Uncovering Biases in Information Choice and its Use: The Role of Strategic Uncertainty*

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Abstract

We study how the presence of strategic uncertainty affects how people choose and use information in a simple game that can be easily transformed into an individual decision task. Despite differences in initial choices, strategic uncertainty has little effect on how people choose information once behavior has stabilized. While the modal precision choice corresponds to the equilibrium prediction, we find that a substantial proportion of subjects overacquires information. In terms of information use, we find substantial overuse of private information in the strategic, but not in the individual decision environment. We argue that the overuse of information is driven by the difficulty to form beliefs about others' signals and strategies. Our analysis also suggests that overacquisition might be driven partially by the overuse of information and partially by an incorrect perception of the underlying information structure. Finally, we characterize the welfare consequences of the information overacquisition and overuse biases.

Keywords: information acquisition, information use, behavioral biases, strategic uncertainty.

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1 Introduction

Information processing biases have been extensively documented in psychology and economics. These systematic departures from the Bayesian paradigm can take different forms, depending on the information that is neglected or overweighed. The vast majority of the evidence that has identified these biases comes from individual decision-making environments where agents have to form beliefs about unknown states.[1] However, we have limited knowledge of how information processing biases arise in strategic environments, which are characterized by a complex architecture of beliefs. In games of incomplete information players face not only fundamental uncertainty, but also strategic uncertainty and thus form beliefs, respectively, about states and about the actions and beliefs of others. Game theoretic models of incomplete information approach these complex belief systems by assuming Bayesian rationality of players. However, in light of our robust understanding of departures from Bayesian rationality in individual decision making, it is an important endeavour to characterize how information processing biases might arise in strategic settings and to identify biases that could be intrinsic to these environments. Our paper is the first one that attempts to broadly characterize biases in the choice and use of information that are driven by strategic uncertainty.

The objective of this paper is to study experimentally how the presence of strategic uncertainty affects the choice and use of information. We use a simple theoretical framework (based on Morris and Shin (2002)) which features a rich information structure that leads to non-trivial trade-offs in the choice and use of different types of information (private and public). Our theoretical setup nests an individual decision making case, which allows us to easily compare behavior across strategic and non-strategic environments, both theoretically and experimentally. Thus, we are able to identify how the presence of strategic uncertainty affects the choice and use of information.[2]

[1]See, for example, Tversky and Kahneman (1974), Kahneman at al. (1991), Griffin and Tversky (1992), Moore and Cain (2007), or Enke and Zimmermann (2018). Rabin (1998) and Kahneman et al. (2001) provide overviews of this literature.

[2]Dale and Morgan (2012) and Cornand and Heinemann (2014) test the pre-

In terms of information choices, we find that, overall, subjects tend to make similar choices in the strategic and individual environments characterized by both high frequencies of equilibrium precision and overacquisition of information. However, in the initial rounds we see a much stronger tendency for subjects to overacquire information in the presence of strategic uncertainty and it is learning dynamics that eventually lead to similar distribution of choices across environments. In terms of how subjects use the information acquired, we find that strategic uncertainty has a strong effect. While subjects use information similarly to the Bayesian paradigm in the individual decision task, they strongly overuse private information in the strategic setup, particularly those subjects who overacquire information. We show that this overuse is not driven by uncertainty about the rationality of others and explore how specific components of belief formation in strategic environments can affect this result. We analyze the payoff consequences of the overacquisition and overuse biases and find that overacquisition of information is responsible for the observed losses.

As our theoretical benchmark we develop a two-player version of the beauty contest model of Morris and Shin (2002), extended to feature an initial stage of costly information acquisition.[3] In the model, agents want to choose an action

dictions of Morris and Shin (2002) and Angeletos and Pavan (2007) regarding the equilibrium use of information in games without information acquisition and find that subjects underreact to changes in the precision of public information. Heinemann et al. (2007), Szkup and Trevino (2020), and Trevino (2020) investigate how exogenous changes in the information structure affect subjects play in global games. Few papers study information acquisition experimentally in strategic environments, such as Szkup and Trevino (2022), Gretschko and Rajko (2015), and Battacharya et al. (2017). Baeriswyl et al. (2020) study endogenous attention allocation and show that subjects allocate less attention to most common and least private signal than predicted the theory. None of these papers study how the strategic environment affects the way subjects choose or use information.

[3]We focus on a two-player setup since this is the simplest departure from the individual setup. This also avoids some of the difficulties that experimental subjects face when coordinating in larger groups (see Weber (2006)).

that is close to an unknown state of fundamentals and to the other agent's action. We model this utility via a quadratic loss function where the mismatch can occur along these two dimensions. Before choosing their actions, agents observe two noisy signals about the fundamental: a public signal, with an exogenously given precision, and a private signal whose precision is endogenously chosen by each agent at the information acquisition stage, at a cost. The presence of coordination motives and incomplete information implies that agents face a substantial amount of strategic uncertainty as they have to make inferences not only about the action of their opponent, but also about their information and beliefs. At the same time, they face a trade-off between matching the fundamental and matching the action of the other agent. This leads to a rich set of testable theoretical predictions. The equilibrium action rules in the baseline model and in its various behavioral extensions are always linear combinations of private and public information, which makes this model an ideal benchmark to characterize departures from the Bayesian paradigm in terms of biases.

