependent strategies than subjects who choose a lower precision. The latter subjects, in turn, are more likely to set degenerate strategies that never attempt to coordinate on the risky action. These results are consistent with the observation from Figure 3 that shows a larger dispersion of actions for subjects who choose lower precisions. It is important to stress that this pattern in the use of threshold strategies is not found when information is exogenously given (see Szkup and Trevino (2020)), suggesting that this self-selection takes place via the information stage..

We now turn our attention to pairs that weakly converge in precision choices. We first characterize how the individual coordination attempts and successful coordination in the DA treatment are determined by precision choices within a pair. Table 6 reports the number of individual coordination attempts (risky choices) and the number of instances of successful coordination as proportions of the total number of situations when the state falls in the coordination region, $\theta \in [0, 100)$, for each combination of precision choices within pairs.[20] The number of individual coordination attempts corresponds to the instances when at least one of the pair members decides to take the risky action and $\theta \in [0, 100)$, while the number of successful coordination attempts corresponds to the number of cases where both subjects take the risky action and $\theta \in [0, 100)$.

From Table 6 we see that both the individual coordination attempts and instances of successful coordination increase with precision choices when subjects coordinate on precision (diagonal entries). This supports our earlier observation that subjects who choose a higher precision choose the risky action more often. In addition, we can also see a clear monotonicity off the main diagonal, implying that subjects who choose a lower precision are more likely to take the risky action in response to their pair member's higher precision choice. This suggests that subjects are less (more) cautious when they expect their pair member to choose a high (low) precision as they expect them to choose the risky action more (less) often, ceteris paribus. Thus, subjects' information choices have non-trivial spillover effects in the coordination stage. In addition, we see that instances of successful coordination are increasing in precision choices, implying that the tendency to take the risky action more often as subjects get better information is justified. Note that these observations are in contrast to the theoretical predictions from Section 2.4.

We provide further evidence for this result in Table A.4 in the Appendix where we report the results of a random effects probit regression that shows how successful coordination is more likely to occur when pair members converge to a higher precision.

We are now ready to test directly the theoretical predictions by turning our attention to the estimation of thresholds. In Table 7 we compare, for high, medium, and low precision

[20] We restrict our attention to the DA treatment because individual coordination attempts are clearly identified when subjects explicitly choose an action after observing their signals, as opposed to the indirect action choice that happens when subjects choose a threshold in the SM treatment.

	Individual coordination attempts / Total situations			Successful coordination / Total situations		
	High	Medium	Low	High	Medium	Low
H	$\frac{117}{146}$ (80.14%)	$\frac{155}{196}$ (79.08%)	$\frac{26}{46}$ (56.52%)	$\frac{108}{146}$ (73.97%)	$\frac{144}{196}$ (73.47%)	$\frac{10}{46}$ (21.74%)
M		$\frac{211}{308}$ (68.51%)	$\frac{75}{160}$ (46.88%)		$\frac{184}{308}$ (59.74%)	$\frac{56}{160}$ (35%)
L			$\frac{22}{68}$ (32.35%)			$\frac{10}{68}$ (14.71%)

Table 6: Coordination attempts and successes by precision choices in a pair, DA treatment

levels, the theoretical equilibrium thresholds to the thresholds estimated using a random effects logit and to the Mean Estimated Threshold (MET) for the DA treatment.[21] For the SM treatment, we compute the Mean Reported Thresholds (MRT) that subjects reported using the strategy method. Recall that we define weak convergence to high precision as pairs that converge to precision levels 1 or 2, medium precision as pairs that converge to precision levels 3 or 4, and low precision as pairs that converge to precision levels 5 or 6. Therefore, for each precision level (high, medium, low) we report equilibrium thresholds corresponding to both precision levels. We also include the risk dominant threshold of the underlying complete information game, since the theory predicts that this is what thresholds should converge to as the signal noise converges to zero. Standard deviations are reported in parenthesis.

	High precision		Medium precision		Low precision	
Logit (RE) (DA)	17.88		26.39		58.45	
	(8.61)		(8.03)		(21.76)	
MET (DA)	15.52		29.84		50.65	
	(18.37)		(26.79)		(20.12)	
MRT (SM)	20.32		34.34		31.27	
	(3.56)		(16.39)		(21.4)	
Theoretical prediction x^*	Info 1	Info 2	Info 3	Info 4	Info 5	Info 6
	35.31	33.88	31.61	28.31	22.82	18.73
Risk dominant threshold	36		36		36	

Table 7: Estimated thresholds and equilibrium predictions, DA and SM treatments

Table 7 shows that the estimated thresholds for high precision in both the DA and SM treatments are significantly lower than the threshold predicted by the theory and significantly lower than the estimates for medium and low precisions. In contrast, in both the DA and

[21] We use two different methods to estimate thresholds. First, for each precision level, we pool the data of all the subjects who use thresholds in each treatment and fit a logistic function with random effects to determine the probability of taking the risky action as a function of the observed signal. We estimate the mean threshold of the group by finding the value of the signal for which subjects are indifferent between taking both actions, i.e., for which the logistic function gives a value of $\frac{1}{2}$. For the second method we take the average, individual by individual, between the highest value of the signal for which a subject chooses the safe action and the lowest value of the signal for which he chooses the risky action. This number approximates the switching point for the subject. We then take the mean and standard deviation of the thresholds in the group and refer to this estimate as the Mean Estimated Threshold (MET).

SM treatments, the estimated thresholds for medium precision are not statistically different from the equilibrium predictions for precisions 3 or 4, so the subjects that coordinate on a medium precision behave on average in accordance to the unique equilibrium predicted by the theory.[22] Finally, in the DA treatment the estimated thresholds for low precision are statistically larger than the equilibrium predictions. The estimated thresholds for low precision in the SM treatment are also larger than equilibrium but the difference is not statistically significant.[23] Overall, these results indicate two departures from the theory. First, we see a reversal in comparative statics because the estimated thresholds tend to be decreasing rather than increasing in precision choices, as predicted by the theory. Second, we see a level effect because subjects who choose a high precision set lower thresholds than equilibrium while the opposite is true for subjects who choose a low precision. In what follows we refer to the these empirically observed departures from the theory as the *threshold-level effect* of increasing precision.

The departures from the theory in our data are qualitatively similar to departures observed when the signal precision is set exogenously, as documented in Szkup and Trevino (2020), but they are starker under endogenous information. The observation of the threshold-level effect under exogenous information allows us to rule out that subjects fail to see the precision cost as a sunk cost as an explanation for this effect in our setup. As shown in Table A.5 in the Appendix, when the precision of information is exogenous thresholds are non-increasing in precision. In addition, the estimated threshold for high precision is significantly smaller than the theoretical threshold and significantly lower than the estimates for medium and low precisions, but the medium and low precision thresholds are not statistically different from one another. In contrast, in our setup where the information structure is endogenous, thresholds are strictly decreasing in precision choices and are statistically different from each other. These observations indicate that while the threshold-level effect is also present when information is exogenous, it is amplified when subjects choose the precision of their signals. Szkup and Trevino (2020) propose a mechanism based on sentiments about the perception of strategic uncertainty to explain the threshold-level effect, where players' subjective beliefs about the probability that the other player takes the risky action are affected by the fundamental uncertainty in the environment. In their data, subjects in environments with high precision perceive lower strategic uncertainty than subjects in environments with low precision. They also argue that popular models such as QRE, Level-k, or Cursed Equilibrium cannot explain this result. A similar phenomenon might be at play in our experiment with self-selection possibly affecting the way subjects form beliefs.

[22] This result suggests that these subjects understand the trade off between precision and cost by choosing a medium level of precision and then apply this information optimally in the coordination game by choosing the threshold that maximizes their expected profits, given that their opponent also chooses a medium level of precision.

[23] Note that the strategy method might affect the way subjects play the game by imposing the use of thresholds, thus putting more structure on their thinking. This might explain the differences between the DA and SM treatments.

In what follows we focus on our DA treatment to make comparisons with the results of Szkup and Trevino (2020) who only use direct action choice in their experiment. The amplification of the threshold-level effect can be clearly seen in Figure 4, which plots the estimated thresholds in the DA treatments under exogenous and endogenous information, together with shaded standard deviation intervals, as well as equilibrium and first-best thresholds. As we discuss in Section 4, this amplification has non-trivial effects on outcomes in the game and on payoffs.

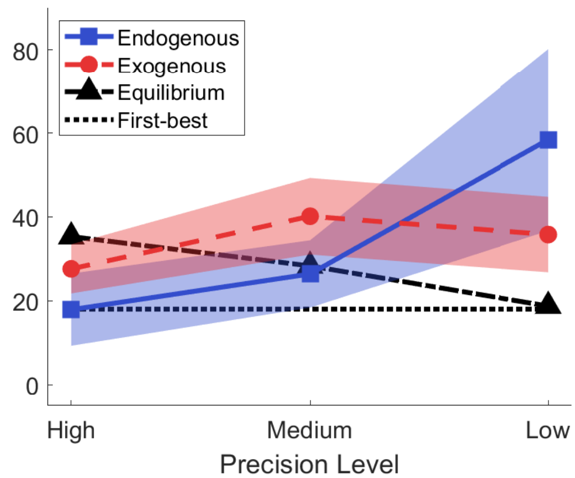


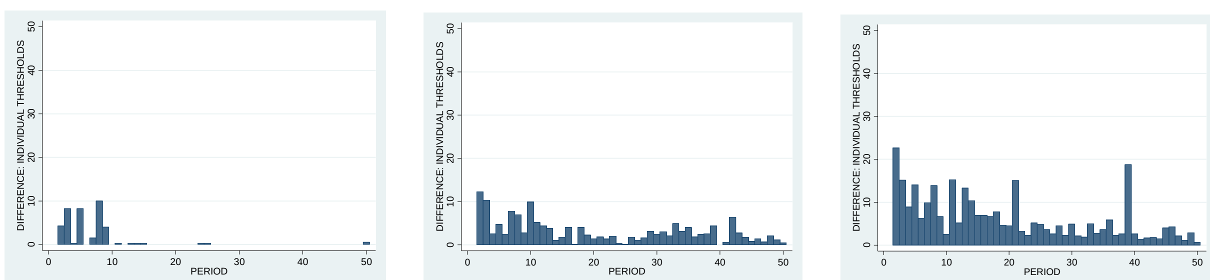
Figure 4: Equilibrium, first-best, and estimated thresholds, DA treatment

These results suggest a specific channel for self-selection that becomes available when information is endogenized. Subjects seem to self-select on a given precision level based on their individual inclination to take the risky action. Subjects who are more inclined to take the risky action choose to purchase a higher precision for their signal in order to minimize the probability of choosing the risky action when it would be unsuccessful and to better coordinate with their opponent by increasing the correlation between their signals. We can think of these as subjects who are more "invested" in the game in the sense that they pay a high price to get a better signal in the information stage and try to extract as much payoff as possible in the coordination stage. On the other hand, subjects who are more inclined to take the safe action choose a lower precision in anticipation of "playing it safe" during the coordination stage by either setting a high threshold or by using the degenerate strategy of always choosing the safe action, for all signal realizations (see Table 5). In terms of the self-selection mechanism, these subjects seek to pay the lowest possible cost of precision since they are less "invested" in the coordination game.

Figure 4 also indicates that the dispersion of thresholds is relatively stable under exogenous information as we move across precisions. However, we see a clear increase in dispersion of thresholds for subjects who self-select into a low precision. This suggests that, regardless of the threshold level, different choices of precision give rise to different levels of dispersion or predictability of strategies within a precision group. We refer to this as the *threshold-quality effect*. This effect was first suggested in Figure 3 when we observed an increase in dispersion of individual actions as subjects chose lower precisions. The threshold-quality effect could also affect the stability of thresholds over time. However, this effect cannot be properly studied in the DA treatment because the thresholds estimated in Table 7 and Figure 4 rely on observing signals and actions throughout many rounds to estimate a single threshold. Therefore, to investigate how subjects' strategies evolve over time we turn to the SM treatment where subjects report a cutoff strategy before observing the realization of their signal.

3.2.4 Stability of thresholds and their convergence within pairs

We first analyze the evolution of individual thresholds in all 50 periods of the experiment for subjects in pairs that coordinate on high, medium, and low precision. For each subject we calculate the difference in absolute value between the threshold reported in one period with respect to the threshold reported in the previous period. Then we compute the average of these differences across all subjects from pairs that coordinate on a given precision level. This is portrayed in Figure 5. The vertical bar at period t illustrates how much, on average, a subject changed the value of his own threshold in period t with respect to the threshold he reported in period $t-1$.

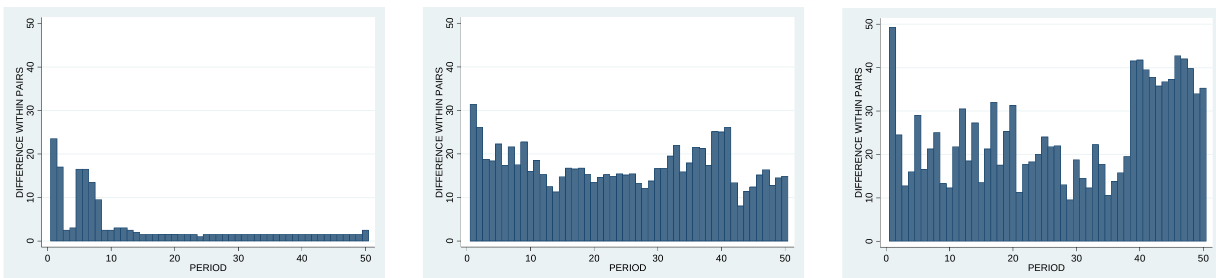


High precision Medium precision Low precision

Figure 5: Convergence of individual thresholds, SM treatment

Figure 5 shows a lot of stability in individual thresholds for subjects in pairs that coordinate on high precision levels. We observe some experimentation in the initial periods (as expected), but thresholds quickly stabilize after 10 rounds and in the last 25 rounds the mean difference is consistently zero, except for the last round. For medium precision levels the mean difference of individual thresholds is less than 7 after the first 10 rounds. For low precision levels it takes a longer time for individual thresholds to stabilize and in the last 25 rounds the mean difference is less than 7 in all but one period. Therefore, we see that subjects who choose a high precision exhibit more stable threshold behavior and that this individual stability seems to decrease as subjects choose lower precisions. This is in line with Figure 3 and illustrates how self-selection gives rise to the threshold-quality effect by showing that the individual thresholds are more stable and predictable over time for subjects who choose higher precisions.

