Sentiments, strategic uncertainty, and information structures in coordination games*

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Abstract

We study experimentally how changes in the information structure affect behavior in coordination games with incomplete information (global games). We find two systematic departures from the theory: (1) the comparative statics of equilibrium thresholds and signal precision are reversed, and (2) as information becomes very precise subjects' behavior approximates the efficient equilibrium of the game, not the risk dominant one. We hypothesize that sentiments in the perception of strategic uncertainty could drive our results. To formalize this hypothesis we extend the standard model by introducing sentiments and we test this mechanism experimentally by eliciting beliefs. We find empirical support for our hypothesis: Subjects are over-optimistic about the actions of others when the signal precision is high and over-pessimistic when it is low. Thus, we show how changes in the information structure can give rise to sentiments that drastically affect outcomes in coordination games.

Keywords: global games, coordination, information structures, strategic uncertainty, sentiments, biased beliefs.

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1 Introduction

Many economic phenomena can be analyzed as coordination problems under uncertainty. Investment decisions, currency attacks, bank runs, or political revolts are situations where decision makers

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would like to coordinate with others to attain certain outcomes but may fail to do so. In addition to the strategic uncertainty that arises from not knowing the actions of others, in these environments decision makers also face uncertainty about the fundamentals that determine the state of the economy (e.g., the profitability of the investment, the strength of the currency peg, or the strength of the political regime). In these environments the information structure characterizes both the degree of fundamental and strategic uncertainty faced by the players, and hence determines the coordination outcome. Thus, a key question is whether better information leads to less coordination failure (see e.g., Angeletos and Lian (2016), Veldkamp (2011), or Vives (2010)).

In this paper we investigate theoretically and experimentally how changes in the information structure affect behavior in coordination games with incomplete information. We use global games as the setup to perform our analysis because they offer a unique framework to study explicitly the effects of fundamental and strategic uncertainty on behavior (see Carlsson and Van Damme (1993), Morris and Shin (1998, 2003)). Global games are coordination games with incomplete information where players observe noisy private signals about payoffs. This perturbation in the information structure leads to a unique equilibrium under mild conditions on parameters that is characterized by coordination failure. In these games, the precision of the signals determines the degree of fundamental and strategic uncertainty. This leads to sharp theoretical predictions: in two-player settings, as the signal precision increases (fundamental uncertainty decreases), equilibrium play is driven more by strategic uncertainty and less by fundamental uncertainty. This eventually leads to convergence in the limit towards the risk-dominant equilibrium of the underlying complete information game.

The contribution of our paper is twofold. First, we document two systematic deviations from the predictions of the theory of global games: (1) the comparative statics of thresholds with respect to signal precision are reversed and (2) as the signal noise decreases, subjects' behavior tends towards the efficient threshold, not the risk-dominant one. These results are robust to a number of variations in the experimental setting. Second, we argue that these departures can be understood by sentiments in the perception of strategic uncertainty and provide evidence supporting this notion.

We begin the paper by setting up a standard global game model that we use as the theoretical benchmark for our experiment. We characterize its unique equilibrium and comparative statics and explain how the equilibrium is affected by the degree of fundamental and strategic uncertainty. In the experiment we vary fundamental uncertainty by exogenously manipulating the signal precision. We find that the vast majority of subjects use threshold strategies, as suggested by the theory and consistent with existing experimental evidence (see Heinemann, Nagel, and Ockenfels (2004, 2009)). However, as mentioned above, we find two systematic departures in the way that subjects respond to changes in the information structure, which have significant welfare effects. These departures suggest that, contrary to the theoretical predictions, the perception of strategic uncertainty might be directly aligned to the degree of fundamental uncertainty in the environment. That is, subjects behave as if they were more certain about the action of their opponent when signals become more precise.

To explain our findings we hypothesize that the departures from the theory are driven by sentiments in the perception of strategic uncertainty. Our hypothesis is based on the observation that the expected payoffs in our model can be approximated by the product of the expected value of the state (determined by the degree of fundamental uncertainty) and the probability that the other player takes the risky action (which captures strategic uncertainty). This simple observation suggests that departures from the theory can be driven by sentiments related to fundamental uncertainty (subjective beliefs about the state of fundamentals) or to strategic uncertainty (subjective beliefs about the likelihood of the opponent taking a specific action). However, when information is very precise, signals convey very accurate information about the state, suggesting that, at least in the case of high precision, departures from the theory are unlikely to be driven by a subjective perception of fundamental uncertainty. Therefore, we hypothesize that our experimental findings are a result of sentiments that affect the perception of strategic uncertainty and that these sentiments are directly affected by the degree of fundamental uncertainty in the environment.

To test this hypothesis we extend our baseline model to allow for situations where the perception of strategic uncertainty can be influenced by sentiments. The extended model allows us to formalize our hypothesis. We then test this hypothesis empirically by eliciting subjects' beliefs about the state and about the actions of others to explicitly test the two main assumptions behind the extended model: that subjects do not exhibit sentiments related to fundamental uncertainty and that sentiments related to strategic uncertainty become positive and stronger as private information becomes more precise. We find support for both of these assumptions in the data. On average, subjects form accurate first-order beliefs, especially with high and medium precision, supporting the idea that sentiments related to fundamentals are an unlikely driver of our results. On the other hand, elicited beliefs about the actions of others indicate that subjects are overly optimistic about the desire of their opponent to coordinate when information is very precise, and pessimistic when the signal noise increases. This suggests that subjects anchor their perception of strategic uncertainty to the degree of fundamental uncertainty. In other words, that fundamental uncertainty determines the sign and magnitude of the sentiments that affect the subjects' perception of strategic uncertainty in the way suggested by the extended model.

The results of our experiments show how a bias in belief formation that has been extensively studied in individual decision making can crucially affect outcomes in strategic environments by altering the way in which players respond to changes in the information structure. The sentiments that we identify, however, are different to the biases that are typically studied in the behavioral literature (which focuses mainly on individual decision making) because they affect the perception of strategic uncertainty, which is unique to strategic environments.[1] We find that the sign and magnitude of the sentiments we identify depend on the informativeness of the environment, which has important welfare implications for coordination problems. For example, more successful coordination can be attained when information is very precise because it leads players to become overoptimistic about the likelihood of a successful coordination. On the other hand, coordination failures are likely to occur under very noisy information because players tend to become overly pessimistic about the likelihood of a successful coordination.

Our results not only provide novel qualitative insights about behavior in coordination problems with incomplete information, but they may also shed light on some of the recent findings in macroeconomics and finance. For example, in the context of business cycles, Bloom (2009) suggests that recessions are accompanied by an increase in uncertainty, while Angeletos and La'O (2013) and Benhabib, Wang, and Wen (2015) argue that recessions can be driven by sentiments. Our framework provides a natural connection between these two seemingly unrelated ideas. As our results show, an increase in uncertainty leads to negative sentiments about the likelihood of profitable risky investment (via pessimism about others investing). This leads to lower levels of aggregate investment, which leads to, or amplifies, a recession. Our results could also inform policy making qualitatively. For example, the mechanism we identify could help to make the case for greater transparency in financial regulation. If we interpret rolling over loans as a risky choice in environments with strategic complementarities (as in Diamond and Dybvig (1983) or Goldstein and Pauzner (2005)), an increase in transparency (better information) can lead to positive sentiments about the likelihood of others rolling over, which decreases coordination failure by having fewer early withdrawals and can result in greater financial stability.[2]

Our baseline setup is a discrete version of Morris and Shin (1998) and the corresponding experimental treatments are related to Heinemann, Nagel, and Ockenfels (2004) who test the predictions of Morris and Shin (1998). Our model with sentiments is related to Izmalkov and Yildiz (2010) who study theoretically how sentiments affect outcomes in global games of regime change. However, we use a different notion of sentiments which is driven by our experimental findings. In Izmalkov and Yildiz (2010) sentiments arise because of a non-common prior assumption, and thus affect the perception of fundamental uncertainty. In our setup all subjects share the same prior belief and

[1] If the sentiments were related to the perception of fundamental uncertainty (beliefs about the state), they would be closer to the biases studied in the individual decision making literature (see Weinstein (1980) or Camerer and Lovallo (1999)).

[2] Macroeconomic environments, clearly, deal with a much larger number of agents than our stylized 2-player case. As has been shown in the experimental literature, coordiantion is more easily sustained with a small number of players. However, we find that as fundamental uncertainty increases it becomes harder to coordinate, even with only 2 players. This result could be amplified by increasing the number of players. It is important to stress that our results are qualitative since they focus on comparative statics, not point predictions. That is, we posit that agents might be more likely to coordinate when they have better information, but we are agnostic about actual rates of coordination (which might depend on number of players, cultural differences, experience, etc.)

sentiments arise because of the uncertainty that a player faces regarding the interpretation and/or use of information by his opponent.

Our paper contributes to the literature that studies coordination games both experimentally and theoretically. Harsanyi and Selten (1988) define risk dominance and payoff dominance as two contrasting equilibrium refinements for coordination games with multiple equilibria. They suggest that in the presence of Pareto ranked equilibria risk dominance is irrelevant since "collective rationality" should select the payoff-dominant equilibrium. However, experimental evidence highlights how strategic uncertainty can lead to coordination failure in games with complete information and Pareto ranked equilibria (see Van Huyck, Battalio, and Beil (1990, 1991), Cooper, DeJong, Forsythe, and Ross (1990, 1992), or Straub (1995)). More recent literature studies how the information structure affects equilibrium in coordination games. Theoretical contributions include Angeletos and Pavan (2007), Bannier and Heinemann (2005), Colombo, Femminis and Pavan (2014), Hellwig and Veldkamp (2009), Iachan and Nenov (2015), Pavan (2016), Szkup and Trevino (2015), and Yang (2015), among others. Darai, Kogan, Kwasnica, and Weber (2017) and Avoyan (2017) use a global game setting to study experimentally how different types of public signals and cheap talk affect coordination outcomes, respectively. Cornand and Heinemann (2014) and Baeriswyl and Cornand (2016) propose alternative ways of thinking about coordination in experiments about the closely related family of beauty contest games. This paper also contributes to the literature that incorporates aspects of bounded rationality to propose alternative equilibrium notions, such as Nagel (1995), Costa-Gomes and Crawford (2006), McKelvey and Palfrey (1995), Eyster and Rabin (2005), or Koessler and Jehiel (2008).

The paper is structured as follows. Section 2 presents the theoretical benchmark for the experiment and discusses the role of fundamental and strategic uncertainty in the unique equilibrium of a global game. Section 3 presents the experimental design and the theoretical predictions for the parameters used in the experiment. In Section 4 we present our experimental findings and characterize the main departures from the theory. In Section 5 we propose an extension to the model of Section 2 that allows for sentiments in the perception of strategic uncertainty in an effort to reconcile our findings with the theory and we show the results of additional experiments that we provide evidence to support it. Section 6 discusses and compares alternative explanations for the observed departures from the theory and concludes.

2 The model

In this section we describe the theoretical model that serves as a benchmark for our experiment, which is similar to Carlsson and van Damme (1993), Frankel, Morris, and Pauzner (2003), and Morris and Shin (2003).

2.1 The setup

There are two identical players in the economy, $i \in \{1, 2\}$, who simultaneously choose whether to take action A or action B. Action B is safe and always delivers a payoff of 0. Action A is risky and has a cost T associated to it. Action A delivers a payoff of $\theta - T$ if it is successful and $-T$ if it fails, where $\theta \in \mathbb{R}$ is a random variable that reflects the state of the economy.[3] Action A succeeds if both players choose action A and $\theta > \underline{\theta}$ (the state is high enough to make action A profitable), or if $\theta \geq \overline{\theta}$ (the state is high enough that the success of action A does not depend on player j's choice). The parameters $\overline{\theta}$ and $\underline{\theta}$ define, respectively, upper and lower dominance regions for the fundamental. Thus, players face the following payoffs:

	Success	Failure
Action A	$\theta - T$	$-T$
Action B	0	0

The state variable θ follows a normal distribution with mean μ_θ and variance σ_θ^2. Players do not observe the realization of θ. However, each player $i = 1, 2$ observes a noisy private signal about it:

$$x_i = \theta + \sigma \varepsilon_i,$$

where $\sigma > 0$ and $\varepsilon_i \sim N(0, 1)$. The noise ε_i is *i.i.d.* across players and we denote by $\phi(\cdot)$ its probability density function, and $\Phi(\cdot)$ its cumulative distribution function. The precision of the signal that each player receives is determined by its standard deviation, σ.[4]

2.2 Equilibrium

The equilibrium of our game can be computed using the standard approach. Since the solution method has been well established in the literature, we refer an interested reader to papers such as Carlsson and van Damme (1993), Morris and Shin (2003), or Hellwig (2002) for details. As shown in these papers, player $i = 1, 2$ uses a monotone threshold strategy by which he chooses action A if his signal x_i is larger than a threshold x^*, and he chooses action B otherwise. Given the use of threshold x^* by all players, the threshold x^* is determined by the solution to the following

[3] One can think of a number of applications where θ represents the relevant fundamentals. For example, it can represent the return to a risky investment or the gain from overthrowing an oppressive regime. In the first case, the action of players would be to invest (A) or not (B). In the second case, the action would be to engage in a political protest (A) or not (B). We can think of T as an investment cost in the first case, or as an opportunity cost in the second.

[4] The precision of a random variable that follows normal distribution is defined as the inverse of its variance. In what follows we use higher precision, lower variance, or lower standard deviation interchangeably to describe the informativeness of signals.

indifference condition:

$$E\Big[\theta \Pr(x_j > x^*|\theta)\,\Big|x^*, \theta \in [\underline{\theta}, \overline{\theta}]\,\Big] + E\Big[\theta\Big|x^*, \theta > \overline{\theta}\Big] - T = 0 \tag{1}$$

which states that at the threshold signal x^* player i is indifferent between taking action A and taking action B.

Proposition 1 is standard in the literature and establishes that the coordination game has a unique equilibrium as long as the standard deviation of the prior, σ_θ, is high enough. Moreover, as the precision of the signals increases, the optimal thresholds converge to the risk-dominant threshold for the underlying complete information game, which in our case is equal to $2T$.

Proposition 1 *Fix $\sigma_\theta > 0$. There exists $\overline{\sigma} > 0$ such that for all $\sigma < \overline{\sigma}$ there exists a unique, dominance solvable equilibrium in which both players use threshold strategies characterized by $x^*(\sigma)$. Moreover as $\sigma \searrow 0$ this equilibrium threshold converges to the risk-dominant equilibrium of the complete information game.*

Even though this result has been established in the literature, we provide an indirect proof for our specific setup in the appendix, where this benchmark specification can be easily identified a special case of the extended model of Section 5. We test these predictions experimentally in an effort to understand whether subjects use threshold strategies and how these thresholds depend on the informativeness of signals (determined by σ). In particular, the treatment variations in the signal precision will allow us to approximate the path towards complete information with the objective to document empirically whether thresholds "converge" to a specific equilibrium of the complete information game.

2.3 Strategic uncertainty

We briefly describe how strategic uncertainty in this game depends on the precision of private signals, which is key to understand why the equilibrium threshold converges to the risk dominant equilibrium as the signal noise vanishes. We will revisit these concepts in Section 5 when we extend the model to reconcile the theory with our experimental findings.

One typically refers to strategic uncertainty as the uncertainty about the actions of other players (see Van Huyck et al., (1990) or Brandenburger (1996)). According to this definition, strategic uncertainty is high if a player is very uncertain about the behavior of others. In our model the key object that allows us to measure strategic uncertainty faced by player i is $\Pr(x_j > x^*|x_i)$, which represents the probability that player i assigns to player j taking action A. If $\Pr(x_j > x^*|x_i)$ is close to $1/2$ then player i deems each action by player j almost equally likely and hence faces high strategic uncertainty. On the other hand, if $\Pr(x_j > x^*|x_i)$ is close to 0 or 1 then he expects player j to take a particular action, thus he faces little strategic uncertainty.

The extent of strategic uncertainty that players face in the game varies with σ. As σ decreases, player i's signal is closer to the state θ, so he is able to better estimate player j's signal. Thus, for signals $x_i \neq x^*$ if he receives a high (low) signal, he believes that player j also receives a high (low) signal and he assigns a higher probability to player j choosing action A (action B). Consider now the case where player i observes the signal $x_i = x^*$, that is, a signal equal to his opponent's threshold. In this case, an increase in the precision of x_i will increase strategic uncertainty since $\Pr(x_j > x^*|x_i = x^*)$ converges monotonically to 1/2. Thus, for signals around x^*, the strategic uncertainty faced by player i increases as σ decreases. This leads to the limit result in Proposition 1, first shown by Carlsson and Van Damme (1993) for homogenous signal distributions.