In the theoretical analysis we characterize the unique Bayesian Nash equilibrium of our model. We also derive the corresponding predictions for the individual decision-making version of our game where we remove strategic motives, i.e., where agents care only about matching the state of fundamentals. Our paper relates to the broader literature that studies theoretically information acquisition and use in coordination games. Following Morris and Shin (2002), the tensions between increased transparency of public information and welfare have been investigated both when private information is exogenously determined (Angeletos and Pavan (2007), Ui and Yoshizawa (2015)) and when it is endogenized (Hellwig and Veldkamp (2009), Myatt and Wallace (2012), Colombo et al. (2014), Pavan (2016)). These papers assume Bayesian rationality of players. Our experiment validates some of the predictions of this literature, such as the crowding out effects of increased transparency of public information on private information acquisition, and hints at welfare mechanisms. However, the biases we identify also suggest that some of the predictions of these papers might need to be reevaluated.

With our theoretical results in hand, we design the experiment to understand how strategic uncertainty affects the choice and use of information. In terms of information choices, we observe no difference in the strategic and non-strategic environments once behavior has stabilized. In both environments the distributions of precision choice are bimodal with peaks corresponding to the equilibrium prediction and overacquisition of information. The overacquisition of private information is sustained across different levels of transparency of public information and, interestingly, its extent is not driven by strategic motives. In the strategic environment information choices are strategic complements, so we could imagine that precision choices off-equilibrium could differ across environments due to this. However, we observe no significant differences in the individual decision making environment and this is robust to varying parameters of the model, such as strength of coordination motives or transparency of public information. We find strong support for the theoretical prediction that an increase in the transparency of public information crowds out private information acquisition (see Colombo et al. (2016)).

While overacquisition of instrumental information has been documented in few instances, it is not clear why subjects are prone to overacquire information.[4] To understand the sources of the observed heterogeneity we analyze the dynamics of information choices in our experiment. We find that subjects who start choosing high precisions rarely experiment with lower precisions, so they do not get to learn the benefits of cheaper and less precise private information, leading to higher precision than the equilibrium becoming an absorbent state. By looking at learning dynamics we find differences across the individual and strategic environments in the initial rounds of the experiment, with relatively uniform choices across different precisions in the individual setup and choices that clearly favor overacquisition in the strategic setup. However, the distribution of precision choices in both environments converge to similar distributions. These results suggest that strategic uncertainty does have an impact on subjects information choices encouraging them initially to overacquire information, but its impact diminishes over time.

In contrast to our results about information choices, we see clear differences in how subjects use information in the strategic and non-strategic environments.

[4]Conlon et al. (2016) and Reshidi et al. (2021) present evidence for overacquisition of information in non-strategic environments, while Gretschko and Rajko (2015) and Battacharya et al. (2017) do so under strategic uncertainty.

While subjects in the individual decision making environment use their signals closely to the Bayesian benchmark, we see a stark overuse of private information in the strategic case, especially for those subjects who overacquire information. We show that this overuse of information is not due to a sunk cost fallacy because the bias is still present in the strategic setup in a treatment without costly information acquisition. Moreover, the overuse of private information is robust to variations in the strength of strategic complementarities. We show that our results cannot be explained by standard models of bounded rationality in games (level-k, regret aversion, quantal response equilibrium, or overconfidence). Likewise, we do not identify any learning dynamics throughout the different rounds of the experiment that could explain our results. Thus, we conclude that strategic uncertainty has a strong and lasting impact on subjects' use of information.

We investigate possible channels through which strategic uncertainty may lead to this disparity in the use of information. We refer to strategic uncertainty broadly as the uncertainty about the behavior of the other player. In our setup this is manifested in three ways. The first is captured by the theoretical model where, in equilibrium, players do not know the other player's signal, so they do not know their action, but they do know the mapping between signals and actions (i.e., the strategy). The second manifestation of strategic uncertainty that can occur in our experiment, but not in the model, is that subjects do not know the strategy of the other player so they have to form beliefs about the mapping between signals and actions. Finally, strategic uncertainty can manifest in our experiment as uncertainty about the rationality of the other player. To understand our results about the overuse of private information in the strategic setting we focus on these 3 manifestations by devising 2 treatments where we shut down the first and third manifestations, respectively, in a simple way.

We explore whether uncertainty about the rationality of others could be responsible for our results by running a treatment where subjects play against a computer that follows the equilibrium action. Subjects do not know the signal or the strategy of their opponent, but they know that their opponent behaves optimally. We find persistent overuse of private information in this case as well, which suggests that the overuse bias we identify in the game is inherent to strategic forces that do not rely on beliefs about rationality. In a different treatment we shut down the strategic uncertainty related to forming beliefs about the signal observed by the other player. This treatment is identical to the strategic baseline, except that subjects are provided with the Bayesian estimates of their opponent's private signal, which is based on the subject's own signals. Qualitatively, we still observe that subjects who overacquire information overuse private information with respect to the Bayesian benchmark, but we see a drastic decrease in the magnitude of this bias, suggesting that the difficulty to form beliefs about the information held by others contributes significantly to the overuse bias. The persistence of this bias in this treatment, however, suggests that it is not just the difficulty to form beliefs about the observations of others, but that forming beliefs about the mapping of these observations to actions might also be behind this bias.