Stronger evidence of self-selection and the threshold-quality effect comes from studying convergence of thresholds within pairs. To analyze convergence of thresholds within pairs we compute the average difference in absolute value between the reported thresholds of pair members, for each period and each precision level. Recall that subjects do not receive feedback about the threshold reported by their pair member, but only observe whether the action A (risky action) was successful or not. Thus, it is not trivial for subjects to converge in their thresholds within pairs. We plot our results in Figure 6.



High precision | Medium precision | Low precision

Figure 6: Convergence of thresholds within a pair, SM treatment

We see from Figure 6 that subjects from pairs that converge to high precision levels coordinated extremely well on their thresholds, and that the extent of coordination of thresholds becomes weaker as the chosen precision level decreases. The observed convergence of thresholds within pairs that choose a high precision illustrates the threshold-quality effect of the self-selection mechanism based on precision choices. The greater stability of individual actions for this group of subjects illustrated in Figure 5 naturally allows subjects in pairs that converge to a high precision to better coordinate their strategies. However, when subjects choose lower precisions individual strategies become less predictable (subjects are not as "invested" in the game as those who choose high precision). Therefore, it becomes harder to coordinate strategies within a pair, leading to large disparities of thresholds and less successful coordination for low precision levels.

We refer to the threshold-quality effect as the mechanism behind the observed disparities in stability and convergence of thresholds that depends on the level of precision chosen in a pair. Unlike the threshold-level effect, which was also present in the case of exogenous information, the threshold-quality effect is unique to the game with endogenous information. This is because adding an initial stage of information acquisition opens the door to self-selection. As a consequence, we see that regardless of the level of thresholds, the quality of individual play improves with precision choices, as illustrated in Figures 5 and 6 (and suggested by Figures 3 and 4 above and by Figure A.1 in Appendix A). Self-selection in this case reflects how subjects who are more "invested" in the game are more likely to choose the highest precision and set a "cleaner" strategy, while those who are not very "invested" in the game (or who are unsure about how to play the game) might opt for the cheapest precision and choose actions in a less predictable way (even not signal-dependent) in the coordination stage.[24]

Finally, note that the threshold-quality effect leads to a more nuanced interpretation of the earlier result that subjects in pairs that choose a higher precision have a higher index of successful coordination (see Table 6). This result might be intuitive since more precise information allows subjects to better estimate each others' signals and, hence, coordinate their actions. However, Figure 6 suggests that at least some of the higher rate of successful coordination achieved by these pairs is due to the convergence of subjects' thresholds, an effect that is absent in the theoretical model.[25] Thus, Figure 6 suggests that self-selection, via the threshold-quality effect, has also non-trivial implications for the extent of coordination subjects achieve.

3.3 Understanding self-selection: Analysis of early rounds

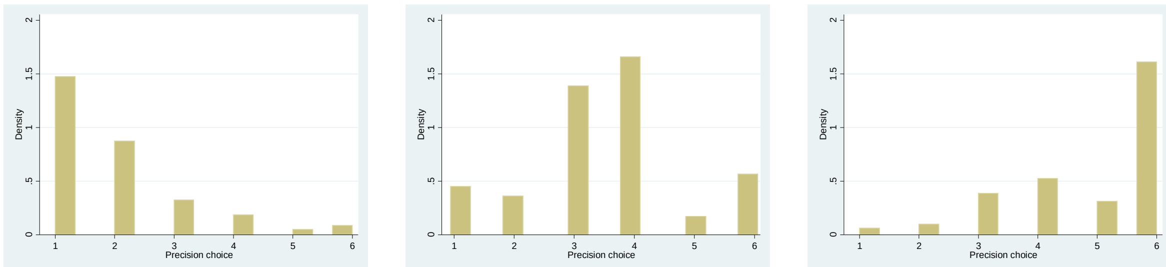
We have argued that the stark difference in behavior across subjects who choose different precision levels can be explained by thinking of information choices as a vehicle for subjects to self-select in the way they play the game. We argue that behind this self-selection is the subjects' subjective valuation of information, which determines their propensity to take the risky action and the quality or "cleanliness" of their strategies. This analysis relied on the last 25 rounds of the experiment because we wanted to allow enough time for individual behavior to stabilize. In this section we look at the initial rounds of the experiment to provide evidence in support of the self-selection mechanism. To do this, we follow the same categorization as before and separate subjects according to the precision type to which they individually converge in the last 25 rounds of the experiment: high (precision levels 1 and 2), medium (precision levels 3 and 4), or low (precision levels 5 and 6). We refer to this as the subject's "type". Evidence of clear differences across the 3 different types of subjects in early rounds would provide strong support for the self-selection mechanism.

Persistence of individual precision choices We first analyze precision choices in early rounds to understand if subjects experiment with precision choices early on, learning from their own experience and the past precision choices of their opponent. Instead, if precision choices are persistent in early rounds with not much experimentation, this would be consistent with the notion of self selection. Figure 7 plots the histograms of precision choices in the

[24] While we discuss the threshold level and quality effects separately, it is likely that these effects are related to each other. For example, when subjects converge to a high precision, being able to better predict the other's behavior might encourage both subjects to set a lower threshold. It is important to note, however, that such an argument goes against the theoretical mechanism of global games where an increase in the precision of information increases strategic uncertainty for signals close to the threshold signal. This is precisely the reason why the equilibrium thresholds are increasing in precision (see Morris and Shin (2003)).

[25] Our theoretical model predicts that all subjects who chose the same level of precision will also set the same threshold, that is, there is no dispersion. From a behavioral point of view, the main benefit of a higher precision is that each subject can better estimate whether his pair member's signal is above or below the threshold signal, leading to better coordination of thresholds.

first 10 rounds of the experiment, by types. We can see clear differences in precision choices in the early rounds by type, suggesting that subjects select an individual preferred precision early on in the experiment, which is consistent with their future choices. Figure A.3 in the Appendix plots the same graphs for the first 25 rounds, showing an even starker pattern.



High precision | Medium precision | Low precision

Figure 7: Histogram of precision choices by precision type, first 10 rounds

To further understand precision choices in early rounds, Figure 8 contains the transition matrix of precision choices by type in the first 10 rounds and the first 25 rounds of the experiment. The entry a_{ij} of each matrix shows the probability of choosing a precision type j in round $t+1$, given that a subject chose precision type i in round t, for $i, j \in \{high, medium, low\}$ and $t \leq 10$ and $t \leq 25$, respectively.

First 10 rounds

	High	Medium	Low
High	**0.78**	0.15	0.07
Medium	0.08	**0.81**	0.11
Low	0.06	0.17	**0.77**

First 25 rounds

	High	Medium	Low
High	**0.85**	0.11	0.04
Medium	0.06	**0.87**	0.07
Low	0.04	0.12	**0.84**

Figure 8: Transition matrix of choices of precision types in early rounds, DA and SM treatments

As we can see, precision types are absorbent states since the early rounds, meaning that subjects are significantly more likely to choose the same precision type (high, medium, low) in consecutive periods than to change to other types, even in the first 10 rounds. This, together with the results from Figure 7, is suggests that subjects might self-select by choosing an individual precision type early on in the experiment, as opposed to arriving at their precision type in the last 25 rounds by experimentation and learning with different precision types.

Similarly, we find no significant evidence that subjects' precision choices in the initial rounds of the experiment are determined by their opponent's past precision choices. First, we observe that only about a quarter of subjects clicked on the button to look at their opponent's past precision choices. In the first 25 rounds, subjects identified as high, medium, and low precision types did so 25.83%, 28%, and 20.67% of the times, respectively.[26] Note that the

[26] In the last 25 rounds these numbers were 12.17%, 14.36%, and 5.67%, for high, medium, and low precision types, respectively.

propensity to look at this information is very similar across precision types, suggesting that the interest in others' past precision choices is not related to the precision level at which subjects self-select. For those subjects who choose to observe their opponent's past precision choices we run regressions, for each group of precision type subjects, to understand how a subject's precision choice in period t depends on their own precision choice in period $t-1$ and on their opponent's precision choice in period $t-1$, which is interacted with a dummy that takes the value of 1 if a subject clicked the button to see their opponent's previous precision choices in period $t-1$. Table A.7 in the Appendix shows that by far the most important determinant of precision choices for these subjects is their own previous precision choice, for all precision types. These results suggest that the persistence of individual precision choices is mainly driven by subjects' own previous choices and not by subjects learning to coordinate precisions with others. Overall, the above results provide support for the self-selection mechanism.

Effect of precision choices on early actions in the game We now analyze how precision types affect the decision to take the risky action in the game in the initial rounds of the experiment. In Table 8 we present the results of a random effects logit regression where the likelihood of taking the risky action depends on the signals observed by subjects that we characterize as high, medium, or low precision type, for the first 25 rounds of the experiment, separately. Using low precision types as the baseline, we include a dummy that takes the value of 1 for subjects that are identified as high precision types and 0 otherwise, and a similar one for subjects that are identified as medium precision types. We interact signals with these dummies. As we can see from Table 8, the insignificant coefficients for the dummies that separate precision types indicate that the higher incidence of risky choices for high precision types is not just a "level" effect (i.e., that different types are inherently more or less aggressive in the game), but instead that different precision types respond differently to their signals, with high types being more responsive to their signals than lower types (all pair wise comparisons of signal coefficients are statistically different to at least the 5% level of significance), which is consistent with the threshold-level effect related to self-selection.

Variable	Risky action {0,1}
D{high type}	0.803
	(0.756)
D{Medium type)	0.302
	(0.667)
Signal	0.0169***
	(0.004)
Signal*high type	0.083***
	(0.01)
Signal*med type	0.058***
	(0.005)
Constant	0.621***
	(0.066)
N	1200

Clustered (by subject) standard errors in parentheses; * significant at 10%; ** significant at 5%; *** significant at 1%

Table 8: Risky choices in the game as a function of observed signals, by precision type, DA treatment

Taken together, these results suggest that the heterogeneity in behavior that we observe in the experiment is a result of an underlying heterogeneity of preferences for information that manifests itself from the very beginning of the experiment. This heterogeneity gives rise to self-selection in the quality of information that subjects seek, which ultimately determines not only how aggressively subjects seek to coordinate with others (threshold-level effect), but also how "cleanly" they do so (threshold-quality effect).

4 Welfare

In this section we investigate the payoff consequences of the threshold-level and the threshold-quality effects that result from self-selection. To do so, we compare the payoffs earned by subjects with the theoretical equilibrium payoffs. In addition, to better quantify the effects of self-selection via information acquisition we compare the payoffs in our experiment to the payoffs of Szkup and Trevino (2020) when information is exogenously determined. Since that experiment only uses direct action choices, in this section we focus only on our DA treatment. To make payoffs comparable across different sessions, treatments, and experiments, we normalize all the payoffs with respect to the first-best payoffs that correspond to the recommendation of a social planner that faces no informational constraints (i.e., if subjects chose the risky action whenever it was profitable, $\theta \geq T$).[27,28] In what follows, we refer to this ratio as the payoff-efficiency index as it captures welfare losses or gains with respect to optimal play.

Payoffs in the Coordination Stage We first investigate whether subjects' deviations from the equilibrium thresholds, conditional on their precision choices, are payoff-enhancing or payoff-reducing in the coordination stage (i.e., abstracting from the cost of information acquisition). That is, we compare the payoff-efficiency index of the observed action with the payoff-efficiency index associated with hypothetical equilibrium payoffs.[29] In addition, to separate the impact of self-selection via information acquisition from the effect of exogenous variations in precision we compare the payoff-efficiency index in our experiment to the one computed using data from Szkup and Trevino (2020). Table 9 presents our results.[30]

Precision	High	Medium	Low
Endogenous Information			
Realized	0.96	0.88	0.49
Equilibrium	0.93	0.95	0.95
Exogenous Information			
Realized	0.94	0.83	0.75

Table 9: Payoff-efficiency index for observed and equilibrium payoffs, DA treatment

We focus first on the payoff-efficiency indices in our data with endogenous information. From Table 9, we see that the payoff-efficiency index for subjects who converged to high precision is higher than if they had followed the equilibrium strategies. In contrast, the payoff-efficiency index for subjects that chose medium precision is lower than the equilibrium payoff-efficiency index. Finally, for low precision, the payoff-efficiency index is only half of the payoff-efficiency index that subjects would have earned had they followed the equilibrium

[27] We normalize payoffs to control for the fact that subjects could earn a high payoff even if they used inefficient and poorly coordinated strategies in sessions where the average realization of θ is high. First-best payoffs have the advantage of being computed directly using the observed realizations of θ and do not depend on precision choices. In contrast, equilibrium payoffs in the coordination game depend on precision choices, so we could only compute the expected equilibrium payoffs of the full game. The same issue applies to constrained efficiency payoffs.