To summarize, a reduction of fundamental uncertainty (an increase in the precision of private information) increases strategic uncertainty for intermediate signals (i.e., signals in the neighborhood of x^*) and decreases strategic uncertainty for high or low signals. In fact, it is the increase in strategic uncertainty for intermediate signals that determines the convergence of player i's threshold towards risk dominance in the limit, as fundamental uncertainty decreases. Our experimental test of this model will allow us to investigate whether this relation between strategic and fundamental uncertainty is consistent with the behavior of subjects.

3 Experimental design

In this section we describe our experimental design and the predictions of the model that we test. We implement a between subjects design that allows us to directly compare the behavior of subjects across treatments.

There are two main dimensions in which our treatments vary: the precision of the private signals and whether beliefs are elicited or not. Table 1 summarizes our experimental design. We vary the precision of the private signals in the following way: complete information (standard deviation of 0), high precision (standard deviation of 1), medium precision (standard deviation of 10), and low precision (standard deviation of 20). In the treatments with belief elicitation, we elicit subjects' beliefs about the state θ after observing their signal and, once they have chosen an action, we elicit their beliefs about the probability they assign to their opponent having chosen actions A and B. Both elicitations of beliefs are incentivized using a quadratic scoring rule.[5]

[5] Given our interest in studying fundamental and strategic uncertainty one could think of only running session with belief elicitation. However, we first follow a revealed preference approach and focus only on choice data to study how the behavior of the game varies with the information structure. We do not elicit beliefs in the first set of treatments (1-4 in Table 1) because we do not want to alter the individual reasoning of subjects by drawing attention to fundamental and strategic uncertainty. However, once we identify systematic deviations from the theory, we hypothesize that subjective beliefs might be behind these results and test this hypothesis directly with these

TREATMENT	SIGNAL PRECISION	BELIEF ELICITATION	# SUBJECTS
1. COMPLETE INFORMATION	PERFECT (sd = 0)	NO	22
2. HIGH PRECISION	HIGH (sd = 1)	NO	38
3. MEDIUM PRECISION	MEDIUM (sd = 10)	NO	40
4. LOW PRECISION	LOW (sd = 20)	NO	44
5. HIGH PRECISION / BELIEFS	HIGH (sd = 1)	YES	22
6. MEDIUM PRECISION / BELIEFS	MEDIUM (sd = 10)	YES	20
7. LOW PRECISION / BELIEFS	LOW (sd = 20)	YES	16

Table 1: Experimental design

Each session of the experiment consists of 50 independent and identical rounds. The computer randomly selects five of the rounds played and subjects are paid the average of the payoffs obtained in those rounds, using the exchange rate of 3 tokens per 1 US dollar. In the treatments with belief elicitation, a sixth round was randomly selected for payment for belief accuracy only. It was made clear to subjects that the round selected for payment for beliefs would be a different round from the ones chosen for payment in the game to avoid hedging.

Subjects are randomly matched in pairs at the beginning of the session and play with the same partner in all rounds. To avoid framing effects the instructions use a neutral terminology. To avoid bankruptcies, subjects were endowed with 24 tokens at the beginning of each round.

Before starting the first paying round subjects have access to a practice screen where they can generate signals and they are given an interactive explanation of the payoffs associated to each possible action, given their signal and the underlying state θ.

In each round, a state θ is randomly drawn but not shown to subjects. They privately observe their own private signal of θ and simultaneously choose whether to take action A or B. After both actions have been recorded, subjects receive feedback about their own private signal, their choice of action, the realization of θ, how many people in their pair chose action A, whether A was successful or not, and their individual payoff for the round.

The experiment was conducted at the Center for Experimental Social Science at New York University using the usual computerized recruiting procedures. Each session lasted from 60 to 90 minutes and subjects earned on average $30, including a $5 show up fee. All subjects were undergraduate students from New York University.[6] The experiment was programmed and conducted with the software z-Tree (Fischbacher, 2007). There were a total of 14 sessions and 202 participants.

Our experiment is related to the work of Heinemann et al. (2004) (HNO04 henceforth) who test the predictions of the model by Morris and Shin (1998) in the laboratory. Unlike HNO04, our focus is to understand how behavior in a global game changes as we vary the precision of the

additional treatments.

[6] Instructions for all treatments can be found at http://econweb.ucsd.edu/~itrevino/pdfs/instructions_st.pdf.

private signals.[7] Our experiment is also related to Cabrales, Nagel, and Armenter (2007) who consider a 2-player discrete information version of the global game used in Heinemann et al. (2004) and investigate whether subjects coordinate on the risk-dominant equilibrium of the underlying complete information game. It is also related to Duffy and Ochs (2012) who compare subjects' behavior in a static and dynamic complete information version of the game used in Heinemann et al (2004).

3.1 Theoretical predictions for the experiment

The theoretical model is governed by a set of parameters $\Theta = \left\{\mu_\theta, \sigma_\theta, \underline{\theta}, \overline{\theta}, T, \sigma\right\}$. In the experiment:

- The state θ is randomly drawn from a normal distribution with mean $\mu_\theta = 50$ and standard deviation $\sigma_\theta = 50$.
- The coordination region is for values of $\theta \in [0, 100)$, that is $\underline{\theta} = 0$ and $\overline{\theta} = 100$.
- The cost of taking action A is $T = 18$.

Given these parametric assumptions we characterize the predictions of the model in the form of a hypothesis to be tested with our experiment:

Hypothesis 1

a) Under incomplete information subjects use unique equilibrium threshold strategies.

b) Under complete information subjects behave in accordance to the theoretical prediction of multiplicity of equilibria.

c) Thresholds are increasing in precision and tend towards the risk-dominant threshold.

Given the parameters used in the experiment, the equilibrium threshold when subjects observe signals with high precision (standard deviation of 1) is 35.31, for medium precision (sd of 10) it is 28.31, and with a low precision (sd of 20) it is 18.73. When subjects have complete information about θ, the theory suggests multiple equilibria.[8]

To understand the intuition behind the predictions for comparative statics of thresholds and precisions, note that thresholds are low in general when μ_θ is high relative to T since this makes the

[7] Our experiment also differs from HNO04 in terms of implementation. HNO04 use uniform distributions for the state and for private signals and they give subjects in each round a block of 10 independent situations (signals) and subjects have to choose an action for each signal before getting feedback. They then get feedback about the 10 choices and move on to the next round where they face a new block of 10 decisions. They have 16 rounds of 10 situations each. Additionally, each game of HNO04 consisted of 15 players, as opposed to our two-player case.

[8] This does not rule out the use of threshold strategies, but rather that under multiple equilibria behavior is not predictable. If subjects were to use thresholds, then multiplicity of equilibria would imply not observing a clear pattern toward a specific threshold.

risky action more likely to succeed in expectation.[9] This is stronger when the precision of signals is low, since in this case players assign a high weight to the prior. Thus, for a low precision of signals the model predicts low thresholds for our choice of parameters. An increase in the precision of signals has two effects on players' behavior. First, they assign a lower weight to the information contained in the prior. Second, it increases the correlation between the players' signals, making it harder for a player to predict whether his opponent observes a signal higher or lower than his own, which leads to higher strategic uncertainty for intermediate signals (close to the thresholds). For our choice of parameters, this means that thresholds will increase as the precision of signals increases. In the limit, as signals become perfectly informative, this increase in strategic uncertainty leads players to choose the risk dominant equilibrium of the game, which corresponds to 36 ($2T$), as in the limit they assign probability 1/2 to the other player observing a signal higher or lower than their own.

4 Experimental results

In this section we present the results from the treatments without belief elicitation in order to establish patterns in behavior for sessions where subjects have not been made to think about fundamental and strategic uncertainty. Section 5 presents the results of the treatments with belief elicitation.

We first explain our methodology to estimate thresholds since they are the relevant objects for our hypotheses. We then test our hypothesis and summarize the departures from the theoretical predictions found in the data to facilitate the introduction of the extended model in Section 5.3. Most of the results reported from the experiment pertain to the last 25 rounds to allow for behavior to stabilize, unless otherwise specified.

4.1 Estimation of thresholds

We say that a subject's behavior is consistent with the use of threshold strategies if the subject uses either perfect or almost perfect thresholds. A perfect threshold consists of taking the safe action B for low values of the signal and the risky action A for high values of the signal, with exactly one switching point (the set of signals for which a subject chooses A and the set of signals for which he chooses B are disjoint). For almost perfect thresholds, subjects choose action B for low signal values and action A for high signal values, but we allow these two sets to overlap for at most three observations. These two types of behavior are illustrated in Figure 7 in the appendix.

[9] See Heinemann and Illing (2002), Bannier and Heinemann (2005), Iachan and Nenov (2014), or Szkup and Trevino (2015) for detailed theoretical discussions of comparative statics in global games with respect to the precision of information.

Once we identify the subjects who use threshold strategies, we use two different methods to estimate their thresholds. For the first method we pool the data of all the subjects who use thresholds in each treatment and fit a logistic function with random effects (RE) to determine the probability of taking the risky action as a function of the observed signal. The cumulative logistic distribution function is defined as

$$\Pr(A) = \frac{1}{1 + \exp(\alpha + \beta x_i)}$$

We estimate the mean threshold of the group by finding the value of the signal for which subjects are indifferent between taking actions A and B, that is, the value of the signal for which they would take the risky action with probability $\frac{1}{2}$, which is given by $-\frac{\alpha}{\beta}$. As pointed out by HNO04, the standard deviation of the estimated threshold, $\frac{\pi}{\beta\sqrt{3}}$, is a measure of coordination and reflects variations within the group. We call this the Logit (RE) method.

For the second method we take the average, individual by individual, between the highest signal for which a subject chooses the safe action and the lowest signal for which he chooses the risky action. This number approximates the value of the signal for which he switches from taking one action to taking the other action. Once we have estimated individual thresholds this way, we take the mean and standard deviation of the thresholds in the group. We refer to this estimate as the Mean Estimated Threshold (MET) of the group.

4.2 Hypothesis 1

To study part (a) of Hypothesis 1, we use the procedure described above to determine how many subjects use thresholds strategies. We find that the behavior of 93.44% of subjects is consistent with the use of threshold strategies. In particular, 97.37% of subjects use thresholds in the high precision treatment, 92.5% for medium precision, and 90.91% for low precision. This result is qualitatively similar to HNO04. Table 2 shows the mean estimated thresholds for the different treatments. Standard deviations are reported in parenthesis. While we find that the vast majority of subjects use threshold strategies, these thresholds differ substantially from the thresholds predicted by the theory. In particular, we reject to the 1% and 5% levels of significance that the thresholds estimated for the high precision treatment coincide with the theoretical equilibrium threshold of 35.31, using the MET and logit methods, respectively. For the treatments with medium and low precision, we reject to the 1% level that subjects play the equilibrium thresholds of 28.31 and 18.73, respectively, using both methods.[10,11]

	Complete info	High precision	Medium precision	Low precision
Logit (RE)	22.11	27.61	40.16	35.79
	(7.15)	(5.86)	(9.13)	(9.00)
MET	21.07	27.42	40.37	36.23
	(11.85)	(19.16)	(18.77)	(23.36)
Equilibrium x^*		35.31	28.31	18.73
Risk dominant eq.		36	36	36

Table 2: Estimated thresholds and equilibrium predictions

We also reject part (b) of Hypothesis 1 with our data. In the treatment with complete information all of our subjects not only use threshold strategies, but the vast majority of them set their threshold at the efficient level. Figure 8 and Table 9 in the Appendix show the distribution of individual thresholds under complete information and summary statistics, respectively, illustrating that the thresholds of over 77% of our subjects coincide with the efficient threshold of 18. We interpret the clear use of clean and efficient thresholds by the majority of our subjects in this treatment as supporting the notion of a unique equilibrium. These findings are consistent with the findings of HNO04, who document that subjects play threshold strategies in coordination games with complete information. This is also consistent with Charness et al (2014) who show efficient play in coordination games under complete information.

Finally, we also reject part (c) of Hypothesis 1. The thresholds of subjects that are given either a medium or a low precision are not statistically different from each other, but they are statistically higher than the thresholds under high precision.[12] This means that the estimated thresholds are non-increasing on the precision of information and tend towards the efficient threshold obtained

[10] We include the risk dominant equilibrium of the underlying complete information game as a benchmark to compare the behavior of subjects in the private information game, since the global games solution converges to this equilibrium as the noise in private signals goes to zero (see Carlsson and van Damme, 1993). The thresholds estimated for high precision are different to the risk dominant threshold to the 1% level. For the treatments with medium and low precision we cannot reject the hypothesis that the estimated thresholds coincide with the risk dominant threshold of 36.

[11] The estimations of Table 2 are restricted to subjects who use individual thresholds (93.44% of our sample). If we include the subjects who do not use individual thresholds, our logit estimates for the different precisions are: 26.33 (high), 39.69 (medium), and 31.44 (low). Therefore, our qualitative findings do not change by including the full sample.

[12] To take into account heterogeneity of individual thresholds within each treatment, we also compare the distributions of individual thresholds across treatments, which corroborate our analysis of mean comparisons. Using a Kolmogorov-Smirnov test, we find that the distribution of thresholds for the high precision treatment is significantly different from that of the medium precision and low precision treatments (p-values <0.000 and 0.002, respectively), and that the distributions of thresholds for medium and low precisions are not statistically different at the 5% level of significance (they are at the 10% level, p-value of 0.08). We get similar results using a Wilcoxon ranksum test, whereby we reject the hypothesis that individual thresholds under high precision come from the same distribution as the thresholds under medium precision (p-value < 0.001) and under low precision (p-value <0.05). However, we cannot reject the hypothesis that the thresholds under medium and low precision come from the same distribution (p-value of 0.289).

under complete information, and not the risk dominant, as the theory predicts. That is, a high precision of signals leads to more successful coordination than what is suggested by the theory.[13] Table 10 in the appendix reports estimated thresholds for Rounds 1-25 of the experiment to understand if the results we report are a product of learning. As we can see, our qualitative results are present since the beginning of the experiment.

Table 3 compares the average realized payoffs in each treatment to the expected equilibrium and constrained efficient payoffs that would have arisen if subjects had behaved according to these benchmarks, for the given realizations of θ in each session.[14] As we can see, the deviation from equilibrium under high precision of signals is payoff improving, even with respect to the constrained efficiency benchmark.

Precision	High	Medium	Low
Realized payoffs	34.54	33.3	29.71
	(8.82)	(13.48)	(9.65)
	[32.04, 37.04]	[30.41, 36.19]	[27.26, 32.16]
Expected equilibrium	23.62***	29.1***	31.44***
payoffs	(8.77)	(10.42)	(5.97)
	[20.41, 26.82]	[25.83, 32.38]	[28.9, 33.97]
Expected constrained	29.26***	31.58***	31.68***
efficient payoffs	(7.11)	(10.83)	(5.96)
	[26.32, 32.21]	[28.41, 34.75]	[29.16, 34.21]

Different from realized payoffs at the ***1%; **5%; *10% level; sd in parentheses; CI in brackets

Table 3: Average payoffs

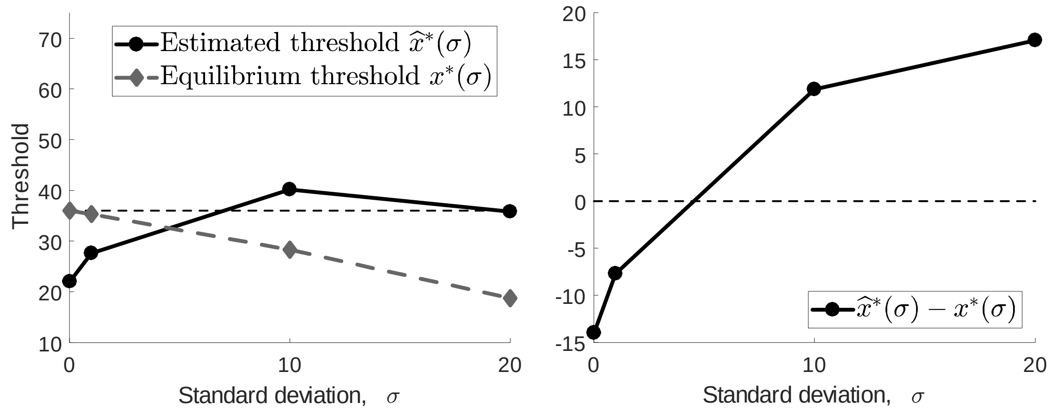
4.3 Summary of departures from the theoretical predictions

Our experimental analysis highlights two main empirical departures from the theoretical predictions: (1) thresholds tend to decrease as signals become more precise, rather than increase as predicted by the theory, and (2) as the signal precision increases, thresholds tend towards the efficient threshold, rather than the risk dominant one. This systematic path to convergence towards efficiency illustrates an underlying force in the game that is not captured by the theory. These two discrepancies in our data with respect to the theoretical predictions are illustrated in the left panel of Figure 1, which plots the estimated thresholds (solid line) and the theoretical predictions for the different noise levels (dashed line) and a horizontal line at the risk dominant equilibrium. The

[13] Similar efficient thresholds are found in a robustness check with high precision and random rematching in every round, suggesting that the efficient play that we observe in the experiment is not due to repeated game effects.