We analyze realized payoffs and compare them to the payoffs that would result if subjects behaved according to the theoretical benchmark to quantify the welfare effects of the biases we identify. We find that the overuse of information has a negligible effect on payoffs, whereas the overacquisition of information leads to significant welfare losses due to the higher cost of more precise information. This result is robust to treatment and parameter variations.

Our results might offer relevant lessons for economists, policy makers, and professionals. The abundance of information in the digital world leads individuals, firms, investors, and policy makers to spend more time than ever gathering, processing, and utilizing information. Our qualitative results, i.e., the identification of biases, are relevant to understand these processes in a variety of settings that feature key trade-offs emphasized in our model, such as firms' technology adoption, investment and pricing decisions, financial investors' portfolio choices, or banks' lending decisions.

2 The model

The model used to derive hypotheses for the experiment is a modified version of Morris and Shin (2002) with two players (as opposed to a continuum) and extended to feature an initial stage of costly information acquisition. Therefore, our model captures the interplay between endogenous information choices and fundamental and strategic uncertainty. In the Online Appendix we discuss the specific relationship between our model and Morris and Shin (2002).

2.1 Preferences

There are two identical agents $i = 1, 2$. Agent i's utility is given by

$$U(a_i, a_j, \theta) = -(1-\alpha)(a_i - \theta)^2 - \alpha(a_i - a_j)^2, \tag{1}$$

where a_i is agent i's action, a_j is the action of the other agent, and θ is a payoff-relevant variable, which we refer to as the fundamental state. The constant $\alpha \geq 0$ captures the degree of strategic complementarity in agents' actions. This utility function indicates that agents would like to choose an action close to the fundamental θ and close to the other agent's action, with α capturing the relative importance assigned to each of these motives. The higher is α the more agents care about matching the other agent's action relative to matching the state. When $\alpha = 0$ the strategic motive is absent, in which case the model corresponds to an individual decision making problem. As is standard in the literature, we set $\alpha < 1$ to ensure the existence of equilibrium (see Angeletos and Pavan (2007)). The fundamental state, θ, is distributed according to a normal distribution with mean μ_θ and variance τ_θ^{-1}, that is $\theta \sim N(\mu_\theta, \tau_\theta^{-1})$. However, the realization of θ is not observed by the agents.

2.2 The Information Acquisition Stage

In the first stage of the model each agent chooses privately the precision of their private signal about θ, x_i, that they receive at the beginning of the second stage, where

$$x_i = \theta + \tau_i^{-1/2}\varepsilon_i,\ \varepsilon_i \sim N(0,1), \tag{2}$$

with ε_i independent of θ and i.i.d. across agents. τ_i is the precision that agent i chooses for his signal in the information acquisition stage. We assume $\tau_i \in [\underline{\tau}, \infty]$ (with $\underline{\tau} \geq 0$), where choosing $\tau_i = \infty$ implies observing θ perfectly. Acquiring more precise information is costly: choosing τ_i is associated with a cost $C(\tau_i)$.

Assumption 1 *The cost function C is continuously twice differentiable and satisfies the following properties: (i) C is strictly increasing in τ_i ($C'(\cdot) > 0$), (ii) C is strictly convex in τ_i ($C''(\cdot) > 0$), (iii) $C(\underline{\tau}) = C'(\underline{\tau}) = 0$, and (iv) $\lim_{\tau_i \to \infty} C'(\tau_i) = \infty$.*

These are standard assumptions in the literature on costly information acquisition (see, for example, Colombo et al. (2014) or Szkup and Trevino (2015)). The first and second part of Assumption 1 imply that more precise information is more costly and that a marginal increase in precision is more costly when the precision is already high; that is $C(\cdot)$ is increasing and convex. The third part states that acquiring no information is associated with no cost and that an infinitesimal improvement in precision is costless. As a consequence, in equilibrium, both agents will choose to improve the precision of their signals. The last property implies that no agent will ever acquire perfectly informative signals.

After making their information choices privately, agents move to the coordination stage.

2.3 The Coordination Stage

In the coordination stage, agents observe noisy signals about θ and choose their actions. Additionally to the private signal with endogenous precision specified in (2), both agents observe a public signal y, given by

$$y = \theta + \tau_y^{-1/2}\varepsilon_y,\ \varepsilon_y \sim N(0,1)\,, \tag{3}$$

where τ_y is the precision of the public signal and ε_y is independent of θ and ε_i.