[28] For completeness, in the Appendix we provide the average realized payoffs that subjects earned in each treatment (see Table A.6).

[29] In order to compute the equilibrium payoff-efficiency index we constructed hypothetical equilibrium payoffs. That is, for each signal observed by a subject, we computed the action that he would choose if he were following the equilibrium strategy (and do the same for his pair member) and then computed the payoff implied by this hypothetical behavior. We then computed the ratio of the average equilibrium payoff and the first-best payoff.

[30] Note that in our DA treatment the coordination stage follows the exact same experimental design that was used in Szkup and Trevino (2020). Thus, the payoffs in two experiments are directly comparable.

strategies. It is worth stressing that the payoff-efficiency index for realized payoffs exhibits a clear monotonicity that is not present in the case of the efficiency index associated with theoretical payoffs. This suggests that information affects subjects' behavior beyond the channels emphasized by the theory (i.e., beyond the ability to better predict fundamentals and pair member's signal).

These results suggest that the threshold-level and the threshold-quality effects described above have significant welfare consequences. Recall that subjects who choose a high precision set thresholds that are closer to efficiency than the equilibrium thresholds and they coordinate their thresholds extremely well. Subjects who choose a medium precision set thresholds close to the equilibrium threshold but do not coordinate their thresholds as effectively. Finally, subjects who choose a low precision in the DA treatment set inefficient thresholds far above the equilibrium threshold, which results in lower payoffs. Therefore, Table 9 suggests that both the threshold-level and threshold-quality effects are also manifested in terms of welfare, leading to an increase in payoff-efficiency for subjects who choose a high precision and a clear decrease in payoff-efficiency for subjects who choose a low precision.

To properly understand the payoff-efficiency consequences of the threshold-level and threshold-quality effects derived from self-selection, we also compare in Table 9 the payoff-efficiency index of our subjects to an analogous index with exogenously set precisions of Szkup and Trevino (2020). As expected, we see that in both experiments the payoff-efficiency index is increasing in precision. This is because the threshold-level effect is present in both scenarios, suggesting that subjects who observe more precise signals make more accurate decisions and better coordinate their actions within a pair.

However, there are two notable differences between the payoff-efficiency indices in the two experiments. First, we see a larger decrease in payoff-efficiency for subjects who choose a low precision than when a low precision is exogenously set. Second, the rate at which the payoff-efficiency index is increasing is higher when subjects choose the precision of their signals. These differences provide further support for the self-selection mechanism and highlight the important welfare consequences of self-selection via information acquisition.

Payoffs in the Information Acquisition Game We now analyze payoffs in the two-stage game, taking into account the costs of information. As above, we focus on the payoff efficiency index, but we now take into account the cost of information acquisition when computing realized payoffs (though we leave the denominator unchanged). We report our results in Table 10. Note that the differences in the payoff-efficiency indices for a given precision reported in Tables 9 and 10 capture the cost of information acquisition.

We can see that subjects who self-select by choosing a high precision end up experiencing a slightly larger loss in terms of efficiency than those subjects who chose medium precision, despite their efficiency gains in the coordination stage. This suggests that the payoff gains in the coordination stage might not be enough to compensate for the cost of precision. On the other hand, the poor performance in the coordination stage for subjects who choose a low

Precision	High	Medium	Low
Payoff-efficiency index	0.76	0.78	0.45

Table 10: Payoff-efficiency index in the two-stage game

precision could not be offset by low precision costs. Thus, subjects that choose a medium precision reported the lowest payoff loss in terms of efficiency, followed closely by those that choose a high precision, while subjects who choose a low precision saw a significant decrease in payoff-efficiency.

5 Conclusion

We have studied how the choice of costly information in coordination games can have non-trivial effects in actions and outcomes that are not captured by theoretical models. Our results suggest that subjects in our experiment self-select via information choices and that this self-selection is manifested in the form of a threshold-level and a threshold-quality effect in the coordination game. In particular, subjects who choose a high precision tend to set lower thresholds, thus taking the risky action more often, and these thresholds tend to be cleaner and more predictable, leading to more stability of individual behavior and coordination within pairs that choose a high precision. We observe the opposite for subjects who choose lower precisions: we estimate higher thresholds with high dispersion and low predictability and, as a result, we observe poor stability of individual choices and of coordination within pairs that choose a low precision. Qualitatively, the threshold-level effect is consistent with previous results when subjects are exogenously provided with high, medium, or low precision of information. However, when subjects endogenously choose their precision this effect becomes more pronounced. The threshold-quality effect, which is responsible for the differences in predictability and stability of strategies, is a novel finding that is specific to our environment with endogenous information choices. The self-selection mechanism is better illustrated with this effect because it suggests that subjects who are more "invested" in the game will self-select into acquiring the best available information and set clean, clear strategies in the game. On the other hand, subjects who are not too invested in the game will not purchase better, more expensive information, and their behavior in the game will be more erratic.

In terms of applications, it has been suggested that wealth inequality is the result of high-income earners also earning higher rates of return on their investment (Becker (1967)). Indeed, this relation has been shown to hold systematically in the data (Yitzhaki (1986)). Arrow (1987) suggested two possible explanations. First, individuals may face different costs of information acquisition, and those with lower costs acquire more information, earn higher returns, and get wealthier over time. Alternatively, high-income individuals might have more incentives to acquire information since they tend to hold a higher share of their wealth in the form of risky assets. Our results suggest yet another possibility. Namely, individuals differ in their intrinsic propensity to acquire information and ability to use it as implied by our self-selection mechanism. As suggested by our welfare analysis, these intrinsic differences then would result in ex-post wealth differences even if individuals initially face the same information costs and had the same wealth. Understanding the importance of self-selection as a driver of wealth inequality is an intriguing avenue for future research.

Another interesting avenue for future research is to understand the psychological motivation behind the self-selection process. We can think of several reasons why some subjects might choose the lowest possible precision and not engage in setting clean or surplus-extracting strategies in the game. One possibility is that they find the cognitive effort of setting such strategies to be too high. This could explain the larger incidence of degenerate strategies that are not signal-contingent for low precision subjects. Similarly, a high cognitive cost to find an optimal strategy could drive these subjects to engage in trial and error type of behavior, which leads to noisier strategies. There are, of course, other possible explanations, such as subjects choosing not to actively engage in the game and signaling this intent by spending as little money as possible in information. Whatever the explanation, our results suggest that the initial stage of costly information choice can help researchers understand subjects' intentions in the coordination game through the signaling mechanism that arises due to self-selection.

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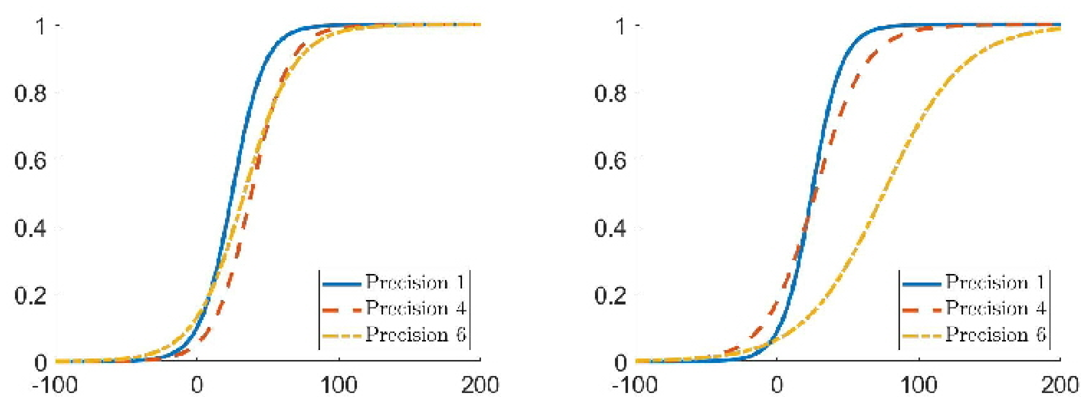
Appendix

A Additional Results

		Choice of pair member 2					
		Prec 1	Prec 2	Prec 3	Prec 4	Prec 5	Prec 6
	Prec 1	**5.77%**	1.92%	7.69%	9.62%	0%	3.85%
Choice	Prec 2		**5.77%**	0%	1.92%	0%	1.92%
of pair	Prec 3			**3.85%**	15.38%	0%	1.92%
member	Prec 4				**13.46%**	3.85%	13.46%
1	Prec 5					**0%**	1.92%
	Prec 6						**7.69%**

Table A.1: Combination of precision choices, DA and SM treatments



Exogenous information | Endogenous Information

Figure A.1: Probability of taking the risky action by precision choices in the experiment with exogenous information (data from Szkup and Trevino (2020)) and with endogenous information.

Effects of precision choices on individual actions

For the DA treatment, we present in Table A.2 the results of a random effects logit where the dependent variable is the decision to take the risky (1) or the safe (0) action and the independent variables are dummies for the six precision levels, interacted with the signal realizations.[31] All the coefficients for the interacted variables are positive and significant to the 1% level and the magnitudes of the coefficients decrease for lower precisions. Positive coefficients imply that subjects are more likely to take the risky action for higher signal

[31] We interact precisions and signals because the decision in the game is determined by the value of the signal, and the information choice affects how precise the signal is.

realizations, for all precision levels, consistent with the monotonicity implied by threshold strategies. The decrease in magnitude of the coefficients for lower precision implies that this effect is stronger when subjects choose very precise signals than when they observe noisier signals. We find similar evidence in the SM treatment. Table A.3 reports the results of a random effects OLS regression where the dependent variable is the threshold reported by subjects and the independent variables are dummies for each level of precision, setting precision 1 as the baseline. Each of these dummies takes the value of 1 if the subject chooses this precision level and 0 otherwise. We find that the reported thresholds depend positively and significantly on the level of precision chosen. The magnitudes of the coefficients for each precision level increase as we move towards less precise information, suggesting that less precise information gives rise to higher thresholds, corroborating the findings of Table A.2 for the DA treatment.

Variable	Choice of risky action {0,1}
Precision 1*signal	0.148***
	(0.021)
Precision 2*signal	0.135***
	(0.018)
Precision 3*signal	0.103***
	(0.011)
Precision 4*signal	0.087***
	(0.007)
Precision 5*signal	0.041***
	(0.013)
Precision 6*signal	0.056***
	(0.006)
Constant	-2.837***
	(0.376)
N	1500

Clustered (by subject) standard errors in parentheses; * significant at 10%; ** significant at 5%; *** significant at 1%

Table A.2: Choice of the risky action as a function of precision, DA treatment

Variable	Reported threshold
Precision 2	6.25
	(4.00)
Precision 3	10.65***
	(3.48)
Precision 4	10.78***
	(3.39)
Precision 5	12.87***
	(3.66)
Precision 6	12.54***
	(3.23)
Constant	20.66***
	(5.14)
N	1100

Clustered (by subject) standard errors in parentheses; * significant at 10%; ** significant at 5%; *** significant at 1%

Table A.3: Reported threshold as a function of precision, SM treatment

Variable	Successful coordination {0,1}
(H, H)*signal	0.176***
	(0.03)
(M, M)*signal	0.075***
	(0.006)
(L, L)*signal	0.041***
	(0.007)
(H, M)*signal	0.117***
	(0.013)
(H, L)*signal	0.051***
	(0.011)
(M, L)*signal	0.054***
	(0.004)
Constant	-3.338***
	(0.41)
N	1500

Clustered (by pairs) standard errors in parentheses; * significant at 10%; ** significant at 5%; *** significant at 1%

Table A.4: Successful coordination as a function of combination of convergent precision choices in a pair, DA treatment

Welfare

Table A.6 compares average realized payoffs of subjects for the DA and SM treatments,

	High precision		Medium precision		Low precision	
Exogenous Information						
Logit (RE) (DA)	27.61		40.16		35.79	
	(5.86)		(9.13)		(9.00)	
MET (DA)	27.42		40.37		36.23	
	(19.16)		(18.77)		(23.36)	
	Info 1	Info 2	Info 3	Info 4	Info 5	Info 6
Theoretical prediction x^*	35.31	33.88	31.61	28.31	22.82	18.73
Risk dominant threshold	36		36		36	

Table A.5: Estimated thresholds and equilibrium predictions with exogenous information, Szkup and Trevino (2020)

separated by precision, to three benchmarks.[32] The first one is the average expected payoffs that would have arisen if subjects had followed the equilibrium strategy for each realization of the state θ observed in the different sessions of the experiment. The second is a constrained efficiency benchmark where players truthfully reveal their signals and jointly choose actions to extract the maximum surplus, for each realization of θ observed in the experiment. Under this benchmark subjects would still face fundamental uncertainty and would have to purchase information for their signals to deal with it, but they would not face strategic uncertainty, similar to a planner who faces the same informational constraints as players but who chooses actions for both pair members to extract the maximum surplus. In this case, the optimal precision choice would be level 6. The payoffs we report for these two benchmarks are expected payoffs built using the observed realizations of θ in each round.[33] The third benchmark corresponds to the average payoffs that would have arisen if subjects had chosen a "first-best" action under complete information, i.e., if they coordinated on the risky action whenever they could get a positive payoff ($\theta > 18$). This first-best corresponds to the case of a social planner who observes the realizations of θ and prescribes the actions that maximize payoffs for both players without informational constraints. Only the realization of θ in the experiment is used to construct this benchmark and no signals are taken into consideration. For this reason, under this benchmark subjects choose the lowest precision. Standard deviations are reported in parenthesis.