[14] The constrained efficiency benchmark corresponds to the case where a social planner faces the same informational constraints as players and chooses actions for both pair members to extract the maximum surplus. Under this benchmark subjects would still face fundamental uncertainty but not strategic uncertainty.

right panel of Figure 1 shows that while estimated thresholds are non-monotone in σ, the difference between estimated and theoretical threshold is a monotonic function of σ.



Panel A: Comparison of $\widehat{x}^*$ and x^* Panel B: Difference between $\widehat{x}^*$ and x^*

Figure 1: Comparison of theoretical thresholds $x^*(\sigma)$ and estimated thresholds $\widehat{x}^*(\sigma)$.

It is worth stressing that these departures from the theoretical predictions are robust in the sense that we observe them across treatments with different ways to estimate thresholds, to belief elicitation (see Section 5.3.1), and they even become starker when the information structure is endogenously determined by subjects (see Szkup and Trevino (2019)).[15]

5 Understanding departures from the theory

We have documented two empirical departures from the theory. In order to understand what could be driving these deviations we go back to the benchmark model of Section 2 and focus on what happens in the limit, as the signal noise vanishes, $\sigma \to 0$. In this case, one can show that the indifference condition of player $i \in \{1, 2\}$ is approximately given by[16]

$$\Pr(x_j \geq x^* | x^*) \, \mathbb{E}[\theta | x^*] = T \tag{2}$$

Equation (2) tells us that when players' signals are very precise their equilibrium behavior is determined by the fundamental uncertainty (captured by $\mathbb{E}[\theta|x^*]$) and the strategic uncertainty (captured by $\Pr(x_j \geq x^*|x^*)$) that players face in equilibrium. This suggests that, under the assumption that players best-respond to their beliefs, deviations from the theoretical predictions can be driven by subjective perceptions of fundamental and/or strategic uncertainty. We refer

[15] Szkup and Trevino (2019) investigate experimentally costly information acquisition in a global game using the same experimental protocol.

[16] One can show that for a sufficiently small σ, the LHS of Equation (1) is approximately equal $E[\theta|x^*] \Pr(x_j \geq x^*|x^*)$. This can be deduced from the fact that in the limit as $\sigma \to 0$, the indifference condition becomes $x^* \Pr(x_j \geq x^*|x^*) = T$.

to these subjective perceptions as sentiments (see Angeletos and La'O, 2013, or Izmalkov and Yildiz, 2010). Note, however, that as $\sigma \to 0$ then $\mathbb{E}[\theta|x_i] \to x_i$, and thus the task of computing the expected value of θ becomes rather simple. This suggests that, at least in the case of high precision, departures from the theory observed in the experiment are unlikely to be driven by a subjective perception of fundamental uncertainty due to sentiments, but rather by sentiments that affect the perception of strategic uncertainty.

Given this observation, one possible explanation for the disparity between the theory and the experiment is that subjects exhibit sentiments related to strategic uncertainty and that these sentiments might vary with the precision of private information. In particular, subjects might be optimistic about the likelihood of others taking the risky action when information is very precise and pessimistic when information is imprecise.[17] This intuition is captured in the following hypothesis.

Hypothesis 2 *The behavior of subjects is driven by sentiments in the perception of strategic uncertainty and these sentiments are positively related to the precision of private signals. That is, subjects exhibit positive sentiments about the likelihood of others taking the risky action when information is precise, and they exhibit negative sentiments when information is imprecise.*

To test this hypothesis we proceed as follows. First, in Section 5.1 we formalize Hypothesis 2 by extending the baseline model of Section 2 to include exogenous sentiments that depend on the precision of private signals. As discussed above, in the extended model we make two main assumptions: (1) players form first order beliefs correctly, and (2) they exhibit sentiments when they form beliefs about the action of their opponent, which are affected by precisions as we hypothesize. We use the extended model to understand whether the presence of such sentiments can rationalize the thresholds observed in the data.

Second, in Section 5.3 we test Hypothesis 2 experimentally by testing explicitly the two main assumptions of the extended model. By eliciting beliefs about the state and about the action of the other player, we can analyze whether subjects form first-order beliefs correctly and whether they exhibit positive sentiments about the behavior of others as we vary the precision of private information.

In Section 6 we discuss the suitability of alternative equilibrium notions that assume bounded rationality (such as QRE, limited depth of reasoning, correlated equilibria, and analogy-based expectation equilibria) and argue that, unlike our extended model, these alternative models cannot fully organize our experimental findings.

[17] This observation about how a higher precision might make subjects more optimistic about successful coordination via a reduction of strategic uncertainty is reminiscent of mechanisms that enhance coordination under complete information that rely on also decreasing strategic uncertainty via, for example, forward induction (see Crawford and Broseta (1998) or Camerer (2003) for a survey).

5.1 Model with sentiments

In this section we extend the baseline model to accommodate for sentiments in the perception of strategic uncertainty. We model sentiments as a specific form of additive bias when estimating the probability that the other player takes the risky action. For each $i = 1, 2$ and $j \neq i$, let $\Pr_i \left(x_j \geq x_j^* | x_i \right)$ denote the subjective probability that player i assigns to player j taking the risky action.[18] We consider the following specific form of subjective beliefs about the action of the other player:

$$\Pr{}_i \left(x_j \geq x_j^* | x_i \right) = \int_{\theta=-\infty}^{\infty} \left[1 - \Phi \left(\frac{x_j^* - \theta}{\sigma} - \alpha_i \right) \right] \phi \left(\theta | x_i \right) d\theta, \tag{3}$$

where $\alpha_i \in \mathbb{R}$ is the sentiment of player i about behavior of player j. Note that when $\alpha_i = 0$, player i forms Bayesian beliefs about player j's choice of action. If $\alpha_i > 0$ then player i exhibits positive sentiments about player j taking the risky action (i.e., his beliefs about the likelihood of player j taking the risky action are larger than the Bayesian beliefs), while if $\alpha_i < 0$ player i exhibits negative sentiments about player j's willingness to coordinate.

The sentiments captured by α_i can arise, for example, if player i believes that player j extracts the information from his private signal x_j in a biased way by computing his posterior mean as if his signal was $x_j = \theta + \sigma\varepsilon_j + \sigma\alpha_i$.[19] Thus, one can interpret α_i as measuring player i's overconfidence (or underconfidence) about player j taking the risky action, which results from player i's beliefs about player j's overestimation (or underestimation) of the value of the state θ.[20] Furthermore, to be consistent with Hypothesis 2, we assume that sentiments are *positively related* to the precision of private signals. That is, $\partial\alpha_i / \partial\sigma < 0$. In other words, we assume that players believe that $\alpha_i > 0$ when information is precise and $\alpha_k < 0$ otherwise.

In Section B of the Appendix we provide a detailed description and characterization of the extended model, including proofs of existence and uniqueness of equilibrium, as well as the characterization of these thresholds in the limit, as noise vanishes. In particular, we show that there exists a unique equilibrium in monotone strategies when private signals are sufficiently precise, similarly to the baseline model. In the unique equilibrium of the extended model the thresholds are decreasing functions of players' sentiments. This follows directly from our assumption that positive sentiments for player i imply that, for any signal x_i, player i assigns a higher probability to player j taking the risky action than in the standard model. Moreover, we show that in the limit, as

[18] To economize on notation we denote the conditional distribution of θ given x_i by $\phi(\theta|x_i)$.

[19] We still refer to α_i as player i's sentiment (and not player j's) because the sentiment α_i is part of player i's belief about player j's beliefs.

[20] The formulation of the subjective probability of Equation (3) is also consistent with other interpretations. For example, it could be the result of introducing uncertainty about the threshold used by player j in equilibrium. If we define the threshold that player j uses as $x_j^* = x^* - \sigma\alpha_i$, then Equation (3) would correspond to the probability that player i ascribes to player j taking the risky action when he believes that player j's equilibrium threshold is x_j^*.

the noise in private signals vanishes, players use the same threshold regardless of the magnitude of their sentiments. Unlike the standard model, the threshold in the limit differs in general from the risk-dominant threshold ($2T$). When α_i's increase when precision increases (as we assume), the limiting threshold tends to the welfare efficient threshold T.

It is important to emphasize that the extended model is meant as a means to understand and explain our experimental results, which is why we make specific assumptions about how sentiments depend on the signal precision. In what follows we explicitly test these assumptions and find empirical support for them. Other authors have stressed the importance of studying sentiments in games of incomplete information. Izmalkov and Yildiz (2010), for example, study sentiments resulting from a non-common prior. In a macroeconomic setup, Angeletos and La'O (2013) interpret sentiments as shocks to first-order beliefs of endogenous outcomes. Unlike these papers, we assume a common prior and correct first-order beliefs, but we allow sentiments to affect the perception of strategic uncertainty and to depend on the informativeness of the environment. This characterization is consistent with the dynamics of overconfidence proposed in the psychology literature, which could be amplified by strategic motives. If we interpret situations of high uncertainty as situations where coordination is "harder" for subjects, the negative sentiments that arise under high fundamental uncertainty are reminiscent of the underconfidence and pessimism that arise in individual decision making when tasks are harder, and the positive sentiments under low uncertainty are similar to the overconfidence or optimism that arise in individual decision making for easier tasks (see Moore and Cain (2007)). However, we stay agnostic about how sentiments arise. The answer to this question would require decision theoretic foundations that are beyond the scope of this paper.

5.2 The extended model and the data

Based on our extended model, Figure 2 depicts the sentiments that rationalize our findings. For this figure we assume that both players have the same bias and then compute the value of the bias that is consistent with the observed average threshold. As we can see, even though estimated thresholds are not monotonic in precision, the estimated sentiments are monotonically increasing in signal precision, which is consistent with the assumption of the extended model. This is because, as shown in the right panel of Figure 1, the distance between estimated and theoretical thresholds is a monotone function of precision.

Figure 2 shows that the presence of sentiments about the action of others can organize our experimental findings. This exercise, however, is silent about the mechanism proposed in Hypothesis 2. In what follows we investigate the empirical validity of the sentiment-based mechanism of Hypothesis 2 by testing whether subjects form accurate first-order beliefs and whether they exhibit

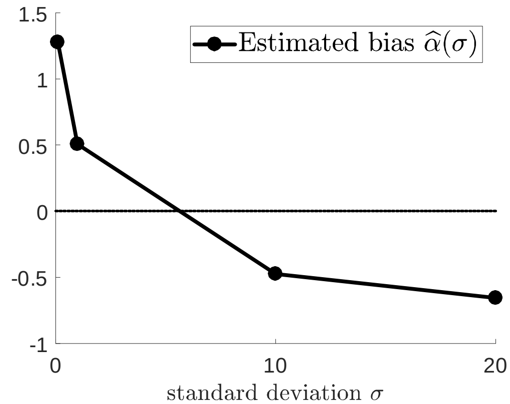


Figure 2: Sentiments consistent with estimated thresholds.

sentiments about the actions of others that are increasing in the precision of information.

5.3 Experimental test of Hypothesis 2

To test the main assumptions of the extended model that give a formal structure to Hypothesis 2 we run additional sessions where we elicit subjects' beliefs. The experimental protocol of these sessions is identical to the previous one (for high, medium, and low precisions), except for two additional questions that asked subjects to report their best guess about the state θ (after observing their private signals) and the probability they assign to their opponent taking each possible action (after choosing their own action and before getting feedback).

The elicited beliefs allow us to understand the subjects' perception of fundamental uncertainty (via their reports about the value of the state) and of strategic uncertainty (via their reports about the probability of their opponent taking the risky action) and thus identify the type of sentiments that might arise under different signal precisions. We first focus first on subjects' first-order beliefs and find support for our assumption that the observed departures from the theory are not driven by sentiments related to fundamentals. We then analyze the elicited beliefs about the actions of others.

5.3.1 Preliminaries

Before we analyze the elicited beliefs we estimate the thresholds for these additional sessions to confirm that the departures from the theory presented in Section 4 are also present in the new data set. As we can see in Table 4, when we elicit beliefs the estimated thresholds are decreasing in precision, contradicting the theoretical predictions. These results are in fact starker than in the treatments without belief elicitation, since now the estimated thresholds are monotonically decreasing in precision.[21] Figure 9 in the appendix plots the mean estimated thresholds under the two treatment conditions. These starker thresholds can be due to the salience created by the elicitation of beliefs, since it has been documented that eliciting beliefs might affect the way subjects play the game by accelerating best-response behavior (see Croson (2000), Gächter and Renner (2010) or Rutström and Wilcox (2009)).[22] This is intuitive because belief elicitation forces subjects to think about fundamental and strategic uncertainty, thus putting more structure to their thought process. The thresholds of Table 4 will serve as the reference thresholds for the analysis that follows.

	High precision	Medium precision	Low precision
Logit (RE)	19.24	28.76	40.72
	(4.71)	(8.76)	(11.02)
MET	19.24	29.36	39.85
	(7.64)	(22.61)	(27.92)
Equilibrium x^*	35.31	28.31	18.73
Risk dominant eq.	36	36	36

Table 4: Estimated thresholds and equilibrium predictions, belief elicitation.

5.3.2 Perception of fundamental uncertainty

We first show that, consistent with our hypothesis, subjects do not seem to exhibit sentiments in their perception of fundamental uncertainty. Let θ_i^B denote the stated belief of subject i about θ. Table 5 shows the mean and median differences in absolute value between the stated beliefs about the state and the true state ($|\theta_i^B - \theta|$), and between the stated beliefs about the state and the observed signal ($|\theta_i^B - x_i|$), for each level of precision. As expected, subjects rely on their signals more and form more accurate beliefs about the state when signals are very precise, and this decreases as signals become noisier.

	$\|\theta_i^B - \theta\|$			$\|\theta_i^B - x_i\|$		
Precision	High	Medium	Low	High	Medium	Low
Mean	1.19	10.41	19.59	0.83	7.02	12.24
Median	0.88	7.38	17.45	0.55	4.81	9.99
St. dev.	3.79	28.33	14.57	3.84	27.5	12.72

Table 5: Differences in absolute value between first order beliefs, the true state, and observed signals, by precision.

To further evaluate the accuracy of subjects' first order beliefs, we estimate the weights assigned

[21] Recall that in the treatments without belief elicitation thresholds were non-increasing in precision and the distance between the estimated and theoretical thresholds was monotonically decreasing in precision.

[22] For a survey on belief elicitation, see Schotter and Trevino (2015).

to the private signal and to the prior when forming beliefs about θ.[23] We can see in Table 6 that subjects put higher weights on the private signal as its precision increases and that the mean and median weights are very close to the Bayesian weights for high and medium precision. For a low precision subjects seem to assign a higher weight to private signals than Bayes' rule implies.[24]

Precision	High	Medium	Low
w_i^{Bayes}	0.99	0.96	0.86
Mean (w_i)	0.98	0.97	0.93
Median (w_i)	1.00	0.94	0.98
St. dev. (w_i)	0.14	0.53	0.54

Table 6: Estimated weights given to private signals, by precision.

Finally, we directly compare the theory's Bayesian beliefs and subjects' stated beliefs. To do so, in Figure 3 we divide the space of private signals into intervals of length 10 and calculate the average reported belief of subjects (solid line) and the average theoretical belief (dashed line) about θ for the signals that fall into each interval.[25] We find that the average reported and theoretical beliefs are very close to each other, particularly in the case of high and medium precision. This further suggests that sentiments in the perception of fundamental uncertainty are an unlikely driver of our experimental results.