After observing the private and public signals agents simultaneously choose their actions to maximize their utility, defined in (1). When choosing their actions agents face both fundamental uncertainty (about θ) and strategic uncertainty (about the other agent's action a_j), and hence agents' actions need to balance their desire of matching the fundamental and the action of the other player. Agents have three sources of information to help them make their decisions: (i) the common prior belief $N\left(\mu_\theta, \tau_\theta^{-1}\right)$, (ii) the private signal x_i, and (iii) the public signal y. As has been pointed out by Morris and Shin (2002) and Angeletos and Pavan (2007), public information plays a key role in determining agents' actions. Just as private information, it reduces fundamental uncertainty, but it also serves as a device to reduce strategic uncertainty because it is observed by both agents.

2.4 Equilibrium

We now characterize the equilibrium of our model and derive testable predictions. We solve the model using backward induction. Thus, we first characterize the equilibrium in the coordination stage for any given pair of precision choices, $\{\tau_i, \tau_j\}$, which specifies the optimal use of information. We then move to the information acquisition stage and characterize equilibrium information choices. The detailed derivations of the results reported below can be found in the Online Appendix.[5]

2.4.1 Equilibrium Use of Information

Consider agent i who observes private signal x_i with precision τ_i and public signal y with precision τ_y, and who believes that agent j acquires precision τ_j. Agent i's strategy corresponds to $a_i : \mathbb{R}^2 \rightarrow \mathbb{R}$ that maps agent i's signals $\{x_i, y\}$ into an action. Therefore, taking as given the strategy of agent j, $a_j(x_j, y)$, the optimal strategy of agent i satisfies

$$a_i^*(x_i, y) = \max_{a' \in \mathbb{R}} \mathbb{E}\left[U\left(a', a_j(x_j, y), \theta\right) \middle| x_i, y; \tau_i, \tau_j\right]$$

Given the functional form of the utility function (Equation (1)), the first order condition associated with the maximization problem implies that

$$a_i^*(x_i, y) = (1-\alpha)\,\mathbb{E}\left[\theta | x_i, y\right] + \alpha \mathbb{E}\left[a_j(x_j, y) | x_i, y\right] \tag{4}$$

Let $z \equiv (\tau_y y + \tau_\theta \mu_\theta) / (\tau_y + \tau_\theta)$ and $\tau_z \equiv \tau_y + \tau_\theta$. Agents' common posterior belief (i.e., posterior belief based only on public information) is given by $\theta | y \sim N\left(z, \tau_z^{-1}\right)$. In addition, for each $i = 1, 2$, let $\delta_i = \tau_z / (\tau_z + \tau_i)$ so that $\mathbb{E}[\theta | x_i, z] = \delta_i z + (1 - \delta_i) x_i$, where δ_i is the weight assigned to public information in the posterior of a Bayesian agent after observing both public and private signals. We now characterize the equilibrium strategies of the coordination stage.

Lemma 1 *Let $\boldsymbol{\tau} = \{\tau_i, \tau_j\}$. For each player $i = 1, 2$, the unique linear equilibrium strategy of the coordination stage is given by*

$$a_i(x_i, y) = \beta_i^*(\boldsymbol{\tau})\, x_i + \gamma_i^*(\boldsymbol{\tau})\, z, \tag{5}$$

[5] Colombo et al. (2014) consider information acquisition in a general linear-quadratic Gaussian setup but do so in the model with a continuum of agents. Ui and Yoshizawa (2015) consider a general linear-quadratic Gaussian setup with finitely many players but with exogenous information structure. We contribute to this literature by solving in the Online Appendix a general quadratic-Gaussian model with information acquisition and two players (of which our experimental setup is a special case).

where

$$\beta_i^*(\boldsymbol{\tau}) = (1-\alpha)(1-\delta_i)\frac{1+\alpha(1-\delta_j)}{1-\alpha^2(1-\delta_i)(1-\delta_j)} \tag{6}$$

and $\gamma_i^*(\boldsymbol{\tau}) = 1-\beta_i^*(\boldsymbol{\tau})$. *If* $\alpha = 0$ *then* $\beta_i^*(\boldsymbol{\tau}) = 1-\delta_i$.

This result shows that in a linear equilibrium each agent's action is a weighted sum of his private and public information. However, as first pointed our by Morris and Shin (2002), if $\alpha > 0$ then agents rationally attach a weight β_i^* to private information, which is lower than the weight on private information in the Bayesian posterior, $(1-\delta_i)$. Only when $\alpha = 0$ we have $\beta_i^* = 1-\delta_i$. This result is driven by the presence of strategic complementarities (as measured by α) and by the fact that relying more on public information allows agents to better coordinate their actions. The parameter α plays a key role as it measures the "equilibrium degree of coordination" or, equivalently, the private value that agents assign to aligning their choices.[6] Finally, it is easy to see that β_i^* is decreasing (or, equivalently, γ_i^* is increasing) in α and in τ_z.