We focus on comparisons within columns, since different realizations of θ across pairs can give rise to different magnitudes of payoffs. Subjects who choose a high precision increase their payoff with respect to constrained efficiency and equilibrium play in the DA and SM treatments. For subjects with medium and low precisions equilibrium payoffs are higher than realized payoffs. However, for the DA treatment, realized payoffs for subjects with medium precision are on average 6.9% less than the corresponding equilibrium payoffs,

[32] We look at net realized payoffs substracting the cost of precision and the cost of taking the risky action, if applicable.

[33] We do not use the observed signals from the experiment to calculate payoffs because for each benchmark a specific precision is assumed to be chosen and each precision gives rise to a different distribution of signals.

	High precision		Medium precision		Low precision	
Treatment	DA	SM	DA	SM	DA	SM
Realized payoffs	27.58	40.84	26.05	37.73	17.81	35.09
	(4.82)	(4.46)	(6.47)	(4.07)	(14.02)	(4.5)
Expected equilibrium	26.07***	38.49***	27.97***	38.99***	31.46***	37.91***
payoffs	(5.62)	(0.77)	(5.88)	(1.01)	(5.82)	(2.18)
Expected constrained	26.39**	38.69***	28.25***	39.16***	31.78***	38.09***
efficient payoffs	(5.69)	(0.76)	(6.01)	(0.97)	(6.01)	(2.15)
First-best complete	29.55***	41.55**	30.84***	41.9***	34.05***	40.97***
information payoffs	(5.24)	(0.65)	(5.84)	(0.77)	(6.2)	(1.88)

Statistically different from realized payoffs at the ***1%; **5%; *10% level of significance

Table A.6: Average payoffs and efficiency benchmarks, DA and SM treatments

whereas realized payoffs for subjects with low precisions are on average 43.39% less than the corresponding equilibrium payoffs.[34] Therefore, choosing a low precision leads to the highest loss in individual payoffs with respect to equilibrium.

Some of the differences in the payoffs observed in Table A.6 between treatments DA and SM treatments are due to the different realizations of θ in the experiment. Figure A.2 plots the distributions of realized θ for each treatment. As we can see, the SM treatment featured, on average, higher realizations of θ. Thus, it is not surprising to see higher payoffs for that treatment.

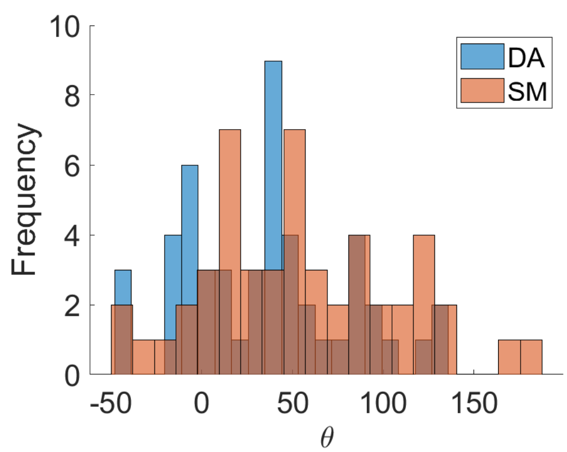
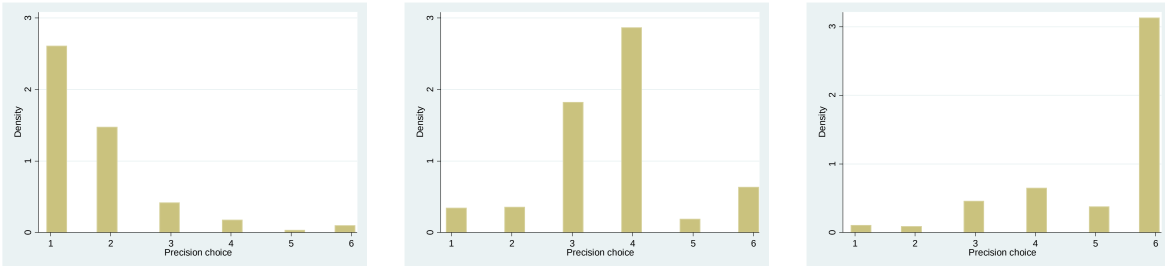


Figure A.2: Theoretical predictions and estimated thresholds for exogenous and endogenous information structures

Understanding self-selection

[34]Even if subjects who choose equilibrium precisions also set equilibrium thresholds, we can still observe differences in realized payoffs with respect to the expected equilibrium payoffs. This is due to the fact that we are comparing payoffs from the realization of θ for a small sample to the expected payoffs that would arise according to the distribution of a population. By a similar argument, constrained efficiency payoffs should, on average, be higher than realized payoffs, but over finite samples this might not be the case.



High precision | Medium precision | Low precision

Figure A.3: Histogram of precision choices by precision type, first 25 rounds

	First 10 rounds			First 25 rounds		
	High	Medium	Low	High	Medium	Low
Own prec choice$_{t-1}$	0.611***	0.626***	0.628***	0.602***	0.65***	0.706***
	(0.055)	(0.033)	(0.051)	(0.032)	(0.02)	(0.028)
Other prec choice$^{\dagger}_{t-1}$	0.074*	0.054**	0.009	0.059***	0.034**	-0.007
	(0.043)	(0.023)	(0.029)	(0.022)	(0.013)	(0.018)
Constant	0.658***	1.316***	1.911	0.621***	1.272***	1.576***
	(0.131)	(0.129)	(0.255)	(0.066)	(0.078)	(0.151)
N	216	504	216	576	1344	576

† Interacted with dummy that takes value of 1 only if subject observed history of other's precision choices in t-1

Clustered (by subject) standard errors in parentheses; * significant at 10%; ** significant at 5%; *** significant at 1%

Table A.7: Choice of precision as a function of own and other's precision choice in previous period, DA treatment

B Proofs of theoretical results

In this section we provide the proofs of the claims stated in Section 2. We first show that the game played by the agents in the second stage belongs to the class of monotone supermodular games as defined by Vives and van Zandt (2007). We then characterize the equilibrium of the second stage game for any feasible players' choice of information. Finally, we establish existence of the equilibrium of the model with information acquisition.

Throughout this appendix, we denote by $\tau_i = 1/\sigma_i^2$, $i = 1, 2$, the precisions of the players' private signals, chosen in period 1. As usual, it is more convenient to work with precisions rather than with standard deviations or variances. However, in several places we switch back to using standard deviations since working with compact intervals (recall that $\sigma_i \in [0, \underline{\sigma}]$) simplifies some of the arguments.

B.1 Relation to monotone supermodular games

Using the notation of Vives and van Zandt (2007), define $N = \{1, 2\}$ as the set of players indexed by i. Let player i's type space be a measurable space $(T_i, \mathcal{F}_i)$ and denote by $(T_0, \mathcal{F}_0)$ the state space that is capturing the residual uncertainty.[35] We let $\mathcal{F}_{-i}$ be the product $\sigma-$algebra $\otimes_{k \neq i} \mathcal{F}_k$. Let player i's interim beliefs be given by a function $p_i : T_i \to M_{-i}$, where M_{-i} is the set of probability measures on $(T_{-i}, \mathcal{F}_{-i})$. Finally, let $A_i = \{0, 1\}$ be the action set of player i, A be the set of action profiles and $u_i : A \times T \to \mathbb{R}$ be the payoff function.

Definition 2 *A game belongs to the class of monotone supermodular games if*

1. *The utility function $u_i(a_i, a_{-i}, \omega)$ is supermodular in own actions, a_i, and has increasing differences in (a_i, a_{-i}) and in (a_i, ω).*
2. *The belief map $p_i : T_i \to M_{-i}$ is increasing with respect to a partial order on M_{-i} of first-order stochastic dominance.*

The following proposition is the key result established by Vives and van Zandt (2007).

Proposition 3 *Assume that a game Γ belongs to the class of monotone supermodular games. Furthermore, assume that the following hold:*

1. *Each T_k is endowed with a partial order.*
2. *A_i is a complete lattice.*
3. *$\forall a_i \in A_i$, $u_i(a_i, \cdot) : T \to \mathbb{R}$ is measurable.*
4. *u_i is bounded.*

[35] In a global games setting, we usually interpret $(T_i, \mathcal{F}_i)$ to be the space of possible signals that agent i receives, while $(T_0, \mathcal{F}_0)$ corresponds to the measurable space of the underlying parameter of the game.

5. u_i is continuous in a_i.[36]

Then, there exist a least and a greatest Bayesian Nash Equilibrium of the game Γ and each one of them is in monotone strategies.

We now show that the game played in the second stage of our model belongs to the class of monotone supermodular games as defined in Definition 2.

Note that in our model the type space is defined as follows: $T_0 = \mathbb{R}$, $T_i = \mathbb{R}$ for $i = 1, 2$, where $t_0 = \theta$, $t_i = x_i$, $t_j = x_j$ and $\mathcal{F}_i = B(\mathbb{R})$, a Borel σ-algebra on $\mathbb{R}$, $i = 0, 1, 2$. The set of probability measures M_{-i} is simply the set of joint normal probability distributions over $(T_{-i}, \mathcal{F}_{-i})$ conditional on the realization of ω_i. The belief mapping $p_i : \mathbb{R} \to M_{-i}$ maps each signal x_i into the posterior distribution of (θ, x_j) using Bayes' rule. Finally, the underlying utility function for agent i is given by

$$u(a_i, a_j, \theta) = 1_{\{a_i=1\}} \left[\theta \left[1_{\{\theta \in [\underline{\theta}, \overline{\theta}]\}} 1_{\{a_j=1\}} + 1_{\{\theta > \overline{\theta}\}}\right] - T\right] \tag{3}$$

and the expected utility of agent i when he takes action a_i is:

$$v_i(a_i) = 1_{\{a_i=1\}} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \theta \left[1_{\{\theta \in [\underline{\theta}, \overline{\theta}]\}} 1_{\{s_j(x_j)=1\}} + 1_{\{\theta > \overline{\theta}\}}\right] f(\theta, x_j | x_i)\, dx_j d\theta\right] - T \tag{4}$$

where $s_j : T_j \to A_j$ is a strategy of player j and $f(\theta, x_j | x_i)$ is the pdf of the conditional distribution of $\{\theta, x_j\}$ given x_i.

The fact that generic global games belong to the class of monotone supermodular games was noted first by Vives and Van Zandt (2007). The following lemma shows that our coordination-stage game satisfies the conditions listed in Definition 2 so that it belongs to the class of monotone supermodular games.

Lemma 1 *The coordination stage of our model belongs to the class of monotone supermodular games.*

Proof. Since A_i is totally ordered it follows that $u(a_i, a_j, \theta)$ is supermodular in a_i (Example 2.6.2(a) in Topkis (1998)). To see that u has increasing differences note that $u(1, 0, \theta) - u(0, 0, \theta) = \theta 1_{\{\theta > \overline{\theta}\}} - T$ and $u(1, 1, \theta) - u(0, 1, \theta) = \theta 1_{\{\theta > \underline{\theta}\}} - T$ so that

$$u(1, 0, \theta) - u(0, 0, \theta) \leqslant u(1, 1, \theta) - u(0, 1, \theta)$$

That u has increasing differences in (a_i, θ) follows from the observation that $u(1, a_j, \theta) - u(0, a_j, \theta) = \theta \left[1_{\{\theta \in [\underline{\theta}, \overline{\theta}]\}} 1_{\{a_j=1\}} + 1_{\{\theta > \overline{\theta}\}}\right]$ which is increasing in θ.

To show that the belief mapping is increasing with respect to first-order stochastic dominance it is enough to show that the pdf of players' posterior belief about $\{x_j, \theta\}$ satisfies

[36] When A_i is finite this condition is vacuous.

the monotone likelihood ratio property. Let $f(\theta, x_j|x_i)$ denote the pdf of player i's posterior belief. Then

$$f(\theta, x_j|x_i) = f(x_j|x_i, \theta) f(\theta|x_i) = f(x_j|\theta) f(\theta|x_i),$$

where the last equality follows from the fact that x_i and x_j are independent conditional on θ. Next, note that

$$f(\theta|x_i) = (\tau_i + \tau_\theta)^{1/2} \phi \left(\frac{\theta - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}} \right),$$

where $\phi(\cdot)$ is the pdf of the standard normal distribution. It is straightforward to see that for any two signal values $x_i'', x_i' \in \mathbb{R}$ with $x_i'' > x_i'$ the ratio $f(\theta|x_i'') / f(\theta|x_i')$ is strictly increasing in θ. This implies that the family of density functions $\{f(\theta|x_i)\}_{x_i \in \mathbb{R}}$ parameterized by x_j satisfies the strict monotone likelihood ratio (MLR) property (Milgrom (1981)). Thus, we conclude that $f(\theta, x_j|x_i)$ satisfies the monotone likelihood ratio property. ■

Next, we would like to apply Proposition 3 to our model to establish existence of equilibrium at the coordination stage for any players' choices of precision. Unfortunately, in our setup the utility function $u(\cdot)$ is not bounded, and, thus, we cannot apply the above proposition to our problem directly. The next result shows, however, that the Proposition 3 can be extended to the case where $u(\cdot)$ is bounded from below and there exists an integrable function h that dominates u. The idea behind this extension of their result is to use the fact that utility function is bounded from below to establish that best response mapping is well-defined and the dominated theorem to establish that best-response dynamics converge.