To summarize, the fact that subjects set significantly lower thresholds than those suggested by equilibrium when the signal precision is high is unlikely to be explained by sentiments about fundamentals, since subjects seem to form first order beliefs relatively well.

5.3.3 Perception of strategic uncertainty

In this section, we study the stated beliefs about the actions of others to test whether sentiments about strategic uncertainty can explain the results presented in Section 4.

In Figure 4 we compare the elicited and theoretical beliefs about the probability of the opponent taking the risky action for a given signal. Just as in Figure 3, we divide the space of private signals

[23] We assume that beliefs are linear in the prior mean and in their signal (as Bayes' rule prescribes) but we allow for the possibility that players use non-Bayesian weights. In particular, we postulate that subject i's stated belief θ_i^B satisfies $\theta_i^B = w_i x_i + (1 - w_i)\,\mu_\theta$. According to Bayes' rule, the correct weight of the private signal is $w_x^{Bayes} = \frac{\sigma^{-2}}{\sigma^{-2}+\sigma_\theta^{-2}}$. To estimate w_i for each subject i, we back out these weights for the last 25 rounds. We then take the average of the implied weights to obtain subject i's average weight w_i. To eliminate reports that seem to be a result of mistakes (for example reporting belief of -586 when signal was -4.02) we only consider weights that belong to the interval $[-2, 3]$. Finally, we consider only reports by subjects that use perfect or almost perfect thresholds.

[24] One might ponder that subjects can exhibit base rate neglect and that this could explain some of the observed departures, but this is not the case. As Table 11 in the Appendix shows, even if all players neglected the prior the corresponding thresholds would still be increasing in the precision of signals and converge to the risk-dominant equilibrium from below.

[25] The gap in the plots of reported and theoretical beliefs for the case of high precision is due to no signals falling into those intervals in the experiment.

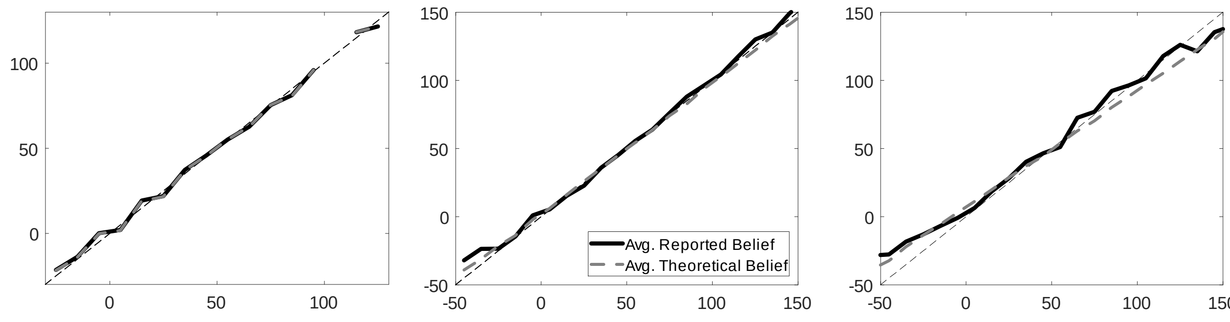


Figure 3: Average beliefs about θ and theoretical beliefs about θ, by signal realization.

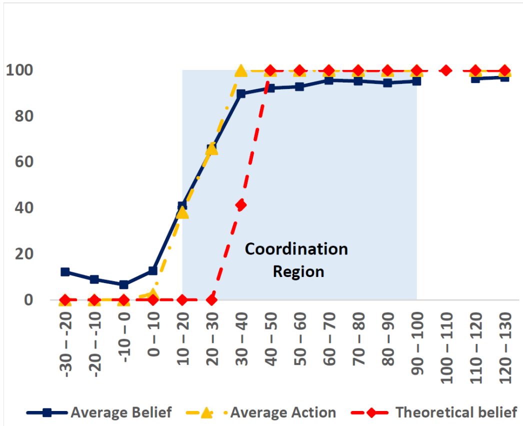
into intervals of length 10 and calculate the average belief reported for the signals that fall into each interval (solid line with squares). The theory's Bayesian beliefs are portrayed in the dashed line with diamond). We also plot the average action of the subjects that observed those signals, which approximates the true probability of taking the risky action (dashed line with triangles). That is, we look at the actions taken by the subjects that observe the signals in each interval and calculate the frequency with which they, as a group, took the risky action.[26]

The graphs in Figure 4 support our hypothesis. Average beliefs are in general higher than equilibrium beliefs for a high signal precision. The opposite is true for low signal precision.[27] Thus, the significantly lower thresholds associated with a high precision in Table 4 can be explained by the positive sentiments that reflect optimistic beliefs about the intention of the opponent to coordinate (left panel of Figure 4).[28] In particular, at the estimated threshold of 19.24 we can see that subjects assign a significantly higher probability to their opponent coordinating than what the theory suggests (measured by the vertical distance between the two lines), thus rationalizing this behavior. Likewise, the significantly higher thresholds associated with a low precision can be explained by negative sentiments that translate into pessimistic beliefs about the intention of the opponent to coordinate (bottom panel of Figure 4). Notice that elicited beliefs are more aligned to equilibrium beliefs for medium precision (center panel of Figure 4). This is also consistent with the estimated thresholds of Table 4, since the mean thresholds of subjects in this treatment are not statistically different from the equilibrium prediction. The distributions of elicited and equilibrium

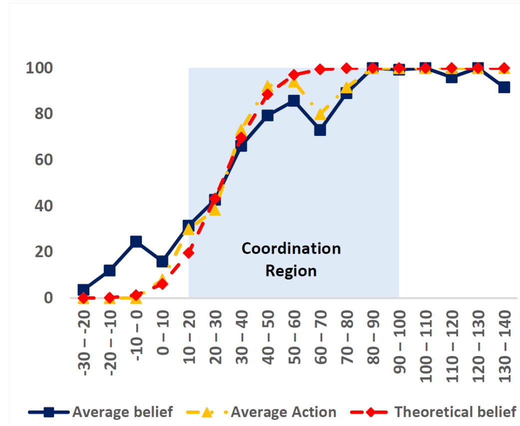
[26] If a subject took the safe action we assign a 0 and if he took the risky action we assign a 1. We then average these numbers across all the signals that fall into each interval and get a number between 0 and 1 that represents the true probability with which subjects took the risky action.

[27] The high reported beliefs for negative signals in the low precision treatment are mainly due to a low number of observations and subjects who observe low signal reporting a 50-50 chance of their opponent taking either action for any signal realization. This supports our hypothesis that a high fundamental uncertainty leads to the perception of high strategic uncertainty.

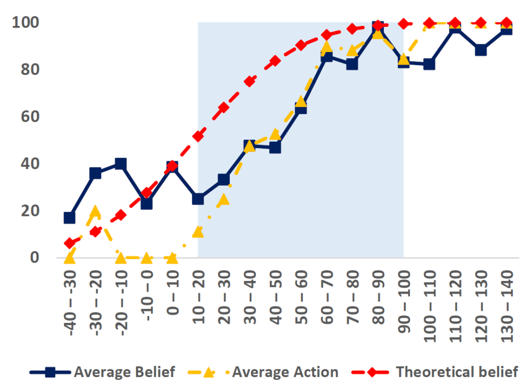
[28] We could also think of the complete information treatment as a way to understand strategic uncertainty under high precision. Subjects might percieve the high precision treatment as being very close to complete information, thus translating the optimistic beliefs that lead them to coordinate on the efficient threshold to this setup.



High precision ($\sigma = 1$)



Medium precision ($\sigma = 10$)



Low precision ($\sigma = 20$)

Figure 4: Average beliefs about the action of the opponent, average action chosen by subjects, and theoretical probabilities, by signal realization.

beliefs are statistically different to each other to the 1% level of significance using a Kolmogorov-Smirnov test, for all levels of precision. Finally, note that the beliefs reported by subjects are very close to the true actions observed in the experiment, suggesting that subjects best-respond to their beliefs and, as a consequence, that their threshold behavior could be rationalized by equilibrium behavior in our extended model. Figure 10 in the appendix reports the same graphs as Figure 4 for rounds 11-50 (as opposed to rounds 26-50 as in Figure 4) to show that this sentiment-based behavior is present early in the experiment, so it is not a product of excessive learning.

Therefore, the results of these additional sessions support Hypothesis 2 and provide evidence for a subjective perception of strategic uncertainty where sentiments switch from negative to positive as we move from an environment with high fundamental uncertainty to one with low fundamental uncertainty.

6 Discussion of alternative models

We have argued that the departures from the theoretical predictions can be explained by sentiments that affect beliefs about the behavior of others. However, one may wonder whether other commonly used models of bounded rationality in games can explain the departures from the theoretical predictions that we observe in our experiment. In this section we discuss and estimate popular bounded rationality models in the context of our game. We focus on cursed equilibrium, quantal response equilibrium, analogy-based expectation equilibrium, and level-k and cognitive hierarchy. We also investigate whether modifying the standard global games model by assuming that players are risk-averse, as opposed to risk neutral, could help to reconcile the model with the data. Details about these models can be found in Sections C to F in the Appendix.

In summary, none of these models can fully explain the observed departures from the theory; at least, not without strong assumptions. The main issue is that most of these models still predict that as information becomes precise the equilibrium converges to the risk-dominant equilibrium. The level-k model could potentially explain our results, but only if one assumes a particular relationship between signal precisions and the primitives of the level-k belief hierarchy, which is hard to justify.

Cursed equilibrium Cursed equilibrium (CE) (Eyster and Rabin (2005)) relaxes the assumption that players form correct beliefs about their opponents in a very particular way. In this model, λ measures the degree of "cursedness" of a player, that is, with probability λ a player believes that his opponent randomizes his action choice uniformly and with the remaining probability he forms Bayesian beliefs. Therefore, $\lambda = 0$ corresponds to full rationality and $\lambda = 1$ to full cursedness.

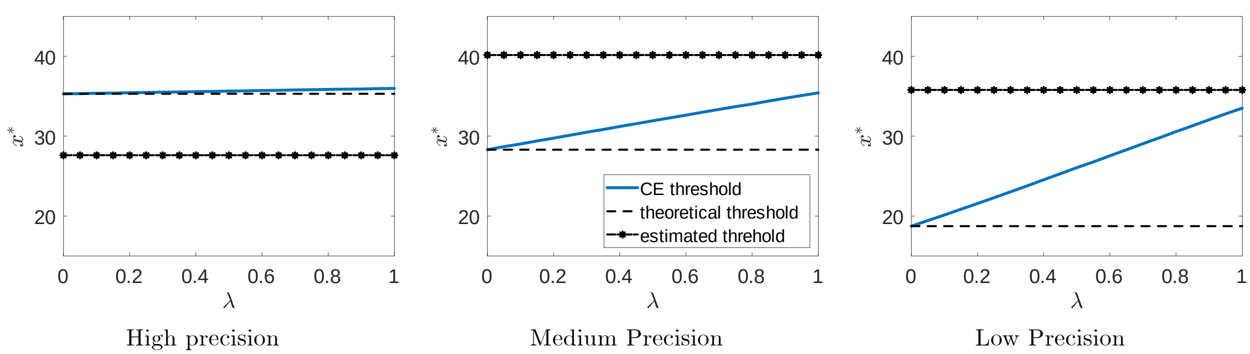


Figure 5: Comparison of cursed equilibrium thresholds for different values of λ with the theoretical and the estimated thresholds

Figure 5 shows the predictions of CE thresholds in our game for different values of λ, for each precision level. As we can see, this specific departure from Bayesian beliefs cannot organize our findings. In particular, regardless of the value of λ, when precision is high or medium the CE threshold (solid line) lies far from the thresholds estimated in our data (dashed line with markers). For low precision the CE threshold lies close to the estimated threshold for high values of λ. However, high levels of λ are hard to justify empirically as high λ implies that subjects to believe that with high probability the other subjects behave randomly. This is in contrast to elicited beliefs which suggest that the majority of subjects expect their opponents to adjust their behavior with their signal (and, thus, behave consistently with use of threshold strategies).

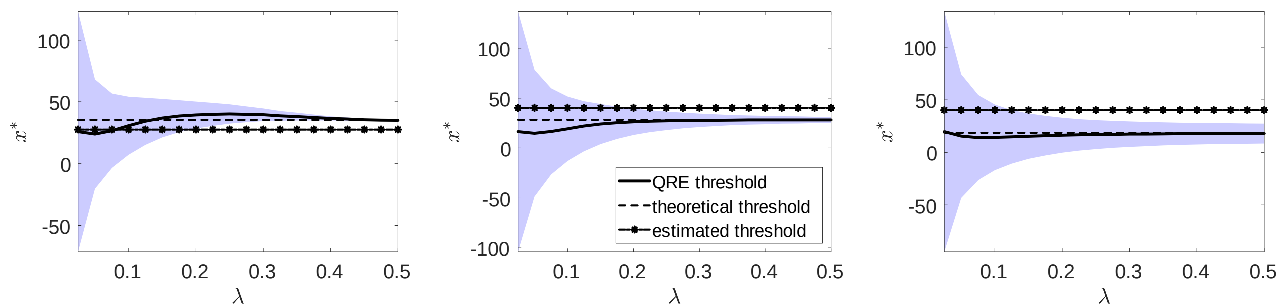
CE predicts that thresholds converge to the risk dominant equilibrium in the limit, which is in contrast to our finding that thresholds tend toward the efficient equilibrium as signals get very precise. The intuition is simple. When $\lambda = 0$ we recover the standard model where equilibrium thresholds converge to the risk dominant threshold as $\sigma \to 0$. On the other hand, when $\lambda = 1$ players expect their opponents to randomize their actions uniformly, so the CE thresholds coincide with the risk dominant threshold.

Quantal response equilibrium An equilibrium concept that is widely used to organize experimental data is quantal response equilibrium (QRE) as introduced to the literature by McKelvey and Palfrey (1995) (see also Goeree, Holt, and Palfrey (2016)). Unlike our extended model that allows for sentiments in belief formation but where players best-respond to their beliefs, QRE assumes that players form correct beliefs, but that they sometimes make mistakes when choosing an action. According to QRE, deviations from the Nash Equilibrium actions are less likely to occur when the cost of making a mistake increases.

To see whether our empirical results can be explained by this concept we compute numerically the symmetric logit QRE for different values of λ, which captures the likelihood of mistakes ($\lambda = 0$ implies random behavior, $\lambda \to \infty$ converges to Nash Equilibrium). Then, for each λ, we feed the observed signal realizations from the experiment into the logit QRE best-response function to derive QRE predictions for the realized signals in the experiment.[29] Figure 6 compares the simulated QRE thresholds to the theoretical thresholds and the thresholds estimated in the experiment.

Each panel in Figure 6 depicts, each precision level, the estimated QRE threshold using our simulated data (the solid line), the theoretical threshold (the dashed line), and the estimated threshold from the experiment (the dashed-dotted line with markers). The the shaded area depicts one standard deviation bounds for the simulated QRE thresholds, which is a measure of coordination (wide bounds imply poor coordination of actions across agents). Figure 6 shows that the logit QRE predictions cannot explain our findings. As shown in the middle and right panels, there does not exist a value of λ when precision is medium or low that could rationalize our findings. In both cases, the estimated QRE threshold using our simulated data lies below the threshold we estimate with

[29] Since each simulation includes random elements (whether an agent chooses the action A or B given his signal is random), we repeat this simulation 100 times. For each simulation we compute QRE thresholds using the Logit method (see Section 4) and then take the average.



High precision Medium Precision Low Precision

Figure 6: Comparison of estimated QRE threshold for different values of λ with the theoretical and estimated thresholds

our experimental data, for any λ. Only in the case of high precision (the left panel), there exists a value of λ ($\lambda \approx 0.05$) that can rationalize the threshold observed in our experiment. However, this QRE threshold is associated a lot of random action choices, illustrated by the large standard deviation in the shaded area. In contrast, in our experimental data we see very clear and clean individual thresholds under high precision and a low standard deviation.

Level-k and cognitive hierarchy Another possibility to explain our findings is to think about how different signal precisions can affect the level of reasoning of subjects according to models like level-k and cognitive hierarchy (see, for example, Nagel (1995), Costa-Gomes and Crawford (2006)). These models assume that players have limited depths of reasoning, with a player who can perform k iterations called a level-k type. Level-0 are assumed to be non-strategic types who randomize their actions according to a pre-specified distribution, and all players of level-k with $k > 1$ best respond to their exogenous belief distribution over lower types (see Strzalecki (2014) for a general formulation of this model).