2.4.2 Equilibrium Choice of Information

We now consider choices in the information acquisition stage. The ex-ante utility of agent i given precision choices $\{\tau_i, \tau_j\}$ is

$$\mathbb{E}\left[U(a_i, a_j, \theta)\middle|\, \tau_i, \tau_j\right] - C(\tau_i), \tag{7}$$

where expectations are taken over possible realizations of θ and the signals. Agent i's problem is to choose precision τ_i to maximize the his ex-ante utility. The first order condition associated with this problem is given by

$$\frac{\partial}{\partial \tau_i}\mathbb{E}\left[U(a_i, a_j, \theta)\middle|\, \tau_i, \tau_j\right] - C'(\tau_i) = 0 \tag{8}$$

Equation (8) determines agent i's optimal choice of precision, for each precision choice of player j. The next result characterizes the unique equilibrium precision.

[6] In our simple model, the equilibrium degree of coordination is simply the weight in the utility function attached to minimizing the distance between agents' choices. In a more general model, the equilibrium degree of coordination is measured by the slope of agents' best-response function (see the Online Appendix or Angeletos and Pavan (2007)).

Lemma 2 *In the unique equilibrium of the model, both agents choose precision τ^*, which is the unique solution to*

$$\tau^* = \sqrt{\frac{1}{C'(\tau^*)}} - \frac{1}{1-\alpha}\tau_z \tag{9}$$

Lemma 2 establishes that in the unique equilibrium, both agents choose τ^*. Therefore, at the coordination stage they use information symmetrically, i.e., $\delta_i = \delta_j$.

Corollary 1 *Consider the equilibrium precision choice τ^*.*

1. *The equilibrium precision choice τ^* is decreasing in the precision of public information τ_z, that is $\partial\tau^*/\partial\tau_z < 0$.*

2. *The equilibrium precision choice τ^* is decreasing in the degree of strategic complementarities α, that is $\partial\tau^*/\partial\alpha < 0$.*

Corollary 1 states that, ceteris paribus, agents will choose to acquire less private information if the precision of public information, τ_z, is high or if the degree of strategic complementarities, α, is high. These predictions are intuitive and aligned with the literature. When τ_z is high, agents already receive highly informative signals and hence the marginal value of acquiring more precise private signal is lower. When α is high, agents care more about coordinating their actions, rather than matching the state. Since private information is relatively more useful for estimating the state than for coordinating actions, it follows that the marginal value of private information decreases. Thus, Corollary 1 predicts that agents have the strongest incentive to acquire private information when $\alpha = 0$ (given our restriction that $\alpha \geq 0$).

Notice that this model corresponds to an individual decision making environment when we remove strategic motives by setting $\alpha = 0$. These theoretical results serve as hypotheses for our experiment. That is, our benchmark is one where agents are Bayesian and, thus, we characterize biases in terms of departures from the Bayesian paradigm of the model.

2.5 Potential Mechanisms for Biases in the Strategic Environment

The above equilibrium analysis imposes strong assumptions both on the behavior of agents and on their beliefs. Rationality assumptions have been challenged by a large literature in economics and psychology. This literature motivates us to understand whether such departures from rationality are present in our setup and how strategic forces affect them. Therefore, in this section we decompose best responses and utility functions in the strategic environment to understand how different channels of bounded rationality can lead to biases in our experiment.

Use of Information Our starting point is the first order condition that determines optimal choices in the coordination stage. Equation (4) shows that agent i needs to form beliefs about three different objects: (i) the fundamental state, θ, (ii) agent j's private signal, x_j, and (iii) agent j's strategy, $a_j^i(x_j, y)$. For each of these beliefs, the theory dictates that agents use Bayesian updating to determine the weights given to their private and public signals. From a behavioral standpoint, deviations from these weights can lead to biases in the use of information, which can lead to actions that depart from equilibrium predictions. To better understand these three possible channels for departures, we rewrite agent i's best response function explicitly in terms of each of these weights. We still assume that agent i expects agent j's strategy to be a linear combination of public and private signals and that agent i's expectations are a linear combination of the public posterior belief and his private signal. Under these assumptions, we can write the optimal action of player i as

$$\widehat{a}_i^* = (1-\alpha)\underbrace{\left[\widehat{\delta}_i z + \left(1-\widehat{\delta}_i\right)x_i\right]}_{\mathbb{E}^i[\theta|x_i,y]} + \alpha\underbrace{\{\widehat{\gamma}_j z + \left(1-\widehat{\gamma}_j\right)\overbrace{\left[\widehat{\eta}_i z + \left(1-\widehat{\eta}_i\right)x_i\right]}^{\mathbb{E}^i[x_j|x_i,y]}\}}_{\mathbb{E}^i\left[a_j^i(x_j,y)\middle|x_i,y\right]}, \tag{10}$$

where $\mathbb{E}^i[\cdot]$ denotes agent i's subjective expectations, $a_j^i(x_j, y)$ denotes agent i's belief about the strategy used by agent j, and $\widehat{a}_i^*$ denotes agent i's optimal action, given these subjective beliefs. We define subjective non-Bayesian weights as follows: $\widehat{\delta}_i$ is the subjective weight assigned by agent i to public information in his posterior belief about θ, $\widehat{\eta}_i$ is the subjective weight assigned by agent i to public information in his posterior belief about x_j, and $\widehat{\gamma}_j$ is the weight that agent i believes that agent j assigns to public information when agent j chooses his action. Note that in the equilibrium of Section 2.4, $\widehat{\delta}_i = \widehat{\eta}_j = \delta_i$ and $\widehat{\gamma}_j = \gamma_j$.