Proposition 4 *Assume that the game to the class of monotone supermodular games and assume that assumptions* $(1) - (3)$ *of Proposition 1 are satisfied. Furthermore, assume that:*

1. *u is bounded from below*
2. *There exists measurable function h such that $h \geq |u|$ and h is integrable with respect to $p(t_{-i}|t_i)$ for all t_i.*

Then there exists a least and a greatest Bayesian Nash Equilibrium of the game Γ *and each one of them is in monotone strategies.*

Proof. We prove this result in two steps. First, assuming that the greatest best reply mapping $\overline{\beta}_i$ is well-defined, increasing, and monotone, we show that under the assumptions of Proposition 4 the greatest Bayesian Nash Equilibrium (BNE) exists. Then, we show that under the assumptions of Proposition 4 $\overline{\beta}_i$ is indeed well-defined, increasing, and monotone.

Step 1: Suppose that $\overline{\beta}_i$, is well-defined, increasing and monotone and u satisfies of Proposition 4. Then we can repeat the argument of Lemma 6 in van Zandt and Vives (2007) to show that there is a greatest and least BNE in monotone strategies. We can relax the boundedness assumption, since under assumptions of Proposition 4 we can interchange the order of limit and integration by applying dominated convergence theorem. Since this is the only step in that proof of Lemma 6 that requires boundedness of the utility function, we are done.

Step 2: Here we need to establish that $\overline{\beta}_i$ is well-defined and increasing. Then, the monotonicity of $\overline{\beta}_i$ will follow from Proposition 11 in van Zandt and Vives (2007). The sensitive part of this step is to show that $\overline{\beta}_i$ is well-defined, and more precisely that it is a measurable function of t_i. For this purpose we extend the proof of Lemma 9 in Ely and Peski (2006) to cover general measurable functions. The rest of argument follows from van Zandt (2010).

Fix $a_i \in A_i$ and define $U_i(t_i, t_{-i}) := u_i(a_i, s_j(t_j), t_i, t_{-i})$. We need to show that a function $v_i : A_i \times \Omega_i \to \overline{\mathbb{R}}$ defined by

$$v_i(a_i, t_i) = \int_{\Omega_{-i}} U_i(t_i, t_{-i}) dp(t_{-i}|t_i)$$

is measurable in t_i. To establish this fact, van Zandt (2010) uses the following Lemma from Ely and Peski (2006).

Lemma (Ely and Peski (2006)) *Let A and B be measurable sets and $g : A \times B \to [0,1]$ be a jointly measurable map. If $m : A \to \Delta B$ (where ΔB denotes the set of probability measures defined on B) is measurable, then the map $L^g : A \to R$ defined as $L^g(a) = \int g(a, \cdot) dm(a)$ is measurable.*

Note however, that the proof of their lemma is unchanged if we allow $g : A \times B \to \overline{\mathbb{R}}$, as long as g is measurable and bounded from below. In this case, there exists an increasing sequence of simple functions g_n such that $g_n \uparrow g$, so by the extended Monotone Convergence Theorem (Ash, 2000) we have $\int g_n d\mu \to \int g d\mu$ for a measure μ defined on $A \times B$. Hence we conclude that $v_i : A_i \times \Omega_i \to \overline{\mathbb{R}}$ is a measurable function of t_i. The rest of the proof follows directly from van Zandt (2010) section 7.5. Monotonicity of $\overline{\beta}_i$ follows from Proposition 11 in van Zandt and Vives (2007). ■

Finally, we show that the utility function u defined in Equation (3) satisfies the additional conditions listed in Proposition 4. In what follows let μ_{x_i} be the probability measure implied $F(\theta, x_j | x_j)$, player i's posterior belief CDF.

Lemma 2 *Utility function u defined in Equation* (3) *satisfies*

1. *u is bounded from below;*

2. *There exists a function h, integrable w.r.t. μ_{x_i}, such that $|u| < h$.*

Proof. It is easy to see that u is bounded from below by $-T$. To prove part 2, we note that

$$\begin{aligned}
\int |u| \, d\mu_{\theta|x_i} &= \int \left| 1_{\{a_i=1\}} \left[\theta \left(1_{\{\theta \in [\underline{\theta}, \overline{\theta}]\}} 1_{\{s_j(x_j)=1\}} + 1_{\{\theta > \overline{\theta}\}} \right) - T \right] \right| d\mu_{x_i} \\
&\leqslant \int |\max\{0, \theta\} - T| \, d\mu_{x_i} \leqslant \int |\theta| \, d\mu_{x_i} + \int |-T| \, d\mu_{x_i} < \infty
\end{aligned}$$

Thus, the function

$$h(a_i, a_j, \theta) = |\max\{0, \theta\} - T|$$

is integrable and dominates the utility function u. This establishes the claim. ■

Corollary 1 *The game played by the agents in coordination stage possess a least and a greatest Bayesian Nash Equilibrium, each one of them is in monotone strategies.*

B.2 Coordination Stage

The expected payoff to player i who takes a risky action after observing signal x_i and who expects that a player j will follow a monotone strategy with threshold x_j^* is given by

$$\begin{aligned} v\left(x_i, x_j^*; \tau_i, \tau_j\right) &= \int_{\underline{\theta}}^{\overline{\theta}} \theta \left[1 - \Phi\left(\frac{x_j^* - \theta}{\tau_j^{-1/2}}\right)\right] (\tau_i + \tau_\theta)^{1/2} \phi\left(\frac{\theta - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}}\right) d\theta \\ &\quad + \int_{\overline{\theta}}^{\infty} \theta (\tau_i + \tau_\theta)^{1/2} \phi\left(\frac{\theta - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}}\right) d\theta - T \end{aligned} \tag{5}$$

It is convenient to make the following change of variables in the above equation

$$z = \frac{\theta - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}}$$

After performing this change variables, Equation (5) becomes

$$\begin{aligned} v\left(x_i, x_j^*; \tau_i, \tau_j\right) &= \int_{L(\underline{\theta}, x_i)}^{L(\overline{\theta}, x_i)} \left[\frac{z}{\sqrt{\tau_i + \tau_\theta}} + \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}\right] \left[1 - \Phi\left(\frac{x_j^* - \frac{z}{\sqrt{\tau_i + \tau_\theta}} - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{\tau_j^{-1/2}}\right)\right] \phi(z)\, dz \\ &\quad + \int_{L(\overline{\theta}, x_i)}^{\infty} \left[\frac{z}{\sqrt{\tau_i + \tau_\theta}} + \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}\right] \phi(z)\, dz - T, \end{aligned} \tag{6}$$

where $L(\theta, x_i)$ is defined as

$$L(\theta, x_i) \equiv \frac{\theta - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}} \tag{7}$$

Lemma 3 *Payoff $v\left(x_i, x_j^*; \tau_i, \tau_j\right)$ has the following properties:*

1. *$v\left(x_i, x_j^*; \tau_i, \tau_j\right)$ is increasing in x_i*
2. *$v\left(x_i, x_j^*; \tau_i, \tau_j\right)$ is decreasing in x_j^*.*
3. *For any $x_j^* \in \mathbb{R}$, we have $\lim_{x_i \to -\infty} v\left(x_i, x_j^*; \tau_i, \tau_j\right) = -T$*

4. *For any* $x_j^* \in \mathbb{R}$*, we have* $\lim_{x_i \to \infty} v\left(x_i, x_j^*; \tau_i, \tau_j\right) = \infty$

Proof. Differentiating $v\left(x_i, x_j^*; \tau_i, \tau_j\right)$ w.r.t. x_i we obtain

$$\begin{aligned}
\frac{\partial v\left(x_i, x_j^*; \tau_i, \tau_j\right)}{\partial x_i} &= \int_{L(\underline{\theta}, x_i)}^{L(\overline{\theta}, x_i)} \frac{\tau_i}{\tau_i + \tau_\theta} \left[1 - \Phi\left(\frac{x_j^* - \frac{z}{\sqrt{\tau_i + \tau_\theta}} - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{\tau_j^{-1/2}}\right)\right] \phi(z)\, dz \qquad (8)\\
&+ \int_{L(\underline{\theta}, x_i)}^{L(\overline{\theta}, x_i)} \left[\frac{z}{\sqrt{\tau_i + \tau_\theta}} + \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}\right] \frac{\tau_i}{\tau_i + \tau_\theta} \tau_j^{1/2} \phi\left(\frac{x_j^* - \frac{z}{\sqrt{\tau_i + \tau_\theta}} - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{\tau_j^{-1/2}}\right) \phi(z)\, dz\\
&- \frac{\tau_i}{\sqrt{\tau_i + \tau_\theta}} \overline{\theta} \left[1 - \Phi\left(\frac{x_j^* - \overline{\theta}}{\tau_j^{-1/2}}\right)\right] \phi\left(\frac{\overline{\theta} - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}}\right)\\
&+ \frac{\tau_i}{\sqrt{\tau_i + \tau_\theta}} \underline{\theta} \left[1 - \Phi\left(\frac{x_j^* - \overline{\theta}}{\tau_j^{-1/2}}\right)\right] \phi\left(\frac{\underline{\theta} - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}}\right)\\
&+ \int_{L(\overline{\theta}, x_i)}^{\infty} \frac{\tau_i}{\tau_i + \tau_\theta} \phi(z)\, dz\\
&+ \frac{\tau_i}{\sqrt{\tau_i + \tau_\theta}} \overline{\theta} \phi\left(\frac{\overline{\theta} - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}}\right)
\end{aligned}$$

In the above expression, the first four terms corresponds to the derivative of the first integral appearing in Equation (6) while the last two terms terms corresponds to the derivative of the second integral appearing in that Equation. Simplifying Equation (8) we obtain

$$\begin{aligned}
\frac{\partial v\left(x_i, x_j^*; \tau_i, \tau_j\right)}{\partial x_i} &= \int_{L(\underline{\theta}, x_i)}^{L(\overline{\theta}, x_i)} \frac{\tau_i}{\tau_i + \tau_\theta} \left[1 - \Phi\left(\frac{x_j^* - \frac{z}{\sqrt{\tau_i + \tau_\theta}} - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{\tau_j^{-1/2}}\right)\right] \phi(z)\, dz \qquad (9)\\
&+ \int_{L(\underline{\theta}, x_i)}^{L(\overline{\theta}, x_i)} \left[\frac{z}{\sqrt{\tau_i + \tau_\theta}} + \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}\right] \frac{\tau_i \tau_j^{1/2}}{\tau_i + \tau_\theta} \phi\left(\frac{x_j^* - \frac{z}{\sqrt{\tau_i + \tau_\theta}} - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{\tau_j^{-1/2}}\right) \phi(z)\, dz\\
&+ \frac{\tau_i}{\sqrt{\tau_i + \tau_\theta}} \overline{\theta} \Phi\left(\frac{x_j^* - \overline{\theta}}{\tau_j^{-1/2}}\right) \phi\left(\frac{\overline{\theta} - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}}\right)\\
&+ \frac{\tau_i}{\sqrt{\tau_i + \tau_\theta}} \underline{\theta} \left[1 - \Phi\left(\frac{x_j^* - \overline{\theta}}{\tau_j^{-1/2}}\right)\right] \phi\left(\frac{\underline{\theta} - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}}\right)\\
&+ \int_{L(\overline{\theta}, x_i)}^{\infty} \frac{\tau_i}{\tau_i + \tau_\theta} \phi(z)\, dz
\end{aligned}$$

Note that since $\underline{\theta} \geq 0$, it follows that

$$(\tau_i + \tau_\theta)^{-1/2} z + \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta} \geq 0 \text{ for all } z \in \left[L\left(\underline{\theta}\right), L\left(\overline{\theta}\right)\right]$$

and so the second term in Equation (9) is strictly positive. The remaining terms are clearly positive. Therefore, we conclude that

$$\frac{\partial v\left(x_i, x_j^*; \tau_i, \tau_j\right)}{\partial x_i} > 0$$

The derivative of $v\left(x_i, x_j^*; \tau_i, \tau_j\right)$ w.r.t. x_j^* is much simpler. In particular, we have

$$\frac{\partial v\left(x_i, x_j^*; \tau_i, \tau_j\right)}{\partial x_j^*} = -\int_{L(\underline{\theta})}^{L(\overline{\theta})} \left[\frac{z}{\sqrt{\tau_i + \tau_\theta}} + \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}\right] \tau_j^{1/2} \phi\left(\frac{x_j^* - \frac{z}{\sqrt{\tau_i + \tau_\theta}} - \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{\tau_j^{-1/2}}\right) \phi(z)\, dz < 0$$

which is negative since

$$\frac{z}{\sqrt{\tau_i + \tau_\theta}} + \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta} \geq 0 \text{ for all } z \in \left[L\left(\underline{\theta}, x_i\right), L\left(\overline{\theta}, x_i\right)\right]$$

Finally, to show that $\lim_{x_i \to -\infty} v\left(x_i, x_j^*; \tau_i, \tau_j\right) = -T$ and $\lim_{x_i \to -\infty} v\left(x_i, x_j^*; \tau_i, \tau_j\right) = \infty$ we note that

$$\int_{L\left(\overline{\theta}, x_i\right)}^{\infty} \left[\frac{z}{\sqrt{\tau_i + \tau_\theta}} + \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}\right] \phi(z)\, dz - T \leq v\left(x_i, x_j^*; \tau_i, \tau_j\right) \leq \int_{L\left(\underline{\theta}, x_i\right)}^{\infty} \left[\frac{z}{\sqrt{\tau_i + \tau_\theta}} + \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}\right] \phi(z)\, dz - T$$

where

$$\begin{aligned} \int_{L\left(\underline{\theta}, x_i\right)}^{\infty} \left[\frac{z}{\sqrt{\tau_i + \tau_\theta}} + \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}\right] \phi(z)\, dz - T &= \frac{\phi\left(L\left(\underline{\theta}, x_i\right)\right)}{\sqrt{\tau_i + \tau_\theta}} + \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}\left[1 - \Phi\left(L\left(\underline{\theta}, x_i\right)\right)\right] - T \\ \int_{L\left(\overline{\theta}, x_i\right)}^{\infty} \left[\frac{z}{\sqrt{\tau_i + \tau_\theta}} + \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}\right] \phi(z)\, dz - T &= \frac{\phi\left(L\left(\overline{\theta}, x_i\right)\right)}{\sqrt{\tau_i + \tau_\theta}} + \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}\left[1 - \Phi\left(L\left(\overline{\theta}, x_i\right)\right)\right] - T \end{aligned}$$

We note that for $\theta \in \left\{\underline{\theta}, \overline{\theta}\right\}$

$$\begin{aligned} \lim_{x_i \to -\infty} \frac{\phi\left(L\left(\theta, x_i\right)\right)}{\sqrt{\tau_i + \tau_\theta}} + \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}\left[1 - \Phi\left(L\left(\theta, x_i\right)\right)\right] - T &= -T \\ \lim_{x_i \to \infty} \frac{\phi\left(L\left(\theta, x_i\right)\right)}{\sqrt{\tau_i + \tau_\theta}} + \frac{\tau_i x_i + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}\left[1 - \Phi\left(L\left(\theta, x_i\right)\right)\right] - T &= \infty \end{aligned}$$

Thus, we conclude that $\lim_{x_i \to -\infty} v\left(x_i, x_j^*; \tau_i, \tau_j\right) = -T$ and $\lim_{x_i \to -\infty} v\left(x_i, x_j^*; \tau_i, \tau_j\right) = \infty$. ■

The above results implies the following corollary.