In the context of global games, Kneeland (2016) shows that the results from HNO04 are more in line with a model of limited depth of reasoning than with equilibrium play. In particular, level-k explains the use of threshold strategies under complete information (as is found in our data). In order for this model to organize our findings we would need to allow the distribution of players with different levels of reasoning to vary with the precision of information. We find that, holding the behavior of level-0 players constant across treatments, a level-k model where the distribution of depths of reasoning varies with precision cannot rationalize the thresholds observed in the experiment. If we were to allow the behavior of level-0 players to also vary with precision then a level-k model could rationalize our findings. However, this would imply that level-0 players would be somewhat strategic, contradicting a standard assumption about level-0 types.[30] Thus,

[30] Alaoui and Penta (2015) interpret the level-k model as an optimization problem where players perform a cost-benefit analysis to determine their level of reasoning in the game. In the context of our experiment, one could imagine

while a level-k model could potentially explain our results, it would require a number of strong assumptions about the relationship between signal precision and the primitives of the level-k belief hierarchy and it would be hard to justify its use other than by saying that it matches the data.

Analogy-based expectation equilibrium Jehiel (2005) and Koessler and Jehiel (2008) argue that it is implausible to assume that players understand their opponents' strategy state-by-state. Instead, they argue that it is more likely that players bundle nearby state into bins (which they refer to as analogy partitions) when forming their beliefs about other players' strategies and they best respond to the opponents' average action in each bin. This leads to the concept of analogy-based expectation equilibrium (ABEE).

The ABEE has intuitive appeal and can be applied to global games. Koessler and Jehiel (2008) suggest that using the coarsest equilibrium partition can help to explain why players are likely to use thresholds that a closer to welfare optimal threshold than predicted by the theory of global games. We follow Koessler and Jehiel (2008) and compute the implied threshold when players use the coarsest analogy partition in our model. However, as Table 7 shows the ABEE with coarsest analogy partition cannot explain the fact that subjects tend to use high thresholds when information is imprecise and low thresholds otherwise.

	High	Medium	Low
ABEE threshold	26.40	25.27	20.58

Table 7: ABEE predictions under the coarsest analogy partition, by precision level

If we allow for finer partitions, two issues arise. First, in a model with a continuum of states such as ours there is an uncountable number of different partitions that one can consider, so it is not clear what would constitute a "reasonable" partition.[31] Second, multiple equilibria tend to emerge when we consider partitions that are coarse, but finer than the coarsest possible partition. Notwithstanding these issues, in Section D of the Appendix, we propose an intuitive way to compute finer analogy partitions for our model and show that for any partition we consider (1) thresholds are increasing in precision and (2) the ABEE implied threshold for low precision is always lower than the threshold observed in the data.

that different precisions can affect the distribution of levels played because the signal precision affects the expected payoff, and thus the benefit of taking the risky action. Even if this could imply a different proportion of level-0 players for different precisions, it should not affect the way in which a level-0 player behaves. If level-0 behavior depended on the informativeness of the environment, then level-0 players would be somewhat strategic, thus violating the main postulate in the literature about level-0 players being non-strategic.

[31] Jehiel (2005) and Jehiel and Koessler (2008) develop ABEE for games with finitely many states only. One would need to develop a theory of what constitutes a reasonable partition with a continuum of states.

Risk aversion Standard global games models assume that players are risk neutral. One could wonder if assuming risk aversion (RA) could help explain our experimental findings. In Section F of the Appendix we extend the standard model of Section 2 to allow for RA and show that if agents have CRRA utility then the individual thresholds are increasing in the level of RA. Thus, RA would push thresholds above the risk-dominant threshold in the limit as information becomes perfectly precise, which is not consistent with our findings. Nevertheless, we estimate what levels of RA would rationalize our findings.

Table 8 reports the coefficients of RA needed to predict the estimated thresholds using the Logit and MET methods. As we see, in order to explain our experimental findings, not only would subjects have to be risk-averse when their signal are imprecise and risk-loving when their signals are precise, but also exhibit an extremely high degree of risk-seeking behavior when the noise in the signal vanishes (i.e., $\sigma \to 0$). That is, in order to rationalize the observed thresholds when $\sigma \to 0$ the required CRRA coefficient would have to be -3.42 for the logit and -4.65 for the MET. Such large changes in risk attitudes and such an extreme degree of risk-seeking behavior are hard to justify given the typical estimations of risk attitudes in the literature.

Precision level	Logit	MET
Complete info	-3.42	-4.65
High	-0.83	-0.87
Medium	0.47	0.48
Low	0.55	0.56

Table 8: Estimated risk aversion coefficients needed to rationalize the estimated thresholds in the experiment for different precision levels

7 Conclusions

In this paper we have studied how changes in the information structure affect behavior in global games. We identify two main departures from the theoretical predictions: (1) the comparative statics of thresholds with respect to signal precisions are reversed, and (2) as the signal noise decreases, subjects' behavior tends towards the efficient threshold, not the risk-dominant one. These departures have significant welfare effects.

To reconcile our findings with the theory, we hypothesize that the observed departures are due to sentiments in the perception of strategic uncertainty and that these sentiments are anchored to the degree of fundamental uncertainty in the environment. This is in contrast to the mechanism of the theory where strategic uncertainty increases for intermediate signals as fundamental uncertainty decreases. To give a formal structure to our hypothesis, we extend the baseline global games model to account for sentiments in the formation of beliefs about the action of the other player that are directly influenced by the precision of private signals. In particular, players are optimistic about the desire of their opponent to coordinate when information is very precise, and pessimistic when the signal noise increases.

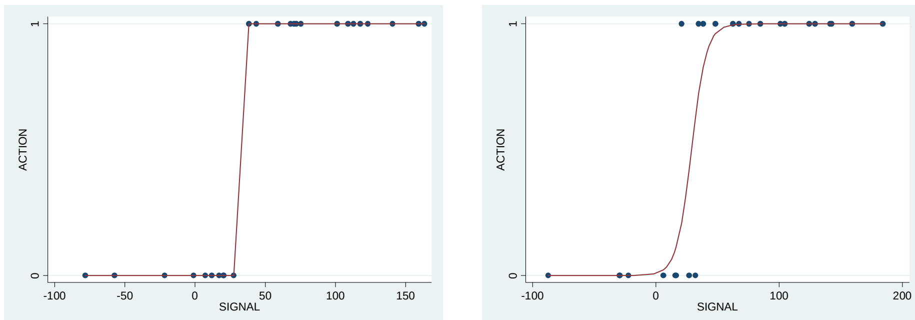
We validate our hypothesis by explicitly testing the two main assumptions of our extended model using observations from additional experiments where we elicit subjects' beliefs about the state and about the actions of others. On the one hand, subjects form first order beliefs accurately, given a level of precision, suggesting that sentiments related to fundamental uncertainty are not the driver of our results. On the other hand, we show that the disparity between stated and theoretical beliefs about the actions of others responds directly to the precision of private signals. Moreover, subjects best-respond to these subjective beliefs since the elicited beliefs coincide with the observed behavior in the experiment. Thus we document how sentiments about the perception of strategic uncertainty can crucially affect outcomes in a game. Our extended model and the evidence that supports it propose a novel mechanism to understand behavior in environments characterized by strategic complementarities and incomplete information.

The optimism/pessimism that drives the sentiments that we identify and characterize in this paper is different to the one typically studied in the behavioral literature, which focuses mainly on individual decision making environments. Our sentiments are intrinsic to strategic environments, since they affect the perception of strategic uncertainty. However, the relationship between sentiments and uncertainty in the environment is consistent with the characterization in the psychology literature on the dynamics of overconfidence if we interpret situations of high fundamental uncertainty as "harder" environments for subjects to coordinate. It is also possible that changes in the perception of strategic uncertainty are related to ambiguity aversion. Subject might perceive the behavior of others as a source of hard-to-quantify uncertainty, particularly when their signals are imprecise. Understanding the mechanisms that lead to the observed deviations from the theoretical predictions from first principles is an important avenue for future research.

Our results can also shed some light on recent stylized facts in macroeconomics. For example, our sentiments-based mechanism can help explain how recessions are associated with heightened uncertainty by noticing that an increase in uncertainty leads to negative sentiments about the likelihood of a profitable risky investment (via pessimism about others investing). This leads to lower levels of aggregate investment, which amplify a recession. This observation reconciles the views of Bloom (2009) and Angeletos and La'O (2013) and Benhabib et al. (2015). In terms of policy making, our results suggest that greater transparency in financial regulation might be beneficial in environments with strategic complementarities, like bank runs, since an increase in transparency can lead to positive sentiments about the likelihood of others rolling over their loans, which leads to less early withdrawals and greater financial stability.

Appendix

A Additional experimental results



(a) Perfect threshold

(b) Almost perfect threshold

Figure 7: Examples of perfect and almost perfect thresholds.

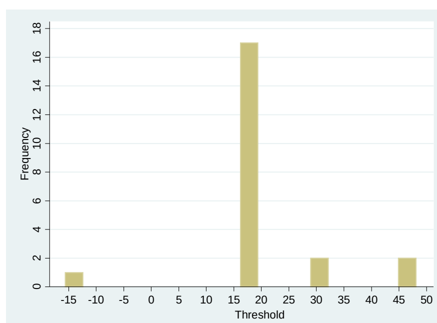


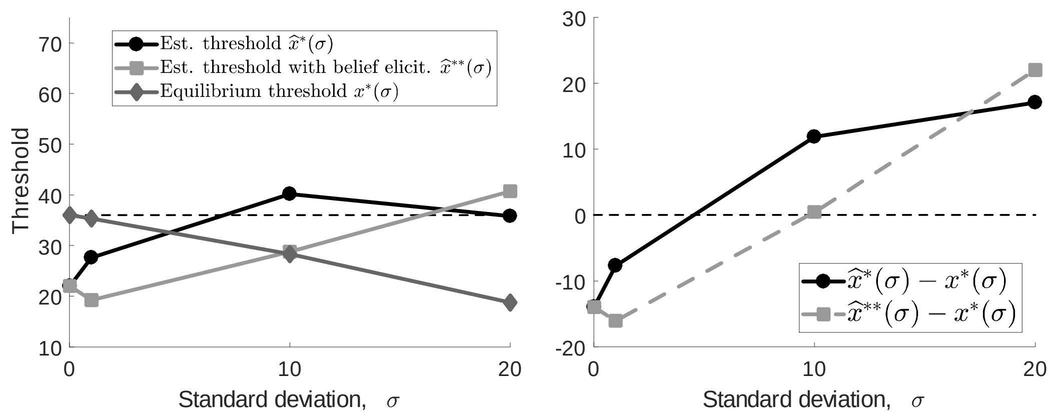
Figure 8: Distribution of individual thresholds under complete information.

Mean	21.07
Median	18.87
St. dev.	12.13

Table 9: Summary statistics of individual thresholds under complete information

	Complete info	High precision	Medium precision	Low precision
Logit (RE)	18.22	19.78	35.49	33.82
	(12.65)	(31.45)	(26.9)	(28.68)

Table 10: Estimated thresholds for Rounds 1-25

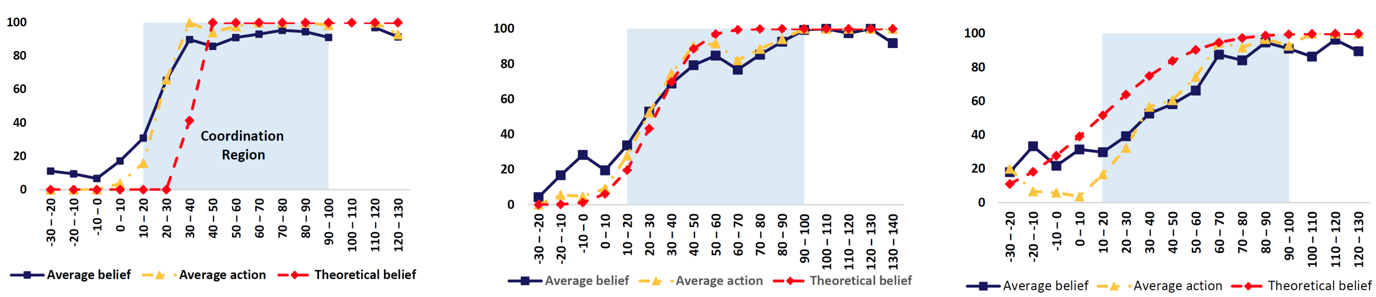


Panel A: Comparison of $\widehat{x}^*$ and x^* Panel B: Difference between $\widehat{x}^*$ and x^*

Figure 9: Comparison of theoretical thresholds $x^*(\sigma)$, estimated thresholds $\widehat{x}^*(\sigma)$, and estimated thresholds with belief elicitation $\widehat{x}^{**}(\sigma)$.

Precision	High	Medium	Low
Thresholds with base rate neglect	35.43	30.36	24.65
Thresholds without base rate neglect	35.31	28.31	18.73

Table 11: Thresholds predicted by the theory when players neglect the prior and when they do not, by precision



High precision ($\sigma = 1$) Medium precision ($\sigma = 10$) Low precision ($\sigma = 20$)

Figure 10: Average beliefs about the action of the opponent and theoretical probabilities for rounds 11-50, by signal realization.we do not take into account the first 10 rounds because there is a lot of noise in the data, possibly due to subjects being acquainted with the interface and understanding the game.

B Model with sentiments in belief formation - FOR ONLINE PUBLICATION

In this section we formally present the extended model with sentiments. We consider a more general version of the model than presented in Section 5.1 by relaxing the assumption that players's sentiments are common knowledge. Instead, in what follows, we assume that player i believes that player j's sentiment takes one of the possible values from the set $\mathcal{A} \in \{\alpha_1, \alpha_2, ..., \alpha_N\}$. Similarly, player j believes that player i's sentiment takes one of the possible values $\{\alpha_1, \alpha_2, ..., \alpha_N\}$.

Note that both the baseline model presented in Section 2 and our extended model presented in Section 5.1 are special cases of this general model. In particular, the baseline model corresponds to the case when $\mathcal{A} = \{0\}$; that is, players form unbiased beliefs about other player's action. Therefore, Proposition 1 stated in Section 2.2 is an immediate corollary of Proposition 2 proved below. The extended model presented in Section 5.1 corresponds to the case when it is common knowledge that sentiment of player i is α_i and the sentiment of player j is α_j.

B.1 The model

For each player $i = 1, 2$ and $j \neq i$, expression 3 in the main text defines the subjective probability that player i assigns to player j taking the risky action. For simplicity, we assume that the set of possible sentiments is finite and ordered, that is, $\alpha_k \in \{\alpha_1, \alpha_2, ..., \alpha_N\}$ where, $\alpha_k < \alpha_{k+1}$. We denote the types of players i and j as the tuples $\{\{x_i, \alpha_k\}, \{x_j, \alpha_l\}\}$ and assume that the presence of the opponent's sentiment is common knowledge but not its magnitude. Instead, player i believes that $\alpha_l \in \{\alpha_1, \alpha_2, ..., \alpha_N\}$ and assigns probability $g(\alpha_l)$ to player j's sentiment being equal to α_l, for each $l = 1, ..., N$. Thus, each player is uncertain not only about the signal that the other player observes but also about the magnitude of his opponent's sentiment. Finally, while player i takes into account the possibility that player j has positive or negative sentiments about the probability that he himself takes the risky action, player i thinks that his own assessment of the probability that player j's takes the risky action is objective.[32]

Let $x_i^*(\alpha_k)$ denote the threshold above which player i takes the risky action when his sentiment is equal to α_k.

Definition 1 (Sentiments equilibrium) *A pure strategy symmetric Bayesian Nash Equilibrium in monotone strategies is a set of thresholds $\{x_i^*(a_k)\}_{k=1}^N$ for each player $i = 1, 2$ such that for each i and each $k = 1, ..., N$, the threshold $x_i^*(a_k)$ is the solution to*

$$\sum_{l=1}^{N} g(\alpha_l) \int_{\underline{\theta}}^{\overline{\theta}} \theta \left[1 - \Phi\left(\frac{x_j^*(\alpha_l) - \theta}{\sigma} - \alpha_k\right)\right] \phi(\theta | x_i^*(\alpha_k))\, d\theta + \int_{\overline{\theta}}^{\infty} \theta \phi(\theta | x_i^*(\alpha_k))\, d\theta = T \tag{4}$$

where $x_j^*(\alpha_l)$ is the threshold of player $j \neq i$ when he exhibits sentiment α_l, $l \in \{1, ..., N\}$.[33]

[32] This is similar to the literature on overconfidence (see for example García et al. (2007)).