Equation (10) provides a systematic way to discuss deviations from equilibrium actions. Specifically, deviations can be due to biases in the estimation of θ

($\widehat{\delta}_i \neq \delta$, which can also arise in the individual decision environment), biases when forming beliefs about the other agent's signal ($\widehat{\eta}_i \neq \delta$), or biased beliefs about the strategy used by the other player ($\widehat{\gamma}_j \neq \gamma_j$). We revisit this decomposition in our experimental analysis.

Acquisition of Information Consider now agents' information choices and assume that agent i correctly understands that the signals are unbiased and the noise components of signals are uncorrelated.[7] Under these mild assumptions we can write agent i's expected ex-ante utility in the coordination stage as

$$\begin{aligned} E\left[U\right] = &-\left(1-\widehat{\gamma}_i\right)^2 Var^i\left(\theta - x_i\right) - \widehat{\gamma}_i^2 Var^i\left(\theta - z\right) - C\left(\tau_i\right) \\ &-\alpha\left\{\widehat{\gamma}_j^2 Var^i\left(z-\theta\right) + \left(1-\widehat{\gamma}_j\right)^2 Var^i\left(x_j - \theta\right) - 2\widehat{\gamma}_i\widehat{\gamma}_j Var^i\left(z-\theta\right)\right\}, \end{aligned} \tag{11}$$

where $\widehat{\gamma}_i$ is the weight that agent i assigns to the public signal when choosing an action in the coordination stage (which is given by $\hat{\gamma}_i = (1-\alpha)\widehat{\delta}_i + \alpha\widehat{\gamma}_j + \alpha(1-\widehat{\gamma}_j)\widehat{\eta}_i)$, $\widehat{\gamma}_j^i$ is the weight that agent i believes that agent j assigns to public information, and $Var^i\left(\cdot\right)$ captures subjective beliefs about the variances of relevant random variables.[8]

Equation (11) helps us identify three mechanisms for possible departures from equilibrium precision choices. First, biases in the use of information ($\widehat{\gamma}_i \neq \gamma_i^*$) may lead agents to choose a non-equilibrium level of precision. Second, biased beliefs about the weight assigned by agent j to the public signal may further distort agents information choices. Third, agents may have an incorrect perception of the joint distribution of $\{\theta, x_i, x_j\}$ meaning that they incorrectly asses how a more precise private signal improves their ability to estimate θ, and hence also the signal of the other player ($Var^i\left(\cdot\right) \neq Var\left(\cdot\right)$). We use Equation (11) to discuss the potential mechanisms behind the biases in information choices that we identify in our experiment.

[7]Alternatively, agents may simply ignore taking such considerations into account due to, for example, correlation neglect. See Section A.4 of the Appendix for more details.

[8]To keep notation simple, we suppress the dependence of $\widehat{\gamma}_i$, $\widehat{\gamma}_j$, and $Var^i\left(\cdot\right)$ on agent i's precision choice and his belief about precision choice of agent j.

In what follows we refer to precision parameters in terms of standard deviations, since this is the more intuitive language that we use in the experiment. This implies the following notation changes: $\tau_\theta^{-1/2} = \sigma_\theta$, $\tau_i^{-1/2} = \sigma_i$, and $\tau_y^{-1/2} = \sigma_y$.

3 Experimental Design

The experiment was conducted using the usual computerized recruiting procedures. All subjects were undergraduate students from the University of British Columbia and the University of California, San Diego. Sessions lasted between 60 and 90 minutes and subjects earned $25 on average. A total of 466 subjects participated in the experiment. The experiment was programmed and conducted using z-Tree (Fischbacher, 2007). We implemented a between-subjects design in order to directly compare the behavior of subjects across treatments. Each session consisted of 40 independent and identical rounds. For the strategic treatment subjects were randomly matched in pairs in every round. In each round, subjects made decisions simultaneously without a preselected action.[9]

Treatments varied in 3 main dimensions: the strength of strategic motives, α (individual decisions, mild complementarities, and strong complementarities), the nature of private signal precision (exogenous or endogenous), and the transparency of public information, σ_y (high or low). Our baseline treatments correspond to the case where private information is endogenously determined, the transparency of public information is high ($\sigma_y = 1$), and we vary the nature of strategic motives to have an environment of individual decision making ($\alpha = 0$) and one with mild strategic complementarities ($\alpha = 0.25$).