Corollary 2 *For any $x_j^* \in \mathbb{R}$, there exists unique $x_i^* \in \mathbb{R}$ such that $v\left(x_i, x_j^*; \tau_i, \tau_j\right) \geq 0$ if and only if $x_i^* \geq 0$. Moreover, $\partial x_i^* / \partial x_j^* > 0$.*

The equilibrium in monotone strategies is characterized by a pair of threshold $\{x_1^*, x_2^*\}$ that solve simultaneously the following two equations

$$v\left(x_1^*, x_2^*; \tau_1, \tau_2\right) = 0 \quad (10)$$

$$v\left(x_2^*, x_1^*; \tau_2, \tau_1\right) = 0 \quad (11)$$

Equation (10) states that when player 1 receives signal x_1^* then he is indifferent between taking the risky action and taking the safe action given that player 2 follows a monotone strategy with threshold signal x_2^*. Equation (11) in turn states that when player 2 receives signal x_2^* then he is indifferent between taking the risky action and safe actions given that player 1 follows a monotone strategy with threshold signal x_1^*. That there exists a pair of thresholds $\{x_1^*, x_2^*\}$ that simultaneously satisfy Equations (10) and (11) follows from Corollary 1.

Lemma 4 *In the limit as $\tau_1 \to \infty$, $\tau_2 \to \infty$, with $\frac{\tau_1}{\tau_2} \to c \in \overline{\mathbb{R}}$ the game has a unique equilibrium in monotone strategies characterized by thresholds $\left\{x_1^{*,\lim}, x_2^{*,\lim}\right\}$, where*

$$x_1^{*,\lim} = x_2^{*,\lim} = 2T$$

Proof. Let

$$x_1^{*,\lim} \equiv \lim_{\substack{\tau_i,\tau_j \to \infty \\ \tau_i/\tau_j \to c}} x_1^* \text{ and } x_2^{*,\lim} = \lim_{\substack{\tau_i,\tau_j \to \infty \\ \tau_i/\tau_j \to c}} x_2^*$$

First, note that agents will never use a threshold strictly larger than $\overline{\theta}$ as in that case they would be strictly better off using thresholds equal to $\overline{\theta}$. Similarly, agents will never use threshold strictly less than $\underline{\theta}$ as in that case they would be strictly better off using thresholds equal to $\underline{\theta}$.

Next, we argue that in the limit both agents have to use the same thresholds, that is

$$x_1^{*,\lim} = x_2^{*,\lim} = x^{*,\lim}$$

for some $x^{*,\lim} \in \mathbb{R}$.[37] To see this note that as τ_i,$\tau_j \to \infty$ (with $\tau_i/\tau_j \to c$) player i's payoff

[37] In what follows, we assume that the all limits are well defined. It is a standard exercise to show that this is indeed the case (see for example Szkup and Trevino (2020)).

indifference condition converges to

$$x_i^{*,\lim}\int_{\underline{L}_i}^{\overline{L}_i}\left[1-\Phi\left(\lim_{\substack{\tau_i,\tau_j\to\infty\\ \tau_i/\tau_j\to c}}\frac{x_j^*-x_i^*}{\tau_j^{-1/2}}\right)\right]\phi(z)\,dz+x_i^{*,\lim}\int_{\overline{L}_i}^{\infty}\phi(z)\,dz-T=0 \tag{12}$$

where

$$\overline{L}_i\equiv\lim_{\substack{\tau_i,\tau_j\to\infty\\ \tau_i/\tau_j\to c}}\frac{\overline{\theta}-\frac{\tau_i x_i^*+\tau_\theta\mu_\theta}{\tau_i+\tau_\theta}}{(\tau_i+\tau_\theta)^{-1/2}}\quad\text{and}\quad\underline{L}_i\equiv\lim_{\substack{\tau_i,\tau_j\to\infty\\ \tau_i/\tau_j\to c}}\frac{\underline{\theta}-\frac{\tau_i x_i^*+\tau_\theta\mu_\theta}{\tau_i+\tau_\theta}}{(\tau_i+\tau_\theta)^{-1/2}}$$

Similarly, the payoff indifference condition of player j converges to

$$x_j^{*,\lim}\int_{\underline{L}_j}^{\overline{L}_j}\left[1-\Phi\left(\lim_{\substack{\tau_i,\tau_j\to\infty\\ \tau_i/\tau_j\to c}}\left(\frac{\tau_i}{\tau_j}\right)^{1/2}\frac{x_i^*-x_j^*}{\tau_j^{-1/2}}\right)\right]\phi(z)\,dz+x_j^{*,\lim}\int_{\overline{L}_j}^{\infty}\phi(z)\,dz-T=0, \tag{13}$$

where $\overline{L}_j$ and $\underline{L}_j$ are defined analogously to $\overline{L}_i$ and $\underline{L}_i$. WLOG, suppose that $x_j^{*,\lim}>x_i^{*,\lim}$. In that case, Equation (12) becomes

$$x_i^{*,\lim}\int_{\overline{L}_i}^{\infty}\phi(z)\,dz-T=0 \tag{14}$$

while Equation (13) becomes

$$x_j^{*,\lim}\int_{\underline{L}_j}^{\infty}\phi(z)\,dz-T=0 \tag{15}$$

where

$$\overline{L}_i=\lim_{\substack{\tau_i,\tau_j\to\infty\\ \tau_i/\tau_j\to c}}\frac{\overline{\theta}-\frac{\tau_i x_i^*+\tau_\theta\mu_\theta}{\tau_i+\tau_\theta}}{(\tau_i+\tau_\theta)^{-1/2}}\geq\lim_{\substack{\tau_i,\tau_j\to\infty\\ \tau_i/\tau_j\to c}}\frac{\underline{\theta}-\frac{\tau_j x_j^*+\tau_\theta\mu_\theta}{\tau_j+\tau_\theta}}{(\tau_j+\tau_\theta)^{-1/2}}=\underline{L}_j$$

since $\overline{\theta}>\underline{\theta}$ and $x_j^{*,\lim}>x_i^{*,\lim}$. But since $\overline{L}_i\geq\underline{L}_j$, Equations (14) and (15) imply that $x_i^{*,\lim}\geq x_j^{*,\lim}$, a contradiction. Thus, we conclude that in the limit we have $x_i^{*,\lim}=x_j^{*,\lim}=x^{*,\lim}$.

It is straightforward to see that $x^{*,\lim}\in\left[\underline{\theta},\overline{\theta}\right]$. We now argue that $x^{*,\lim}\neq\overline{\theta}$ and $x^{*,\lim}\neq\underline{\theta}$. To see this $x^{*,\lim}\neq\overline{\theta}$ define

$$\varkappa\equiv\lim_{\substack{\tau_i,\tau_j\to\infty\\ \tau_i/\tau_j\to c}}\frac{x_j^*-x_i^*}{\tau_j^{-1/2}} \tag{16}$$

It follows that

$$\lim_{\substack{\tau_i,\tau_j\to\infty\\ \tau_i/\tau_j\to c}}\frac{x_i^*-x_j^*}{\tau_i^{-1/2}}=\lim_{\substack{\tau_i,\tau_j\to\infty\\ \tau_i/\tau_j\to c}}-\frac{x_j^*-x_i^*}{\tau_j^{-1/2}}\frac{\tau_j^{-1/2}}{\tau_i^{-1/2}}=-\varkappa\sqrt{c}$$

Given that $x_i^{*,\lim} = x_j^{*,\lim} = x^{*,\lim}$ it follows that player i's and player j's indifference conditions converge to

$$x^{*,\lim} \int_{\underline{L}}^{\overline{L}} \left[1 - \Phi(\varkappa)\right] \phi(z)\, dz + x^{*,\lim} \int_{\overline{L}}^{\infty} \phi(z)\, dz - T = 0 \tag{17}$$

and

$$x^{*,\lim} \int_{\underline{L}}^{\overline{L}} \left[1 - \Phi\left(-\varkappa\sqrt{c}\right)\right] \phi(z)\, dz + x^{*,\lim} \int_{\overline{L}}^{\infty} \phi(z)\, dz - T = 0, \tag{18}$$

respectively, where

$$\overline{L} \equiv \lim_{\substack{\tau_i,\tau_j\to\infty \\ \tau_i/\tau_j\to c}} \frac{\overline{\theta} - \frac{\tau_i x_i^* + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}} = \lim_{\substack{\tau_i,\tau_j\to\infty \\ \tau_i/\tau_j\to c}} \frac{\overline{\theta} - \frac{\tau_j x_j^* + \tau_\theta \mu_\theta}{\tau_j + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}}$$

$$\underline{L} \equiv \lim_{\substack{\tau_i,\tau_j\to\infty \\ \tau_i/\tau_j\to c}} \frac{\underline{\theta} - \frac{\tau_i x_i^* + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}} = \lim_{\substack{\tau_i,\tau_j\to\infty \\ \tau_i/\tau_j\to c}} \frac{\underline{\theta} - \frac{\tau_j x_j^* + \tau_\theta \mu_\theta}{\tau_j + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}}$$

Suppose that $\varkappa \neq 0$. Then Equations (17) and (18) can be simultaneously satisfied only if $\underline{L} = \overline{L}$. However, from the definition of $\underline{L} = \overline{L}$ we see that this can happen only if either $\underline{L} = \overline{L} = -\infty$ (which implies that $x^{*,\lim} \geq \overline{\theta}$) or $\underline{L} = \overline{L} = \infty$ (which implies that $x^{*,\lim} \leq \underline{\theta}$). In the former case, Equations (17) and (18) simplify to

$$x^{*,\lim} \int_{-\infty}^{\infty} \phi(z)\, dz - T = 0$$

implying that

$$\overline{\theta} \leq x^{*,\lim} = T,$$

which contradicts the fact that $\overline{\theta} \geq 2T$. In the latter case, we obtain $T = 0$, which is a contradicts the assumption of positive costs of investment. Thus, we conclude that $\varkappa = 0$, in which case Equations (17) and (18) simplify

$$x^{*,\lim} \frac{1}{2} \int_{\underline{L}}^{\overline{L}} \phi(z)\, dz + x^{*,\lim} \int_{\overline{L}}^{\infty} \phi(z)\, dz - T = 0 \tag{19}$$

It is straightforward to see that Equation (19) imply that $x^{*,\lim} \in \left(\underline{\theta}, \overline{\theta}\right)$. Therefore, in the limit, payoff indifference conditions for player i and j are simply given by

$$x^{*,\lim} \int_{-\infty}^{\infty} \frac{1}{2}\phi(z)\,dz - T = 0$$

implying that $x^{*,\lim} = 2T$. This completes the proof. ■

Having established uniqueness of equilibrium in the limit, we now provide conditions for the equilibrium to exist away from the limit. We first show that for any τ_1 and τ_2 there exists $\overline{\tau}_\theta(\tau_1, \tau_2)$ such that given τ_1 and τ_2 the coordination game has unique equilibrium whenever $\tau_\theta < \overline{\tau}_\theta(\tau_1, \tau_2)$. We then show that if we assume that $\tau_1, \tau_2 \geq \underline{\tau}$ for some arbitrary $\underline{\tau} \in \mathbb{R}$ then we can find a uniform bound on τ_θ, which we denote by $\underline{\tau}_\theta$ such that if $\tau_\theta < \underline{\tau}_\theta$ then the equilibrium is unique for *any* $\tau_1, \tau_2 \geq \underline{\tau}$.[38]

Theorem 1 *Let $\tau_1, \tau_2 \geq \underline{\tau}$. Then there exists $\underline{\tau}_\theta > 0$ such that for all $\tau_\theta < \underline{\tau}_\theta$ and for all $\tau_1, \tau_2 > \underline{\tau}$ there exists a unique, dominance solvable equilibrium of the coordination game in which both players use threshold strategies.*

Proof. As shown above, the coordination game belongs to the class of monotone supermodular games and therefore we know that there are the least and the greatest Bayesian Nash Equilibria in monotone strategies. Therefore, to establish the theorem, we only need to show that for sufficiently high τ_θ there exists a unique monotone equilibrium.