[33] Since player i has to take into account that the threshold of player j depends on the magnitude of his own sentiments, the indifference condition of player i with sentiment α_k includes the summation over all possible biases that player j may have.

Notice that we now have $2N$ equilibrium conditions. Any set of thresholds $\left\{x_i^*(\alpha_k), x_j^*(\alpha_l)\right\}_{k,l=1,...,N}$ that solves this system of equations constitutes an equilibrium. The next proposition establishes that the extended model has a unique equilibrium with sentiments when the noise of the private signals is small enough.

Proposition 2 *Consider the extended model.*

1. *There exists $\overline{\sigma} > 0$ such that for all $\sigma \in (0, \overline{\sigma}]$ the extended model has a unique equilibrium in monotone strategies which is symmetric. In this equilibrium $x^*(\alpha_k) > x^*(\alpha_l)$ for all $k < l$ where $k, l \in \{1, ..., N\}$.*

2. *As $\sigma \to 0$ we have*

$$x^*(\alpha_k) \to x^* \text{ for all } k \in \{1, ..., N\} \tag{5}$$

where

$$x^* = \frac{T}{\sum\limits_{k=1}^{N} g(\alpha_k)\,\Phi\left(\frac{\alpha_k}{\sqrt{2}}\right)}.$$[34]

B.2 Proof of Proposition 2

In order to prove Proposition 2 we first show that the equilibrium exists and is unique in the limit as $\sigma \to 0$. Then, we show that there exists $\bar{\sigma} > 0$ such that for all $\sigma < \bar{\sigma}$ the model with sentiments has a unique equilibrium away from the limit. We find it more convenient to work with precisions of signals and the prior, denoted by τ_x and τ_θ, respectively, rather than standard deviations.[35]

We will make use of two results:

Lemma 2 *Consider a sequence $\{c_n\}_{n=1}^{\infty} \subset [a, b]$. If every convergent subsequence of c_n has the same limit L, then c_n is convergent sequence with a limit L.*

Note that this result allows us to focus on convergent subsequences in our analysis of limiting behavior of thresholds and then, if we confirm the hypothesis of the above Lemma, we know this limiting behavior is passed onto the sequence itself.

Lemma 3 *Let $F : \Omega \to \mathbb{R}^n$ where Ω is a rectangular region of $\mathbb{R}^n$. Let $J(x)$ be the Jacobian of the mapping F. If $J(x)$ is a diagonally dominant matrix with strictly positive diagonal then F is globally univalent on Ω.*[36]

[34] More precisely, the above limit applies if and only if $T / \left(\Sigma_{k=1}^{N} g(\alpha_k)\,\Phi\left(\alpha_k/\sqrt{2}\right)\right) \leq \overline{\theta}$. If $T / \left(\Sigma_{k=1}^{N} g(\alpha_k)\,\Phi\left(\alpha_k/\sqrt{2}\right)\right) > \overline{\theta}$ then in the limit $x^*(\alpha_k) = \overline{\theta}$ for all $k = 1, ..., N$.

[35] Recall that precision of a random variable distributed according to $N(\mu, \sigma^2)$ is defined as $\tau = \sigma^{-2}$.

[36] A square matrix A is called strictly diagonally dominant if $|A_{ii}| > \sum\limits_{j \neq i} |A_{ij}|$ for each row i.

Proof. See Parthasarathy (1983). ■

Suppose that players use thresholds $\{x^*(\alpha_l)\}_{l=1}^N$ where $x^*(\alpha_l)$ is the threshold used by a player with sentiment α_l. Given the vector of thresholds $\{x^*(\alpha_k)\}_{k=1}^N$ the indifference condition for a player with sentiment α_k is given

$$V\left(x^*(\alpha_k), \{x^*(\alpha_l)\}_{l=1}^N\right) = 0$$

where

$$V\left(x^*(\alpha_k), \{x^*(\alpha_l)\}_{l=1}^N\right) = \sum_{l=1}^N g(\alpha_l) \int_{\underline{\theta}}^{\overline{\theta}} \theta \left[1 - \Phi\left(\frac{x^*(\alpha_l) - \theta}{\tau_x^{-1/2}} - \alpha_k\right)\right] f(\theta | x^*(\alpha_k))\, d\theta + \int_{\overline{\theta}}^{\infty} \theta f(\theta | x^*(\alpha_k))\, d - T$$

and

$$f(\theta | x^*(\alpha_k)) = (\tau_x + \tau_\theta)^{1/2} \phi\left(\frac{\theta - \frac{\tau_x x^*(\alpha_k) + \tau_\theta \mu_\theta}{\tau_x + \tau_\theta}}{(\tau_x + \tau_\theta)^{-1/2}}\right)$$

Our goal is to show that the system of equations defined by

$$\begin{gathered} V\left(x^*(\alpha_1), \{x^*(\alpha_l)\}_{l=1}^N\right) = 0 \\ \vdots \\ V\left(x^*(\alpha_N), \{x^*(\alpha_l)\}_{l=1}^N\right) = 0 \end{gathered}$$

has a unique solution. The proof boils down to establishing that the mapping $\mathbf{V} : \mathbb{R}^N \to \mathbb{R}^N$ defined by the LHS of the above system of equations is univalent, which, by the Gale-Nikaido Theorem, implies that there exists a unique vector of thresholds that satisfies the above system of indifference conditions. As Lemma 3 indicates, it is enough to show that the Jacobian of $\mathbf{V}$ is diagonally dominant.

B.2.1 Equilibrium in the limit as $\tau_x \to \infty$

We first establish uniqueness of equilibrium in the limit as the precision of private signals tends to infinity. Throughout this section it is convenient to make the following substitution in the players' indifference conditions:

$$z = \frac{\theta - \frac{\tau_x x^*(\alpha_k) + \tau_\theta \mu_\theta}{\tau_x + \tau_\theta}}{(\tau_x + \tau_\theta)^{-1/2}}$$

The payoff indifference condition becomes

$$\sum_{l=1}^{N} g(\alpha_l) \int_{L(\underline{\theta},x^*(\alpha_k))}^{L(\overline{\theta},x^*(\alpha_k))} \left[\frac{z}{(\tau_x+\tau_\theta)^{\frac{1}{2}}} + \frac{\tau_x x^*(\alpha_k)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}\right]\left[1-\Phi\left(\frac{x^*(\alpha_l) - \frac{z}{(\tau_x+\tau_\theta)^{1/2}} - \frac{\tau_x x^*(\alpha_k)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{\tau_x^{-1/2}} - \alpha_k\right)\right]\phi(z)\,dz$$

$$+ \int_{L(\overline{\theta},x^*(\alpha_k))}^{\infty} \left[\frac{z}{(\tau_x+\tau_\theta)^{1/2}} + \frac{\tau_x x^*(\alpha_k)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}\right]\phi(z)\,dz - T = 0 \qquad (6)$$

where

$$L(\underline{\theta},x^*(\alpha_k)) = \frac{\underline{\theta} - \frac{\tau_x x^*(\alpha_k)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{(\tau_x+\tau_\theta)^{-1/2}} \quad \text{and} \quad L(\overline{\theta},x^*(\alpha_k)) = \frac{\overline{\theta} - \frac{\tau_x x^*(\alpha_k)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{(\tau_x+\tau_\theta)^{-1/2}}$$

We now compute the limit as $\tau_x \to \infty$ of the LHS of Equation (6). It is straightforward to show that for a sufficiently high τ_x we have $x^*(\alpha_k) \in [T-\delta, \overline{\theta}+\delta]$ for some $\delta > 0$. Thus, it follows that $x^*(\alpha_k, \tau_x)$ has a convergent subsequence. In what follows we work with this subsequence. However, to keep notation simple we slightly abuse notation and talk about limiting behavior $x^*(\alpha_k, \tau_x)$ as $\tau_x \to \infty$, rather than behavior of $x^*(\alpha_k, \tau_x^n)$ where τ_x^n is an increasing sequence with $\lim_{n\to\infty} \tau_x^n = \infty$.

Claim 4 *We have*

1. $\lim_{\tau_x\to\infty} x^*(\alpha_l, \tau_x) = \lim_{\tau_x\to\infty} x^*(\alpha_k, \tau_x)$ *for all* $k, l \in \{1, ..., N\}$.[37]
2. $\lim_{\tau_x\to\infty} \left[\tau_x^{1/2}(x^*(\alpha_l,\tau_x) - x^*(\alpha_k,\tau_x))\right] \in \mathbb{R}$

Proof. **(Part 1)** We establish this by contradiction. Without loss of generality suppose that $l < k$. It is straightforward to see that for a sufficiently high τ_x we have $x^*(\alpha_l, \tau_x) > x^*(\alpha_k, \tau_x)$ so that a player who has a larger sentiment uses a lower threshold. Therefore, $\lim_{\tau_x\to\infty} x^*(\alpha_l, \tau_x) \geq \lim_{\tau_x\to\infty} x^*(\alpha_k, \tau_x)$.

Now suppose that $\lim_{\tau_x\to\infty} x^*(\alpha_l, \tau_x) > \lim_{\tau_x\to\infty} x^*(\alpha_k, \tau_x)$. It follows that for some $m \in \{1, ..., N\}$ we have

$$\lim_{\tau_x\to\infty} x^*(\alpha_m, \tau_x) - \lim_{\tau_x\to\infty} x^*(\alpha_k, \tau_x) \leq 0 \text{ then } \lim_{\tau_x\to\infty} x^*(\alpha_m, \tau_x) - \lim_{\tau_x\to\infty} x^*(\alpha_l, \tau_x) < 0$$

At this point we have to consider two cases: (1) $\lim_{\tau_x\to\infty} x^*(\alpha_l, \tau_x) < \overline{\theta}$ and (2) $\lim_{\tau_x\to\infty} x^*(\alpha_l, \tau_x) =$

[37] Intutively, if $x^*(\alpha_l) > x^*(\alpha_k)$ then when a player with sentiment α_l receives a high signal he is not only sure that a player with sentiment α_k or lower takes the risky action, but also expects to earn $x^*(\alpha_l)$ when the risky action is successful. On the other hand, a player with sentiment α_k can *at most* expect all players with sentiment α_k or lower to take the risky action and expects to receive $x^*(\alpha_k)$ when the risky action is successful. In the limit, this implies that either a player with sentiment α_l strictly prefers taking the risky action to taking the safe action or a player with sentiment α_k strictly prefers taking the safe action rather than the risky action. Either statement contradicts the definition of threshold signals.

$\overline{\theta}$.[38] Since the argument in both cases is analogous we consider only the case when $\lim_{\tau_x\to\infty} x^*(\alpha_l,\tau_x) < \overline{\theta}$.

If $\lim_{\tau_x\to\infty} x^*(\alpha_l,\tau_x) < \overline{\theta}$ then

$$\lim_{\tau_x\to\infty} L\left(\overline{\theta}, x^*(\alpha_l,\tau_x)\right) = \infty = \lim_{\tau_x\to\infty} L\left(\overline{\theta}, x^*(\alpha_k,\tau_x)\right)$$

Therefore, the indifference condition of a player with sentiment α_k converges to

$$T = \sum_{m=1}^{N} g(\alpha_m) \int_{-\infty}^{\infty} x^*(\alpha_k)\left[1-\Phi\left(-z-\alpha_k+\lim_{\tau_x\to\infty}\left[\tau_x^{1/2}\left(x^*(\alpha_m,\tau_x)-x^*(\alpha_k,\tau_x)\right)\right]\right)\right]\phi(z)\,dz$$

Now, let $n \le k$ denote the lowest sentiment such that $\lim_{\tau_x\to\infty}[x^*(\alpha_k,\tau_x)-x^*(\alpha_n,\tau_x)] \le 0$, that is for all $\alpha_m < \alpha_n$ we have

$$\lim_{\tau_x\to\infty}[x^*(\alpha_k,\tau_x)-x^*(\alpha_n,\tau_x)] > 0$$

Then

$$\begin{aligned}
T &= \sum_{m=1}^{N} g(\alpha_m) \int_{-\infty}^{\infty} x^*(\alpha_k)\left[1-\Phi\left(-z-\alpha_k+\lim_{\tau_x\to\infty}\left[\tau_x^{1/2}\left(x^*(\alpha_m,\tau_x)-x^*(\alpha_k,\tau_x)\right)\right]\right)\right]\phi(z)\,dz \\
&= \sum_{m=n}^{N} g(\alpha_m) \int_{-\infty}^{\infty} x^*(\alpha_k)\left[1-\Phi\left(-z-\alpha_k+\lim_{\tau_x\to\infty}\left[\tau_x^{1/2}\left(x^*(\alpha_m,\tau_x)-x^*(\alpha_k,\tau_x)\right)\right]\right)\right]\phi(z)\,dz \\
&\le \sum_{m=n}^{N} g(\alpha_m) \int_{-\infty}^{\infty} x^*(\alpha_k)\,\phi(z)\,dz \\
&< \sum_{m=n}^{N} g(\alpha_m) \int_{-\infty}^{\infty} x^*(\alpha_l)\,\phi(z)\,dz \\
&= \sum_{m=n}^{N} g(\alpha_m) \int_{-\infty}^{\infty} x^*(\alpha_l)\left[1-\Phi\left(-z-\alpha_l+\lim_{\tau_x\to\infty}\left[\tau_x^{1/2}\left(x^*(\alpha_m,\tau_x)-x^*(\alpha_k,\tau_x)\right)\right]\right)\right]\phi(z)\,dz \\
&\le \sum_{m=1}^{N} g(\alpha_m) \int_{-\infty}^{\infty} x^*(\alpha_l)\left[1-\Phi\left(-z-\alpha_l+\lim_{\tau_x\to\infty}\left[\tau_x^{1/2}\left(x^*(\alpha_m,\tau_x)-x^*(\alpha_k,\tau_x)\right)\right]\right)\right]\phi(z)\,dz = T
\end{aligned}$$

where the strict inequality follows from the assumption that $x^*(\alpha_l) > x^*(\alpha_k)$, and the fifth line

[38] In the limit as $\tau_x \to \infty$ a player will never set a threshold $x^*(\alpha_k)$ strictly above $\overline{\theta}$ since in this case there exists $\varepsilon > 0$ such that $x^*(\alpha_k) - \varepsilon > \overline{\theta}$, and using this lower threshold leads to a strictly higher payoff for all $\theta \in [x^*(\alpha_k)-\varepsilon, x^*(\alpha_k))$.

follows from observation that

$$\lim_{\tau_x \to \infty} x^*(\alpha_m, \tau_x) - \lim_{\tau_x \to \infty} x^*(\alpha_k, \tau_x) \leq 0 \text{ then } \lim_{\tau_x \to \infty} x^*(\alpha_m, \tau_x) - \lim_{\tau_x \to \infty} x^*(\alpha_l, \tau_x) < 0$$

Thus, we arrived at a contradiction.

(Part 2) As before, we need to differentiate between the case where (1) $\lim_{\tau_x \to \infty} x^*(\alpha_l, \tau_x) < \overline{\theta}$ and (2) $\lim_{\tau_x \to \infty} x^*(\alpha_l, \tau_x) = \overline{\theta}$. Since the arguments in both cases are similar, we again only consider here the first case.