Since our goal is to study the different behaviors in environments with and without strategic motives, we choose mild complementarities as a baseline so that the contrast between our treatments in terms of strategic uncertainty would not be too stark. However, we also run treatments with strong strategic complementarities ($\alpha = 0.75$) to test the robustness of our results across different strengths of strategic motives. We choose high transparency of public information in our baseline treatments because this leads to a more pronounced tension with pri-

[9] Instructions can be found at
https://econweb.ucsd.edu/~itrevino/pdfs/instructions_st_baseline.pdf.

vate information acquisition. However, to investigate the effects of changes in the transparency of public information we also run every treatment with a low transparency ($\sigma_y = 15$). In addition, we also run sessions with exogenous private signal precisions to control for any possible effects that private information acquisition could have in the use of information, for all variations of α and σ_y. Finally, we run two additional treatments of our baseline strategic condition ($\alpha = 0.25$, $\sigma_y = 1$), one where subjects interact against computers and one where subjects are provided with the best guess for the private signal of their pair member, given their own private and public signals.

The rest of the parameters used in the experiment are as follows. For the prior about the state θ we set $\mu = 0$, $\sigma_\theta = 18$, so that $\theta \sim N(0, 18^2)$. For the treatments with endogenous private information we provide subjects a menu of four possible precision choices, specified in Table 1.[10]

Precision	σ	$C(\sigma)$
1	0.5	12
2	2	6
3	6	2.5
4	10	1

Table 1: Precision choices in the experiment

For the treatments with exogenous private signal precision we choose two different precisions from Table 1, one corresponding to the equilibrium precision choice and one to a higher precision than equilibrium (the second most popular choice observed in our sessions with endogenous information). Table 2 summarizes our experimental design.

At the beginning of each session, subjects had access to two practice screens. In the information practice screen, subjects could experiment with different levels

[10] In the experiment we use the term "precision" as a qualitative measure. That is, precision level 1 corresponds to the highest precision, precision level 2 to the second highest, and so on. We use the term precision in this qualitative way throughout the rest of the paper.

Treatment	Strategic motives (α)	Transparency of public signal (σ_y)	Private signal precision	Opponent
Baseline I	None ($\alpha = 0$)	High (σ_y= 1)	Endogenous	N/A
Baseline S	Mild ($\alpha = 0.25$)	High (σ_y= 1)	Endogenous	Human
Low public I	None ($\alpha = 0$)	Low (σ_y= 15)	Endogenous	N/A
Los public S	Mild ($\alpha = 0.25$)	Low (σ_y= 15)	Endogenous	Human
Strong strat	Strong ($\alpha = 0.75$)	High (σ_y= 1)	Endogenous	Human
Belief aid S	Mild ($\alpha = 0.25$)	High (σ_y= 1)	Endogenous	Human
Computer S	Mild ($\alpha = 0.25$)	High (σ_y= 1)	Endogenous	Computer
Exogenous 1	Mild ($\alpha = 0.25$)	High (σ_y= 1)	Exogenous ($\sigma = 2$)	Human
Exogenous 2	Mild ($\alpha = 0.25$)	High (σ_y= 1)	Exogenous ($\sigma = 10$)	Human
Exogenous 3	Mild ($\alpha = 0.25$)	Low (σ_y= 15)	Exogenous ($\sigma = 0.5$)	Human
Exogenous 4	Mild ($\alpha = 0.25$)	Low (σ_y= 15)	Exogenous ($\sigma = 2$)	Human

Table 2: Experimental design

of precision and generate as many different signals as they wanted, for any given state, and they could also generate different states. The practice screen for actions was devised with the intention of familiarizing subjects with the payoff function. Subjects could experiment with different hypothetical values of the state, the action of the other player (for the strategic treatments), and their own action to calculate how many points they would earn.

In each paying round, subjects were endowed with 12 points to make up for the cost of information. Each round of the experiment proceeded as follows. First, for the sessions with endogenous private information, subjects privately chose the precision of their private signals from Table 1. Then they stated their beliefs about their opponent's precision choice, which were incentivized by paying subjects an additional \$5 if their guess was correct in the round selected for payment (for the strategic treatments). Then, subjects observed private and public signals and simultaneously chose their action, with the option of using a hypothetical payoff calculator to aid their calculations. Third, they received feedback about the state, their observed signals, actions, precision choices, and their individual points for that round.

One round of the 40 rounds played was randomly selected for payment. The objective function in the second stage of the game was transformed from the specification in Equation (1) by adding 100 points in order for subjects to not think that any outcome would lead to negative points, so in each round the number of points they could earn in the action was determined by $Points = 100 - (1-\alpha)(a_i - \theta)^2 - \alpha(a_i - a_j)^2$. Despite this transformation, it is possible for subjects to have negative payoffs in a round. To circumvent this issue, the points earned in the round selected for payment were converted to probabilities in a lottery that paid \$20 with probability p and \$5 with probability $(1-p)$, where p is a linear function of the points earned.[11] In particular, $p = 1$ for $Points = 112$, which is the maximum number of points a subject can make in a round. For symmetry we set $p = 0$ for $Points \leq -112$ and the increment in probability to earn the high lottery prize was 0.446% for every additional point earned in the experiment. Therefore, the payment in dollars was composed of a \$10 show up fee, the outcome of the lottery whose probabilities were determined by their performance in a randomly selected round, and \$5 if they guessed correctly the precision choice of their opponent in the round selected for payment (in the strategic treatments).