Any equilibrium in monotone strategies is determined by players' indifference equations

$$v(x_1^*, x_2^*; \tau_1, \tau_2) = 0 \quad (20)$$

$$v(x_2^*, x_1^*; \tau_1, \tau_2) = 0 \quad (21)$$

Let $\mathbf{v}(x_1^*, x_2^*; \tau_1, \tau_2) = 0$ denote the above system of equations written in a vector form. We argue that for sufficiently low τ_θ, the mapping $\mathbf{v} : \mathbb{R}^2 \to \mathbb{R}$ is univalent, which implies that the above system of equations has a unique solution.[39]

We argue first that the equilibrium thresholds belong to a bounded interval. In particular, let $\underline{x}_i(\tau_i, \tau_\theta)$ be the solution to

$$\int_{\underline{\theta}}^{\infty} \theta (\tau_x + \tau_\theta)^{1/2} \phi\left(\frac{\theta - \frac{\tau_i x_i^* + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_x + \tau_\theta)^{-1/2}} \right) d\theta = T,$$

that is $\underline{x}_i(\tau_i, \tau_\theta)$ is player i's optimal threshold when player j uses threshold $x_j^* = -\infty$ (i.e.,

[38] Of course the bound $\underline{\tau}_\theta$ depends on $\underline{\tau}$.

[39] A mapping $\mathbf{v} : E \to \mathbb{R}$, where $E \subset \mathbb{R}^n$, is univalent if $\mathbf{v}$ is one-to-one on its domain.

player j always takes the risky action) and $\overline{x}_i(\tau_i, \tau_\theta)$ is the solution to

$$\int_{\overline{\theta}}^{\infty} \theta (\tau_x + \tau_\theta)^{1/2} \phi \left(\frac{\theta - \frac{\tau_i x_i^* + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_x + \tau_\theta)^{-1/2}} \right) d\theta = T,$$

that is $\overline{x}_i(\tau_i, \tau_\theta)$ is player i's optimal threshold when player j uses threshold $x_j^* = \infty$ (i.e., player j always takes the safe action). Since from Corollary 2 we know that x_i^* is increasing in x_j^*, it follows that for given τ_i and τ_θ, $x_i^* \in [\underline{x}_i(\tau_i, \tau_\theta), \overline{x}_i(\tau_i, \tau_\theta)]$. Moreover, it is easy to see that both $\underline{x}_i(\tau_i, \tau_\theta)$ and $\overline{x}_i(\tau_i, \tau_\theta)$ are continuous in τ_i and τ_θ and have finite limit as $\tau_\theta \to 0$ for any $\tau_i > 0$. By symmetry, it follows that $x_j^* \in \left[\underline{x}_j(\tau_j, \tau_\theta), \overline{x}_j(\tau_j, \tau_\theta)\right]$, where $\underline{x}_j(\tau_j, \tau_\theta)$ and $\overline{x}_j(\tau_j, \tau_\theta)$ are defined analogously.

With this result in hand we now argue that $\mathbf{v}$ is univalent. As sufficient condition for $\mathbf{v}$ to be univalent is for the Jacobian of $\mathbf{v}$ to be diagonally dominant (see Parthasarathy (1983)). From the proof of Lemma 3 we know that

$$\begin{aligned} & \frac{\partial v\left(x_i^*, x_j^*; \tau_i, \tau_j\right)}{\partial x_i^*} - \left| \frac{\partial v\left(x_i^*, x_j^*; \tau_i, \tau_j\right)}{\partial x_j^*} \right| \qquad (22) \\ = & \frac{\tau_\theta}{\tau_i + \tau_\theta} \int_{L(\underline{\theta})}^{L(\overline{\theta})} \left[(\tau_i + \tau_\theta)^{-1/2} z + \frac{\tau_i x_i^* + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta} \right] \tau_j^{1/2} \phi \left(\frac{x_j^* - (\tau_i + \tau_\theta)^{-1/2} z - \frac{\tau_i x_i^* + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{\tau_j^{-1/2}} \right) \phi(z)\, dz \\ & + \int_{L(\underline{\theta})}^{L(\overline{\theta})} \frac{\tau_i}{\tau_i + \tau_\theta} \left[1 - \Phi \left(\frac{x_j^* - (\tau_i + \tau_\theta)^{-1/2} z - \frac{\tau_i x_i^* + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{\tau_j^{-1/2}} \right) \right] \phi(z)\, dz + \Gamma \end{aligned}$$

where

$$L(\overline{\theta}) \equiv \frac{\overline{\theta} - \frac{\tau_i x_i^* + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}} \quad \text{and} \quad L(\underline{\theta}) \equiv \frac{\overline{\theta} - \frac{\tau_i x_i^* + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}}$$

and

$$\begin{aligned} \Gamma(\tau_\theta) \equiv & \frac{\tau_i}{\sqrt{\tau_i + \tau_\theta}} \overline{\theta} \Phi \left(\frac{x_j^* - \overline{\theta}}{\tau_j^{-1/2}} \right) \phi \left(\frac{\overline{\theta} - \frac{\tau_i x_i^* + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}} \right) \\ & + \frac{\tau_i}{\sqrt{\tau_i + \tau_\theta}} \underline{\theta} \left[1 - \Phi \left(\frac{x_j^* - \overline{\theta}}{\tau_j^{-1/2}} \right) \right] \phi \left(\frac{\underline{\theta} - \frac{\tau_i x_i^* + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{(\tau_i + \tau_\theta)^{-1/2}} \right) \\ & + \int_{L(\overline{\theta})}^{\infty} \frac{\tau_i}{\tau_i + \tau_\theta} \phi(z)\, dz \\ > & \ 0 \end{aligned}$$

Moreover, $\lim_{\tau_\theta \to 0} \Gamma(\tau_\theta) \geq 0$ while

$$\lim_{\tau_\theta \to 0} \int_{L(\underline{\theta})}^{L(\overline{\theta})} \frac{\tau_i}{\tau_i + \tau_\theta} \left[1 - \Phi \left(\frac{x_j^* - (\tau_i + \tau_\theta)^{-1/2} z - \frac{\tau_i x_i^* + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{\tau_j^{-1/2}} \right) \right] \phi(z) \, dz > 0$$

for all τ_θ. Finally, we note that

$$\lim_{\tau_\theta \to 0} \frac{\tau_\theta}{\tau_i + \tau_\theta} \int_{L(\underline{\theta})}^{L(\overline{\theta})} \left[\frac{z}{(\tau_i + \tau_\theta)^{-1/2}} + \frac{\tau_i x_i^* + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta} \right] \tau_j^{1/2} \phi \left(\frac{x_j^* - \frac{z}{(\tau_i + \tau_\theta)^{-1/2}} - \frac{\tau_i x_i^* + \tau_\theta \mu_\theta}{\tau_i + \tau_\theta}}{\tau_j^{-1/2}} \right) \phi(z) = 0$$

Therefore, we conclude that for any pair of $\{\tau_1, \tau_2\}$ there exists $\overline{\tau}_\theta(\tau_1, \tau_2)$ such that if $\tau_\theta < \overline{\tau}_\theta(\tau_1, \tau_2)$ then for $i = 1, 2$, $j \neq i$, we have

$$\frac{\partial v\left(x_i^*, x_j^*; \tau_i, \tau_j\right)}{\partial x_i^*} - \left| \frac{\partial v\left(x_i^*, x_j^*; \tau_i, \tau_j\right)}{\partial x_j^*} \right| > 0$$

It follows that the Jacobian of $\mathbf{v}$ is diagonally dominant implying that there exists unique equilibrium in monotone strategies. Corollary 1 implies that this is the unique equilibrium of the second stage.

We now show that if $\tau_1, \tau_2 \geq \underline{\tau}$ for some $\underline{\tau} > 0$ then there exists $\overline{\tau}_\theta > 0$ such that for all $\tau_\theta < \overline{\tau}_\theta$ the equilibrium is unique (i.e., we want to establish existence of uniform bound on τ_θ that ensures the uniqueness of equilibrium). Towards this goal define

$$f(\tau_\theta) \equiv \frac{\partial v\left(x_i^*(\tau_\theta), x_j^*(\tau_\theta); \tau_i, \tau_j, \tau_\theta\right)}{\partial x_i^*} - \left| \frac{\partial v\left(x_i^*(\tau_\theta), x_j^*(\tau_\theta); \tau_i, \tau_j, \tau_\theta\right)}{\partial x_j^*} \right|$$

where we express x_i^* and x_j^* explicitly as functions of τ_θ. Let $\overline{\tau}_\theta(\tau_1, \tau_2)$ be the smallest solution to

$$f(\tau_\theta) = 0$$

so that for all $\tau_\theta < \overline{\tau}_\theta(\tau_1, \tau_2)$ we have $f(\tau_\theta) > 0$. We know that $\overline{\tau}_\theta(\tau_1, \tau_2)$ is well defined by the earlier argument. Moreover, note that $\partial v_i / \partial x_i^*$ and $\partial v_i / \partial x_j^*$ are continuous in all their argument, and so are x_i^* and x_j^*. Therefore, it follows that $\overline{\tau}_\theta(\tau_1, \tau_2)$ is a continuous function of τ_1 and τ_2 on $\mathcal{T} = \left\{ \{\tau_1, \tau_2\} \in \mathbb{R}^2 \middle| \tau_i \geq \underline{\tau},\ i = 1, 2 \right\}$, where $\underline{\tau} > 0$.

We now perform everywhere a change of variables $\sigma_1 = 1/\sqrt{\tau_1}$, $\sigma_2 = 1/\sqrt{\tau_2}$ and $\sigma_\theta = 1/\sqrt{\tau_\theta}$, that is we switch from using precisions to using standard deviations as a measure of informativeness. Note that the set of feasible choice of σ_i, $i = 1, 2$, is given by $[0, \underline{\sigma}]$, where $\underline{\sigma} \equiv 1/\sqrt{\underline{\tau}}$. According to the above argument, for each $\{\sigma_1, \sigma_2\} \in [0, \underline{\sigma}] \times [0, \underline{\sigma}]$ there exists $\overline{\sigma}_\theta(\sigma_1, \sigma_2)$ such that for all $\sigma_\theta \geq \overline{\sigma}_\theta(\sigma_1, \sigma_2)$ the equilibrium is unique. Moreover, $\overline{\sigma}_\theta(\sigma_1, \sigma_2)$ is continuous and finite for all $\{\sigma_1, \sigma_2\} \in [0, \underline{\sigma}] \times [0, \underline{\sigma}]$. But since $[0, \underline{\sigma}] \times [0, \underline{\sigma}]$ is compact and $\overline{\sigma}_\theta(\sigma_1, \sigma_2)$ is continuous it follows that $\overline{\sigma}_\theta(\sigma_1, \sigma_2)$ achieves a maximum on $[0, \underline{\sigma}] \times [0, \underline{\sigma}]$, call it $\overline{\sigma}_\theta$, with $\overline{\sigma}_\theta < \infty$. Setting $\overline{\tau}_\theta = 1/\overline{\sigma}_\theta^2$ completes the proof. ■

Lemma 5 *The risk-dominant equilibrium of the coordination game is a strategy-pair $\{\beta_1^{RD}, \beta_2^{RD}\}$ such that*

$$\beta_i^{RD} = \begin{cases} \text{Choose action } A & \text{if } \theta \geq 2T \\ \text{Choose action } B & \text{if } \theta < 2T \end{cases}$$

Proof. Recall that according to Harsanyi and Selten (1988) a strategy A risk-dominates strategy B if A is associated with the larger product of deviation losses.[40] When both players play A then the product of players' deviation losses is equal to $(\theta - T)^2$. If both players play B then the product of players' deviation losses is equal to T^2. Therefore, A risk-dominates B if

$$(\theta - T)^2 \geq T^2,$$

or if

$$\theta \geq 2T$$

■

B.3 Information Acquisition

In Section B.2 we showed that for any pair $\{\tau_1, \tau_2\}$ of players precision choices such that $\tau_i \geq \underline{\tau}$, $i = 1, 2$, the coordination stage has a unique equilibrium in monotone strategies. The goal of this section is to prove existence and uniqueness of equilibrium in the model with information acquisition. Throughout this section we make the following assumption.

Assumption 1 *The precision level $\underline{\tau}$ is large enough so that for all $\{\tau_1, \tau_2\} \geq \underline{\tau}$ we have $\{x_1^*, x_2^*\} \in (\underline{\theta}, \overline{\theta}) \times (\underline{\theta}, \overline{\theta})$. Moreover, τ_θ is large enough so that equilibrium of the coordination game is unique for any feasible choice of precision (i.e., for any $\{\tau_1, \tau_2\} \geq \underline{\tau}$).*

Note that by Lemma 4 and Theorem 1 such $\underline{\tau}$ and τ_θ exist. Having made the relevant assumptions we now analyze players' precision choices. Each player i chooses his precisions privately to maximize his ex-ante expected utility given by

$$U(\tau_i; \tau_j) = B\left(\tau_i; \tau_j, x_j^*\right) - \widehat{C}(\tau_i),$$

where

$$B\left(\tau_i; \tau_j, x_j^*\right) \equiv \int_{x_i^*}^{\infty} v\left(x_i; x_j^*, \tau_i, \tau_j\right) \frac{1}{\sqrt{\tau_\theta^{-1} + \tau_i^{-1}}} \phi\left(\frac{x_i - \mu_\theta}{\sqrt{\tau_\theta^{-1} + \tau_i^{-1}}}\right) dx_i$$

[40] Player 1's deviation loss from $\{A, A\}$ is given by the change in player 1's payoff in case he switches from A to B, when player 2 plays A. Similarly, player 1's deviation loss from $\{B, B\}$ is given by the change in player 1's payoff in case he switches from B to A, when player 2 plays B.

is the expected benefit to player i from acquiring precision τ_i when player i expects the other player to choose precision τ_j and use threshold x_j^* and

$$\widehat{C}(\tau_i) \equiv C\left(\tau_i^{-1/2}\right) = C(\sigma_i)$$

is the cost of precision τ_i. We now establish several useful properties of function $B\left(\tau_i; \tau_j, x_j^*\right)$.