We know that $\lim_{\tau_x \to \infty} x^*(\alpha_l, \tau_x) = \lim_{\tau_x \to \infty} x^*(\alpha_l, \tau_x)$ for all $l, k \in \{1, ..., N\}$. Denote this limit by x^*. Now, suppose that there exist k and l such that

$$\lim_{\tau_x \to \infty} \left[\tau_x^{1/2} \left(x^*(\alpha_k, \tau_x) - x^*(\alpha_l, \tau_x)\right)\right] \in \{-\infty, +\infty\}$$

Without loss of generality, assume that $l < k$. Then we know that for a sufficiently high τ_x we have $x^*(\alpha_k, \tau_x) < x^*(\alpha_l, \tau_x)$. Therefore

$$\lim_{\tau_x \to \infty} \left[\tau_x^{1/2} \left(x^*(\alpha_k, \tau_x) - x^*(\alpha_l, \tau_x)\right)\right] < \infty$$

So suppose that $\lim_{\tau_x \to \infty} \left[\tau_x^{1/2} \left(x^*(\alpha_k, \tau_x) - x^*(\alpha_l, \tau_x)\right)\right] = -\infty$. Since $x^*(\alpha_k, \tau_x) < x^*(\alpha_l, \tau_x)$ for all $m \neq l, k$ we have

$$\lim_{\tau_x \to \infty} \left[\tau_x^{1/2} \left(x^*(\alpha_k, \tau_x) - x^*(\alpha_m, \tau_x)\right)\right] \leq \lim_{\tau_x \to \infty} \left[\tau_x^{1/2} \left(x^*(\alpha_l, \tau_x) - x^*(\alpha_m, \tau_x)\right)\right]$$

and

$$\lim_{\tau_x \to \infty} \left[\tau_x^{1/2} \left(x^*(\alpha_k, \tau_x) - x^*(\alpha_k, \tau_x)\right)\right] = 0 = \lim_{\tau_x \to \infty} \left[\tau_x^{1/2} \left(x^*(\alpha_l, \tau_x) - x^*(\alpha_k l, \tau_x)\right)\right]$$

But then, in the limit we have

$$\begin{aligned} T &= \sum_{m=1}^{N} g(\alpha_m) \int_{-\infty}^{\infty} x^* \left[1 - \Phi\left(-z - \alpha_k - \lim_{\tau_x \to \infty} \left[\tau_x^{1/2} \left(x^*(\alpha_m) - x^*(\alpha_k)\right)\right]\right)\right] \phi(z)\, dz - T \\ &< \sum_{m=1}^{N} g(\alpha_m) \int_{-\infty}^{\infty} x^* \left[1 - \Phi\left(-z - \alpha_k - \lim_{\tau_x \to \infty} \left[\tau_x^{1/2} \left(x^*(\alpha_m) - x^*(\alpha_l)\right)\right]\right)\right] \phi(z)\, dz - T = T \end{aligned}$$

which is a contradiction. Thus, $\lim_{\tau_x \to \infty} \left[\tau_x^{1/2} \left(x^*(\alpha_k, \tau_x) - x^*(\alpha_l, \tau_x)\right)\right] > -\infty$. ■

With the above result we can now compute the limit of $\{x^*(\alpha_k)\}_{k=1}^N$ as $\tau_x \to 0$.

Lemma 5 *Let x^* be the limit of $\lim_{\tau_x \to \infty} x^*(\alpha_k, \tau_x) = x^*$. Then,*

$$x^* = \frac{1}{\sum_{m=1}^{N} g(\alpha_m)\, \Phi\left(\frac{\alpha_m}{\sqrt{2}}\right)} \quad \textit{if} \quad \frac{1}{\sum_{m=1}^{N} g(\alpha_m)\, \Phi\left(\frac{\alpha_m}{\sqrt{2}}\right)} \leq \overline{\theta}$$

$$x^* = \overline{\theta} \textit{ otherwise}$$

Proof. Suppose that

$$\frac{1}{\sum_{m=1}^{N} g(\alpha_m)\, \Phi\left(\frac{\alpha_m}{\sqrt{2}}\right)} \leq \overline{\theta}$$

Define

$$\kappa(k,1) \equiv \lim_{\tau_x \to \infty} \tau_x^{1/2} \left[x^*(\alpha_k) - x^*(\alpha_1)\right] \in \mathbb{R}$$

where the observation that $\kappa(k,1) \in \mathbb{R}$ follows from Claim 4. As $\tau_x \to \infty$ the indifference condition for a player with sentiment α_m, $m \in \{1, ..., N\}$ converges to

$$x^* \sum_{m=1}^{N} g(\alpha_m) \int_{-\infty}^{\infty} \left[1 - \Phi(-z - \alpha_k - \kappa(m,1) - \kappa(1,k))\right] \phi(z)\, dz = 0$$

Note that $\kappa(1,k) = -\kappa(k,1)$, and therefore the indifference condition is given by

$$x^* \sum_{m=1}^{N} g(\alpha_m) \int_{-\infty}^{\infty} \left[1 - \Phi(-z - \alpha_k - \kappa(m,1) + \kappa(k,1))\right] \phi(z)\, dz - T = 0$$

or

$$x^* \sum_{m=1}^{N} g(\alpha_m)\, \Phi\left(\frac{\alpha_k + \kappa(m,1) + \kappa(k,1)}{\sqrt{2}}\right) - T = 0$$

Thus, we arrive at the following system of $N-1$ unknowns (the unknown being $\{\kappa(m,1)\}_{m=1}^{N}$):

$$x^* \sum_{m=1}^{N} g(\alpha_m)\, \Phi\left(\frac{\alpha_1 + \kappa(m,1) + \kappa(1,1)}{\sqrt{2}}\right) - T = 0$$

$$\vdots$$

$$x^* \sum_{m=1}^{N} g(\alpha_m)\, \Phi\left(\frac{\alpha_N + \kappa(m,1) + \kappa(N,1)}{\sqrt{2}}\right) - T = 0$$

where the first equation corresponds to the indifference condition of a player with sentiment α_1 in the limit, the second equation corresponds to the indifference condition of a player with sentiment

α_2 and so on. By inspection, one can verify that the solution to this equation is

$$\kappa(m,1) = \alpha_m - \alpha_1, \text{ for all } m \in \{1, ..., N\}$$

Moreover, this is a unique solution. To see this, note that equations 2 to N above are a set of $N-1$ non-linear equations in $N-1$ unknowns.[39] It is straightforward to show that the Jacobian of this system is diagonally dominant which, by Lemma 3 implies that this system has a unique solution. It follows that

$$x^* = \frac{T}{\sum_{m=1}^{N} g(\alpha_m)\,\Phi\left(\frac{\alpha_m}{\sqrt{2}}\right)}$$

If $T > \overline{\theta} \sum_{m=1}^{N} g(\alpha_m)\,\Phi\left(\alpha_m/\sqrt{2}\right)$ then x^* converges to $\overline{\theta}$. We prove this by contradiction. Suppose that $x^*(\alpha_k)$'s do not converge to $\overline{\theta}$. Since we know that $\lim_{\tau_x \to \infty} x^*(\alpha_k) \in \left[T, \overline{\theta}\right]$ it follows that in this case we must have $\lim_{\tau_x \to \infty} x^*(\alpha_k) < \overline{\theta}$ and so

$$\overline{L} = \lim_{\tau_x} \frac{\overline{\theta} - \frac{\tau_x x^*(\alpha_k) + \tau_\theta \mu_\theta}{\tau_x + \tau_\theta}}{(\tau_x + \tau_\theta)^{-1/2}} = \infty$$

We perform the substitution $z = \left[\theta - \frac{\tau_x x^*(\alpha_k) + \tau_\theta \mu_\theta}{\tau_x + \tau_\theta}\right] / (\tau_x + \tau_\theta)^{-1/2}$ in the indifference equation of a player with sentiment α_k, $k \in \{1, ..., N\}$ and take the limit as $\tau_x \to \infty$ to obtain

$$\sum_{m=1}^{N} g(\alpha_m) \int_{-\infty}^{\infty} x^* \left[1 - \Phi(-z - \alpha_k - \kappa(m,1) - \kappa(1,k))\right] \phi(z)\, dz = 0,$$

where

$$\kappa(k,l) = \lim_{\tau_x \to \infty} \tau_x^{1/2} \left[x^*(\alpha_k) - x^*(\alpha_l)\right] \in \mathbb{R}$$

and so $\kappa(k,l) = \kappa(k,1) - \kappa(1,l) = \kappa(k,1) + \kappa(l,1)$. Evaluating the resulting integrals, we arrive at the system of equations:

$$x^* \sum_{m=1}^{N} g(\alpha_m)\,\Phi\left(\frac{\alpha_1 + \kappa(m,1) + \kappa(1,1)}{\sqrt{2}}\right) - T = 0$$

$$\vdots$$

$$x^* \sum_{m=1}^{N} g(\alpha_m)\,\Phi\left(\frac{\alpha_N + \kappa(m,1) + \kappa(N,1)}{\sqrt{2}}\right) - T = 0$$

[39] There are only $N-1$ independent constants $\kappa(k,l)$ since $\kappa(k,k) = 0$, $\kappa(k,l) = \kappa(l,k)$, and $\kappa(k,l) = \kappa(\kappa,m) + \kappa(m,l)$.

But we showed above that this system has a unique solution

$$x^* = \frac{T}{\sum\limits_{m=1}^{N} g(\alpha_m)\,\Phi\left(\frac{\alpha_m}{\sqrt{2}}\right)} > \overline{\theta}$$

by assumption. This is a contradiction since x^* was supposed to be smaller than $\overline{\theta}$. It follows that $x^* = \overline{\theta}$. ■

The argument presented above establishes that any convergent sequence $x^*(\alpha_k, \tau_x^n) \to x^*$, where $x^* = T/\left[\Sigma_{m=1}^N g(\alpha_m)\,\Phi\left(\alpha_m/\sqrt{2}\right)\right]$ if $T/\left[\Sigma_{m=1}^N g(\alpha_m)\,\Phi\left(\alpha_m/\sqrt{2}\right)\right] < \overline{\theta}$ and $x^* = \overline{\theta}$ otherwise. But then from the Lemma it follows that $x^*(\alpha_k, \tau_x) \to x^*$. Thus, we established Part (2) of Proposition 2.

B.2.2 Uniqueness of equilibrium away from the limit

In this section we prove part 1 of Proposition 2, that is we show that there exists $\overline{\tau}_x$ such that for all $\tau_x > \overline{\tau}_x$ the model with sentiments has a unique equilibrium.

Lemma 6 *There exists $\overline{\tau}_x$ such that if $\tau_x > \overline{\tau}_x$ then the model has unique equilibrium in monotone strategies.*

Proof. To establish this result it is enough to show that for a sufficiently high τ_x

$$\frac{\partial V_k}{\partial x^*(\alpha_k)} - \sum_{l \neq k} \left| \frac{\partial V_l}{\partial x^*(\alpha_k)} \right| > 0 \text{ for all } k \in \{1, ..., N\},$$

which implies that the Jacobian of the mapping V is diagonally dominant. Using the expressions for the derivatives reported above we have

$$
\begin{aligned}
&\frac{\partial V_k}{\partial x^*(\alpha_k)} - \sum_{l\neq k} \frac{\partial V_l}{\partial x^*(\alpha_k)} \\
&= \sum_{l=1}^{N} g(\alpha_l) \frac{\tau_x}{\tau_x+\tau_\theta} \int_{\underline{\theta}}^{\overline{\theta}} \left[1-\Phi\left(\frac{x^*(\alpha_l)-\theta}{\tau_x^{-1/2}} - \alpha_k\right)\right] (\tau_x+\tau_\theta)^{1/2} \phi\left(\frac{\theta - \frac{\tau_x x^*(\alpha_k)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{(\tau_x+\tau_\theta)^{-1/2}}\right) d\theta \\
&+ \sum_{l=1}^{N} g(\alpha_l) \frac{\tau_x}{\tau_x+\tau_\theta} \int_{\underline{\theta}}^{\overline{\theta}} \theta \tau_x^{1/2} \phi\left(\frac{x^*(\alpha_l)-\theta}{\tau_x^{-1/2}} - \alpha_k\right) (\tau_x+\tau_\theta)^{1/2} \phi\left(\frac{\theta - \frac{\tau_x x^*(\alpha_k)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{(\tau_x+\tau_\theta)^{-1/2}}\right) d\theta \\
&- g(\alpha_k) \frac{\tau_\theta}{\tau_x+\tau_\theta} \int_{\underline{\theta}}^{\overline{\theta}} \theta \tau_x^{1/2} \phi\left(\frac{x^*(\alpha_l)-\theta}{\tau_x^{-1/2}} - \alpha_k\right) (\tau_x+\tau_\theta)^{1/2} \phi\left(\frac{\theta - \frac{\tau_x x^*(\alpha_k)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{(\tau_x+\tau_\theta)^{-1/2}}\right) d\theta \\
&- \sum_{l\neq k} g(\alpha_l) \int_{\underline{\theta}}^{\overline{\theta}} \theta \tau_x^{1/2} \phi\left(\frac{x^*(\alpha_l)-\theta}{\tau_x^{-1/2}} - \alpha_k\right) (\tau_x+\tau_\theta)^{1/2} \phi\left(\frac{\theta - \frac{\tau_x x^*(\alpha_k)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{(\tau_x+\tau_\theta)^{-1/2}}\right) d\theta \\
&+ \text{Positive Terms}
\end{aligned}
$$

where $\sum_{l\neq k} \left|\frac{\partial V_l}{\partial x^*(\alpha_k)}\right|$ in the above expression is captured by

$$
\sum_{l\neq k} g(\alpha_l) \int_{\underline{\theta}}^{\overline{\theta}} \theta \tau_x^{1/2} \phi\left(\frac{x^*(\alpha_l)-\theta}{\tau_x^{-1/2}} - \alpha_k\right) (\tau_x+\tau_\theta)^{1/2} \phi\left(\frac{\theta - \frac{\tau_x x^*(\alpha_k)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{(\tau_x+\tau_\theta)^{-1/2}}\right) d\theta
$$

We focus on the first four terms in the above derivative and notice that they simplify to

$$
\begin{aligned}
&\sum_{l=1}^{N} g(\alpha_l) \frac{\tau_x}{\tau_x+\tau_\theta} \int_{\underline{\theta}}^{\overline{\theta}} \left[1-\Phi\left(\frac{x^*(\alpha_k)-\theta}{\tau_x^{-1/2}} - \alpha_k\right)\right] (\tau_x+\tau_\theta)^{1/2} \phi\left(\frac{\theta - \frac{\tau_x x^*(\alpha_k)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{(\tau_x+\tau_\theta)^{-1/2}}\right) d\theta \\
&- \sum_{l=1}^{N} g(\alpha_l) \frac{\tau_\theta}{\tau_x+\tau_\theta} \int_{\underline{\theta}}^{\overline{\theta}} \theta \tau_x^{1/2} \phi\left(\frac{x^*(\alpha_l)-\theta}{\tau_x^{-1/2}} - \alpha_k\right) (\tau_x+\tau_\theta)^{1/2} \phi\left(\frac{\theta - \frac{\tau_x x^*(\alpha_k)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{(\tau_x+\tau_\theta)^{-1/2}}\right) d\theta
\end{aligned}
$$

To show that the above term is strictly positive for a high τ_x, in each of the above integrals we make the substitution

$$
z = \frac{\theta - \frac{\tau_x x^*(\alpha_k)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{(\tau_x+\tau_\theta)^{-1/2}}
$$

Then, the above integrals become

$$\sum_{l=1}^{N} g\left(\alpha_l\right) \frac{\tau_x}{\tau_x+\tau_\theta} \int_{L\left(\underline{\theta},\alpha_k\right)}^{L\left(\overline{\theta},\alpha_k\right)} \left[1-\Phi\left(\frac{x^*\left(\alpha_k\right)-\frac{z}{\left(\tau_x+\tau_\theta\right)^{1/2}}-\frac{\tau_x x^*\left(\alpha_k\right)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{\tau_x^{-1/2}}-\alpha_k\right)\right]\phi\left(z\right)dz$$

$$-\sum_{l=1}^{N} g\left(\alpha_l\right) \frac{\tau_\theta}{\tau_x+\tau_\theta} \int_{L\left(\underline{\theta},\alpha_k\right)}^{L\left(\overline{\theta},\alpha_k\right)} \left\{\left[\frac{z}{\left(\tau_x+\tau_\theta\right)^{1/2}}+\frac{\tau_x x^*\left(\alpha_k\right)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}\right]\times\right.$$

$$\left.\tau_x^{1/2}\phi\left(\frac{x^*\left(\alpha_k\right)-\frac{z}{\left(\tau_x+\tau_\theta\right)^{1/2}}-\frac{\tau_x x^*\left(\alpha_k\right)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{\tau_x^{-1/2}}-\alpha_k\right)\phi\left(z\right)\right\}dz$$

where

$$L\left(\overline{\theta},\alpha_k\right)=\frac{\overline{\theta}-\frac{\tau_x x^*\left(\alpha_k\right)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{\left(\tau_x+\tau_\theta\right)^{-1/2}} \quad \text{and} \quad L\left(\underline{\theta},\alpha_k\right)=\frac{\underline{\theta}-\frac{\tau_x x^*\left(\alpha_k\right)+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{\left(\tau_x+\tau_\theta\right)^{-1/2}}$$