Lemma 6 *Consider the benefit function* $B\left(\tau_i; \tau_j, x_j^*\right)$.

1. $\lim_{\tau_i \to 0} \frac{\partial B\left(\tau_i;\tau_j,x_j^*\right)}{\partial \tau_i} = 0$
2. $\frac{\partial B\left(\tau_i;\tau_j,x_j^*\right)}{\partial \tau_i} > 0$ *for all* $\tau_i < \infty$
3. $B\left(\tau_i; \tau_j, x_j^*\right)$ *is concave in* τ_i.

Proof. (Part 1) Note that $B\left(\tau_i; \tau_j, x_j^*\right)$ is given by

$$\int_{-\infty}^{\infty} \Phi\left(\frac{\theta - x_i^*}{\tau_i^{-1/2}}\right)\left[\theta \mathbf{1}_{\{\theta \geq \overline{\theta}\}} + \theta \mathbf{1}_{\{\theta \in [\underline{\theta}, \overline{\theta}]\}}\left(1 - \Phi\left(\frac{x_j^* - \theta}{\tau_j^{-1/2}}\right)\right) - T\right]\tau_\theta^{1/2}\phi\left(\frac{\theta - \mu_\theta}{\tau_\theta^{-1/2}}\right)d\theta \quad (23)$$

Differentiating the above expression w.r.t. τ_i and performing a change of variables $z = \tau_x^{1/2}(\theta - x_i^*)$ in the resulting expression we obtain

$$\frac{\partial B\left(\tau_i; \tau_j, x_j^*\right)}{\partial \tau_i} = \int_{-\infty}^{\infty}\left[z\left(\frac{z}{\tau_i^{1/2}} + x_i^*\right)\left\{\mathbf{1}_{\{\theta \geq \underline{\Lambda}_i\}} + \mathbf{1}_{\{\theta \in [\underline{\Lambda}_i, \overline{\Lambda}_i]\}}\left(1 - \Phi\left(\frac{x_j^* - \frac{z}{\tau_i^{1/2}} - x_i^*}{\tau_j^{-1/2}}\right)\right)\right\} - T\right] \quad (24)$$

$$\times \frac{1}{2\tau_i^{3/2}}\tau_\theta^{1/2}\phi\left(\frac{\frac{z}{\tau_i^{1/2}} + x_i^* - \mu_\theta}{\tau_\theta^{-1/2}}\right)\phi(z)\,dz,$$

where $\underline{\Lambda}_i = \tau_i^{1/2}(\underline{\theta} - x_i^*)$ and $\overline{\Lambda}_i = \tau_i^{1/2}(\overline{\theta} - x_i^*)$. Applying dominated convergence theorem to interchange the order of limit and integration, we conclude that

$$\lim_{\tau_i \to \infty} \frac{\partial B\left(\tau_i; \tau_j, x_j^*\right)}{\partial \tau_i} = 0$$

(Part 2): Differentiating the expression for $B\left(\tau_i; \tau_j, x_j^*\right)$ in (23) we obtain

$$\frac{\partial B\left(\tau_i; \tau_j, x_j^*\right)}{\partial \tau_i} = \int_{-\infty}^{\infty}\left\{\left[\theta 1_{\{\theta \geq \overline{\theta}\}} + \theta 1_{\{\theta \in [\underline{\theta}, \overline{\theta}]\}}\left(1 - \Phi\left(\frac{x_j^* - \theta}{\tau_j^{-1/2}}\right)\right) - T\right]\right.$$

$$\left. \times\, \tau_\theta^{1/2}\phi\left(\frac{\theta - \mu_\theta}{\tau_\theta^{-1/2}}\right)\phi\left(\frac{\theta - x_i^*}{\tau_i^{-1/2}}\right)\left(\frac{\theta - x_i^*}{2\tau_i^{1/2}}\right)\right\}d\theta \quad (25)$$

Equation (25) can be equivalently written as

$$\begin{aligned}\frac{\partial B\left(\tau_i;\tau_j,x_j^*\right)}{\partial \tau_i} &= \int_{-\infty}^{\infty}\left\{\left[\theta 1_{\left\{\theta\geq\bar{\theta}\right\}}+\theta 1_{\left\{\theta\in\left[\underline{\theta},\bar{\theta}\right]\right\}}\left(1-\Phi\left(\frac{x_j^*-\theta}{\tau_j^{-1/2}}\right)\right)-T\right]\right.\\ &\quad \left.\times\left(\tau_i+\tau_\theta\right)^{1/2}\phi\left(\frac{\theta-\frac{\tau_i x_i^*+\tau_\theta\mu_\theta}{\tau_i+\tau_\theta}}{\left(\tau_i+\tau_\theta\right)^{1/2}}\right)\left(\frac{\tau_\theta\tau_i}{\tau_i+\tau_\theta}\right)^{1/2}\phi\left(\frac{x_i^*-\mu_\theta}{\sqrt{\frac{\tau_i+\tau_\theta}{\tau_\theta\tau_i}}}\right)\left(\frac{\theta-x_i^*}{2\tau_i}\right)\right\} \qquad (26)\end{aligned}$$

Now, let

$$\begin{aligned} f\left(\theta\right) &= \left(\frac{\tau_\theta\tau_i}{\tau_i+\tau_\theta}\right)^{1/2}\phi\left(\frac{x_i^*-\mu_\theta}{\sqrt{\frac{\tau_i+\tau_\theta}{\tau_\theta\tau_i}}}\right)\theta,\\ g\left(\theta\right) &= \theta 1_{\left\{\theta\geq\bar{\theta}\right\}}+\theta 1_{\left\{\theta\in\left[\underline{\theta},\bar{\theta}\right]\right\}}\left(1-\Phi\left(\frac{x_j^*-\theta}{\tau_j^{-1/2}}\right)\right)-T,\\ w\left(\theta\right) &= \left(\tau_i+\tau_\theta\right)^{1/2}\phi\left(\frac{\theta-\frac{\tau_i x_i^*+\tau_\theta\mu_\theta}{\tau_i+\tau_\theta}}{\left(\tau_i+\tau_\theta\right)^{1/2}}\right)\end{aligned}$$

and note that f and g are increasing functions and $w\left(\theta\right)$ is a pdf. Therefore, the continuous version of weighted Chebyshev's sum inequality implies that

$$\begin{aligned}\frac{\partial B\left(\tau_i;\tau_j,x_j^*\right)}{\partial \tau_i} &= \int_{-\infty}^{\infty} f\left(\theta\right)g\left(\theta\right)w\left(\theta\right)d\theta\\ &\geq \int_{-\infty}^{\infty} f\left(\theta\right)w\left(\theta\right)d\theta\int_{-\infty}^{\infty} g\left(\theta\right)w\left(\theta\right)d\theta\end{aligned}$$

Moreover, since f and g are strictly increasing for all $\theta>\underline{\theta}$ and $w\left(\theta\right)>0$ for all θ, we conclude that the above inequality is strict. Finally, note that

$$\int_{-\infty}^{\infty} g\left(\theta\right)w\left(\theta\right)d\theta = v\left(x_i^*,x_j^*;\tau_i,\tau_j\right)=0$$

Therefore, we conclude that

$$\frac{\partial B\left(\tau_i;\tau_j,x_j^*\right)}{\partial \tau_i}>0$$

(Part 3): We have

$$\frac{\partial^2 B\left(\tau_i;\tau_j,x_j^*\right)}{\partial\tau_i^2} = -\frac{1}{4\tau_i^2}\int_{-\infty}^{\infty}\left\{\left[\theta 1_{\{\theta\geq\overline{\theta}\}}+\theta 1_{\{\theta\in[\underline{\theta},\overline{\theta}]\}}\left(1-\Phi\left(\frac{x_j^*-\theta}{\tau_j^{-1/2}}\right)\right)-T\right]\right.$$
$$\left.\tau_\theta^{1/2}\phi\left(\frac{\theta-\mu_\theta}{\tau_\theta^{-1/2}}\right)\phi\left(\frac{\theta-x_i^*}{\tau_i^{-1/2}}\right)\frac{\theta-x_i^*}{\tau_i^{-1/2}}\left[1+\left(\frac{\theta-x_i^*}{\tau_i^{-1/2}}\right)^2\right]\right\}d\theta$$

Note that

$$\tau_\theta^{1/2}\phi\left(\frac{\theta-\mu_\theta}{\tau_\theta^{-1/2}}\right)\phi\left(\frac{\theta-x_i^*}{\tau_i^{-1/2}}\right)=(\tau_\theta+\tau_x)^{1/2}\phi\left(\frac{\theta-\frac{\tau_i x_i^*+\tau_\theta\mu_\theta}{\tau_i+\tau_\theta}}{(\tau_i+\tau_\theta)^{-1/2}}\right)\left(\frac{\tau_\theta}{\tau_i+\tau_\theta}\right)^{1/2}\phi\left(\frac{x_i^*-\mu_\theta}{\sqrt{\frac{\tau_i+\tau_\theta}{\tau_\theta\tau_i}}}\right)$$

Therefore,

$$\frac{\partial^2 B\left(\tau_i;\tau_j,x_j^*\right)}{\partial\tau_i^2}=-\frac{1}{4\tau_i^2}\int_{-\infty}^{\infty} l(\theta)\,k(\theta)\,w(\theta)\,d\theta$$

where

$$k(\theta) \equiv \frac{\theta-x_i^*}{\tau_i^{-1/2}}\left[1+\left(\frac{\theta-x_i^*}{\tau_i^{-1/2}}\right)^2\right]\left(\frac{\tau_\theta}{\tau_i+\tau_\theta}\right)^{1/2}\phi\left(\frac{x_i^*-\mu_\theta}{\sqrt{\frac{\tau_i+\tau_\theta}{\tau_\theta\tau_i}}}\right)$$
$$l(\theta) \equiv \left[\theta 1_{\{\theta\geq\overline{\theta}\}}+\theta 1_{\{\theta\in[\underline{\theta},\overline{\theta}]\}}\left(1-\Phi\left(\frac{x_j^*-\theta}{\tau_j^{-1/2}}\right)\right)-T\right]$$
$$w(\theta) = (\tau_\theta+\tau_x)^{1/2}\phi\left(\frac{\theta-\frac{\tau_i x_i^*+\tau_\theta\mu_\theta}{\tau_i+\tau_\theta}}{(\tau_i+\tau_\theta)^{-1/2}}\right)$$

Moreover, note that both $k(\theta)$ and $l(\theta)$ are both increasing in θ and strictly increasing for all $\theta\geq\overline{\theta}$ while $w(\theta)$ is a pdf. Therefore, the weighted continuous version of weighted Chebyshev's sum inequality and the properties of $k(\theta)$ and $l(\theta)$ imply that

$$\begin{aligned}\frac{\partial^2 B\left(\tau_i;\tau_j,x_j^*\right)}{\partial\tau_i^2} &= -\frac{1}{4\tau_i^2}\int_{-\infty}^{\infty} l(\theta)\,k(\theta)\,w(\theta)\,d\theta \\ &< -\frac{1}{4\tau_i^2}\int_{-\infty}^{\infty} l(\theta)\,w(\theta)\,d\theta\int_{-\infty}^{\infty} k(\theta)\,w(\theta)\,d\theta\end{aligned}$$

Finally, we note that

$$\int_{-\infty}^{\infty} l(\theta)\,w(\theta)\,d\theta = v\left(x_i^*,x_j^*;\tau_i,\tau_j\right)=0$$

Thus, we conclude that $\partial^2 B\left(\tau_i;\tau_j,x_j^*\right)/\partial\tau_i^2<0$, which establishes the claim. ■

With this result in hand we can now prove the existence of equilibrium in our global game model with information acquisition.

Theorem 2 *There exists a symmetric pure-strategy Bayesian Nash Equilibrium of the game with information acquisition.*

Proof. From Lemma 6 we know $B\left(\tau_i;\tau_j,x_j^*\right)$ is a concave increasing function function with $\lim_{\tau_i\to\infty} B\left(\tau_i;\tau_j,x_j^*\right)=0$. Therefore, the best response function

$$\tau_i^*(\tau_j)=\arg\max_{\tau_i\geq\underline{\tau}} U_i\left(\tau_i;\tau_j\right)$$

is well defined and continuous in τ_j. At this point, it is convenient to go back to considering players' choices of standard deviations rather than precisions. In particular, let $\underline{\sigma}=1/\sqrt{\underline{\tau}}$ and define $\sigma_i^*:[0,\underline{\sigma}]\to[0,\underline{\sigma}]$ with $\sigma_i^*\left(\sigma_j\right)=1/\sqrt{\tau_i^*\left(1/\sigma_j^2\right)}$. Thus, σ_i^* is player i's best response to player j's choice of standard deviation. Note that since τ_i^* is continuous so is σ_i^*. Therefore, the best-response $\boldsymbol{\sigma}=\left\{\sigma_i^*,\sigma_j^*\right\}$ is a continuous function that maps $[0,\underline{\sigma}]^2$ into $[0,\underline{\sigma}]^2$. Hence, by Brouwer's Fixed Point Theorem, $\sigma^*(\cdot)$ has a fixed point. This completes the proof. ■