Since $\lim x^*\left(\alpha_k\right) \leq \overline{\theta}$ we have $L\left(\underline{\theta},\alpha_k\right) \to -\infty$ as $\tau_x \to \infty$ and that $\lim_{\tau_x\to\infty} L\left(\overline{\theta},\alpha_k\right) > -\infty$. Therefore,

$$\begin{aligned}
&\lim_{\tau_x\to\infty}\left[\frac{\partial V_k}{\partial x^*\left(\alpha_k\right)}-\sum_{l\neq k}\frac{\partial V_l}{\partial x^*\left(\alpha_k\right)}\right] \\
&\geq \sum_{l=1}^{N} g\left(\alpha_l\right)\int_{-\infty}^{\overline{L}}\left[1-\Phi\left(-z-\alpha_k+\lim_{\tau_x\to\infty}\left[\tau_x^{1/2}\left(x^*\left(\alpha_l\right)-x^*\left(\alpha_k\right)\right)\right]\right)\right]\phi\left(z\right)dz \\
&-\left(\lim_{\tau_x\to\infty}\frac{\tau_\theta}{\tau_x+\tau_\theta}\right)\sum_{l=1}^{N} g\left(\alpha_l\right)\int_{-\infty}^{\overline{L}} x^*\tau_x^{1/2}\phi\left(-z-\alpha_k+\lim_{\tau_x\to\infty}\left[\tau_x^{1/2}\left(x^*\left(\alpha_l\right)-x^*\left(\alpha_k\right)\right)\right]\right)\phi\left(z\right) \\
&> 0
\end{aligned}$$

It follows that there exists a large enough τ_x, call it $\overline{\tau}_{x,k}$, such that if $\tau_x > \overline{\tau}_{x,k}$ then the k-th row of the Jacobian of mapping $\mathbf{V}$ is dominated by its diagonal entry. Define

$$\overline{\tau}_x = \max_{k\in\{1,\ldots,N\}} \overline{\tau}_{x,k}$$

Then for all $\tau_x \geq \overline{\tau}_x$ the Jacobian of mapping $\mathbf{V}$ is diagonally dominant, which, by Lemma 3, establishes the claim. ■

C Quantal Response Equilibrium

C.1 Definition of Equilibrium

We first define a logit quantal response equilibrium (henceforth, logit QRE) in our model following closely McKelvey and Palfrey (1995). Let $\pi_i : \mathbb{R} \to [0,1]$, $i = 1,2$, be a mapping that assigns to each signal x_i a probability that player i takes action A. Let U_{Hi} be the expected utility to player i from taking action $H \in \{A, B\}$. In particular,

$$U_{Ai}\left(x_i;\pi_j\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \theta\pi_j\left(x_j\right) g\left(x_j|\theta\right) f\left(\theta|x_i\right) dx_j d\theta + \int_{-\infty}^{\infty} \theta f\left(\theta|x_i\right) d\theta - T,$$

where $g\left(x_j|\theta\right)$ is the conditional pdf of player j's signal x_j given θ and $f\left(\theta|x_i\right)$ is the conditional pdf of θ given player i's signal, x_i, and

$$U_{Bi}\left(x_i;\pi_j\right) = 0$$

The logistic quantal response function of player i to π_j is then a mapping $\sigma\left(\cdot;\pi_j\right) : \mathbb{R} \to [0,1]$ that assigns to each player i's signal x_i a probability with which player i plays action A according to the quantal function

$$\sigma\left(x_i;\pi_j\right) = \frac{e^{\lambda U_{Ai}\left(x_i;\pi_j\right)}}{e^{\lambda U_{Ai}\left(x_i;\pi_j\right)} + e^{\lambda U_{Bi}\left(x_i;\pi_j\right)}} \tag{7}$$

The parameter λ, as in McKelvey and Palfrey (1995), measures how likely it is that players choose actions that have a higher expected payoff. Note that $\lambda = 0$ corresponds to random behavior while $\lambda \to \infty$ corresponds the situation where players always choose the optimal action (and hence play according to a Nash equilibrium).

Having introduced the necessary notation we now define a logit QRE.

Definition 7 *Given λ, a logit quantal response equilibrium is a tuple of quantal best-response functions $\{\pi_1^*, \pi_2^*\}$ such that for each $i = 1,2$ and $j \neq i$ we have*

$$\pi_i^*\left(x_i\right) = \sigma\left(x_i;\pi_j^*\right)$$

where $\sigma\left(x_i;\pi_j\right)$ is defined in (7). The logit QRE is symmetric if $\pi_1^*(x) = \pi_2^*(x)$ for each $x \in \mathbb{R}$.

C.2 Estimation

To see whether our empirical results can be explained by logit QRE we compute numerically the symmetric logit equilibrium $\{\pi_1^*, \pi_2^*\}$ for different values of λ. Then, for each λ we feed into π^* the actual signal realizations from the experiment to simulate the responses of subjects if they where playing according to a logit QRE with parameter λ. Thus, our simulation assigns to each observed signal x_i in the experiment an a simulated action (that is, the action chosen according to $\{\pi_1^*, \pi_2^*\}$).

D Analogy-Based Expectation Equilibria

In this section, we briefly discuss the analogy-based expectation equilibrium (ABEE). For a more detailed discussion, see Koessler and Jehiel (2008).

D.1 Coarsest analogy partition

Koessler and Jehiel (2008) define the coarsest partition as follows. They assume that players bundle all states together and hence they best respond to the average behavior of their opponent. Let P_j denote the actual ex-ante probability of player j choosing A given player j's strategy. Thus, player i will choose A if and only if

$$E\left[\theta|x_i\right] P_j - T \geq 0$$

From the above equation, it is immediate that players will follow threshold strategies. Therefore, the ABEE is given by the following two equations:

$$E\left[\theta|x_1^*, \theta \in \left[\underline{\theta}, \overline{\theta}\right]\right] P_2 + E\left[\theta|x_1^*, \theta \geq \overline{\theta}\right] - T = 0 \tag{8}$$

$$E\left[\theta|x_2^*, \theta \in \left[\underline{\theta}, \overline{\theta}\right]\right] P_1 + E\left[\theta|x_2^*, \theta \geq \overline{\theta}\right] - T = 0, \tag{9}$$

where

$$P_i = \int_{-\infty}^{+\infty} \left[1 - \Phi\left(\frac{x_j^* - \theta}{\sigma_x}\right)\right] \phi\left(\frac{\theta - \mu_\theta}{\sigma_\theta}\right) d\theta, \quad i = 1, 2, \ j \neq i \tag{10}$$

The thresholds reported in Table 7 in the paper are computed by solving Equation (8) for each $\sigma_x \in \{1, 10, 20\}$.

D.2 Finer Partitions

Koessler and Jehiel (2008) develop the ABEE concept for games with a finite number of states. However, in a model with infinite many states such as ours there exists an uncountable number of possible analogy partitions and it is unclear how to pick one. Below, we propose a simple sequence of subsequent partition refinements that we believe are intuitive partition choices.

In what follows, we consider a sequence of partitions $\{\mathcal{P}_k\}_{k=0}^{\infty}$, where $\mathcal{P}_0$ denotes the coarsest partition (as described above), and each partition $\mathcal{P}_k$ is a refinement of partition $\mathcal{P}_{k-1}$. We define $\{\mathcal{P}_k\}_{k=0}^{\infty}$ iteratively in the following way. First, we assume that $\mathcal{P}_1 = \left\{(-\infty, \underline{\theta}), \left[\underline{\theta}, \overline{\theta}\right), \left[\overline{\theta}, \infty\right)\right\}$.

Next, we assume that each $\mathcal{P}_k$ refines $\mathcal{P}_{k-1}$ by splitting each analogy bin of partition $\mathcal{P}_{k-1}$ that belongs to $[\underline{\theta}, \overline{\theta})$ into two bins of equal length. That is

$$
\begin{aligned}
\mathcal{P}_2 &= \left\{(-\infty, \underline{\theta})\,, \left[\underline{\theta}, \frac{\overline{\theta}+\underline{\theta}}{2}\right)\,, \left[\frac{\overline{\theta}+\underline{\theta}}{2}, \overline{\theta}\right)\ [\overline{\theta}, \infty)\right\} \\
\mathcal{P}_3 &= \left\{(-\infty, \underline{\theta})\,, \left[\underline{\theta}, \frac{3}{4}\underline{\theta}+\frac{1}{4}\overline{\theta}\right)\,, \left[\frac{3}{4}\underline{\theta}+\frac{1}{4}\overline{\theta}, \frac{\overline{\theta}+\underline{\theta}}{2}\right)\ \left[\frac{\overline{\theta}+\underline{\theta}}{2}, \frac{1}{4}\underline{\theta}+\frac{3}{4}\overline{\theta}\right)\,, \left[\frac{1}{4}\underline{\theta}+\frac{3}{4}\overline{\theta}, \overline{\theta}\right)\,, [\overline{\theta}, \infty)\right\} \\
&\ \vdots
\end{aligned}
$$

The appeal of the above family of partitions is that $\mathcal{P}_1$ corresponds to the partition of the state-space according to the number of players that take action A that are required to make action A successful and each subsequent analogy-partition is a refinement of the previous partition.

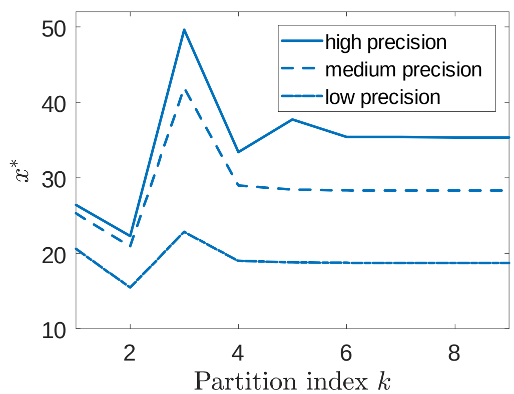


Figure 11: Implied ABEE thresholds, by precision

Figure 11 depicts the implied ABEE threshold for high precision (solid line), medium precision (dashed line), and low precision(dashed-dotted line).[40] We see that as in our baseline model, fixing an analogy partition, the thresholds are increasing in precision. Moreover, for each partition $\mathcal{P}_k$, the threshold for low precision is around 18. Both observations are at odds with the data. Thus, we conclude that ABEE are unlikely to explain our empirical findings.

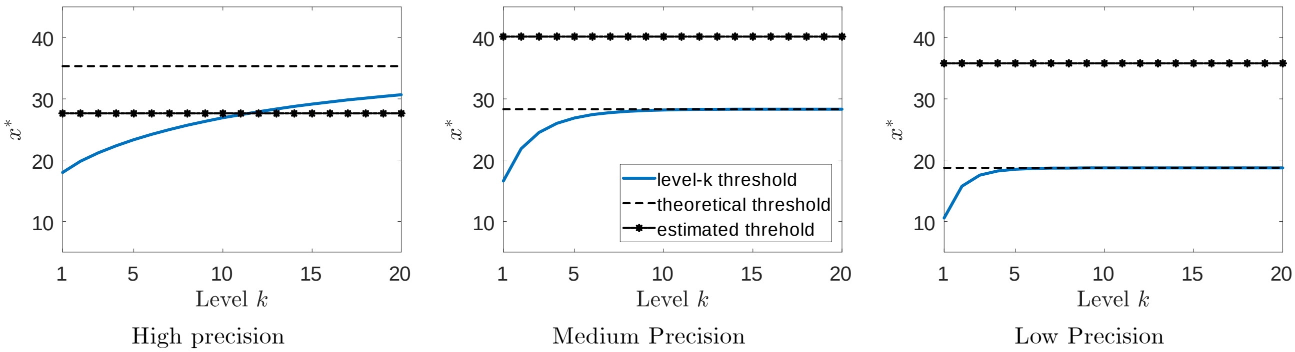


Figure 12: Comparison of level-k threshold (as level of reasoning k varies) when level-0 is always assumed to take the *risky* action with the theoretical and estimated thresholds

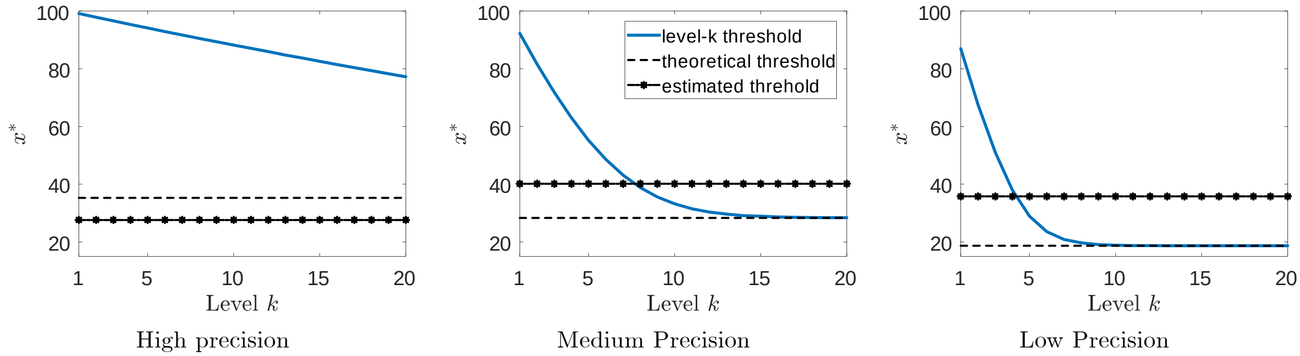


Figure 13: Comparison of level-k threshold (as level of reasoning k varies) when level-0 is always assumed to take the *safe* action with the theoretical and estimated thresholds

E Level-k Model

Figure 12 illustrates that when level-0 players are assumed to always take a risky action then, for the case of medium and low precision, the model predicts thresholds which are lower than those observed in our experiment, regardless of the players' depth of reasoning. On the other hand, when level-0 players are assumed to always choose the safe action (Figure 13) then the implied threshold for high precision is much higher than that observed in the data, regardless of the players' depth of reasoning.

[40] In general, there are multiple equilibrium thresholds when information is precise and the partition is coarse, but finer than the coarsest partition. In our case, for high precision there are multiple equilibrium thresholds when players use partitions $\mathcal{P}_1$ and $\mathcal{P}_2$, while for medium precision there are multiple equilibrium thresholds when players use partition $\mathcal{P}_1$. In these cases we consider the ABEE threshold which is always the closest to the one observed in the data.

F Standard global game with risk aversion

In this section we consider our baseline model (as described in Section 2.2), but assume that players have utility $u(w) = w^{\rho}/\rho$, where w is the payoff they earn in a given round and $\rho \in \mathbb{R}$. Here, $1-\rho$ is the players' coefficient of relative risk aversion.

Let E denote the endowment of a player (recall that in the experiment subjects start with a positive endowment). If action A is successful then the payoff is $\theta + E - T$. If action A is unsuccessful, then the payoff is $E - T$. Thus, the equilibrium thresholds solve

$$
\begin{aligned}
&\int_{\underline{\theta}}^{\overline{\theta}} \frac{(\theta + E - T)^{\rho}}{\rho} \Pr\left(x_j \geq x^* | \theta\right) f\left(\theta | x^*\right) d\theta + \int_{\overline{\theta}}^{\infty} \frac{(\theta + E - T)^{\rho}}{\rho} f\left(\theta | x^*\right) d\theta \\
&+ \int_{\underline{\theta}}^{\overline{\theta}} \frac{(E - T)^{\rho}}{\rho} \Pr\left(x_j < x^* | \theta\right) f\left(\theta | x^*\right) d\theta + \int_{-\infty}^{\underline{\theta}} \frac{(E - T)^{\rho}}{\rho} f\left(\theta | x^*\right) d\theta \\
&= \frac{E^{\rho}}{\rho}
\end{aligned}
$$

The first two terms on the LHS of the above equation correspond to the case where action A is successful, in which case player i earns $\theta + E - T$ if he chooses action A. The last two terms on the LHS correspond to the case when action A is unsuccessful, in which case player i receives payoff $E - T$ if he chooses action A. The payoff from choosing action B is E.

We then use the above equation to compute the coefficient of relative risk aversion needed to rationalize the thresholds observed in the data. As reported in the paper (Table 8) in order to rationalize our findings, subjects would have to be risk-loving when information is precise and risk averse when information is imprecise, which is hard to justify.

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