4 Experimental Results

In this section we present our main experimental results. We first discuss information choices and characterize departures from the equilibrium predictions. Then we study how subjects use the different types of information across environments and characterize biases in the use of information with respect to the theoretical weights predicted by the model. We then quantify welfare effects of the biases we identify by analyzing payoffs. Throughout this section, we present our experimental results by comparing the behavior of subjects in individual and strategic environments. Most of the results of this section relate to the last 30 rounds of the experiment, once subjects' behavior has stabilized, unless otherwise specified.

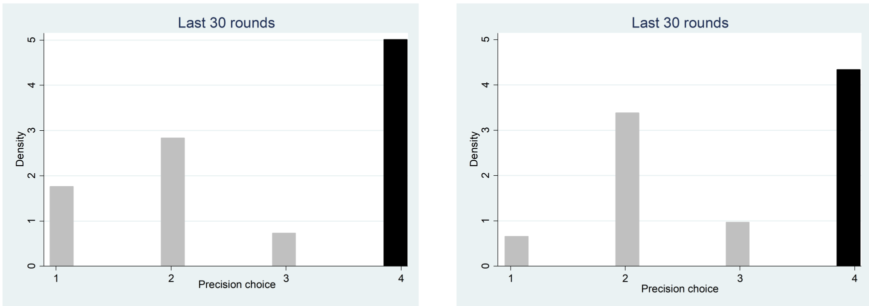
4.1 Choice of Information

To discuss the choice of information we refer to precision levels to be consistent with the language used in the experiment. These levels are a qualitative measure

[11] See Roth and Malouf (1979) for a discussion of paying with probability points.

of signals' informativeness and the specific labels (precision 1, 2, 3, or 4) do not reflect the actual precisions of signals listed in Table 1.

Figure 1 presents the histograms of precision choices for the individual and strategic environments (panels (a) and (b), respectively). Three main observations emerge. First, we see that in both environments precision choices are surprisingly similar. Second, in both environments the modal precision choice corresponds to the equilibrium precision (level 4, black bar). Third, in both environments there are significant departures from the equilibrium prediction, mostly to precision level 2, which represents overacquisition of information.

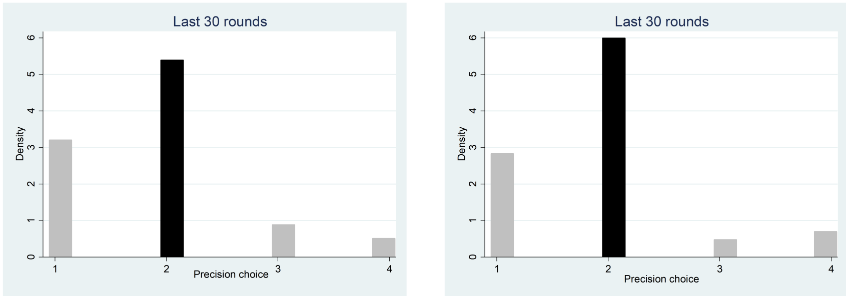


(a) Individual (b) Strategic

Figure 1: Precision choices, baseline treatment

Since the equilibrium precision choice in our baseline treatment is the lowest precision, departures from equilibrium can only imply overacquisition of information. To check the robustness of our results, Figure 2 presents similar histograms for the treatments with the same parameters, except for the transparency of public information, which is now low ($\sigma_y = 15$). In this case, the public signal is very noisy, which increases the incentives to acquire more precise private information. The equilibrium precision choice in this case corresponds to precision level 2 (black bar). Just as in our baseline treatment, the distribution of precision choices in the individual and strategic environment is very similar, the modal precision choice corresponds to the equilibrium prediction, and most of the departures from equilibrium correspond to subjects overacquiring information. Thus, we conclude that our findings are robust to changes in precision of public information and we identify a bias of overacquisition of private information.

Interestingly, these results suggest that the sustained overacquisition of private



(a) Individual (b) Strategic

Figure 2: Precision choices, low transparency

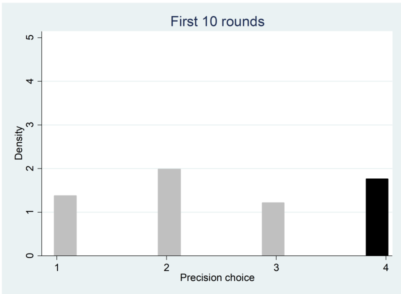
information is not driven by the presence of strategic motives since we see the same choice patterns in the individual decision making environment. Theoretically, in the strategic environment information choices are strategic complements due to the complementarity in actions (Hellwig and Veldkamp (2009)), so we could imagine that strategic considerations could affect precision choices off-equilibrium and lead to different off-equilibrium choices than in the individual decision case. Figures 1 and 2 support the prediction in the theoretical literature for the strategic environment that an increase in the transparency of public information crowds out private information acquisition (see Colombo et al. (2014)). This observation holds both for equilibrium and non-equilibrium choices.

Our results indicate a clear heterogeneity of information choices in the form of two main types of subjects: those who acquire the equilibrium precision and those who overacquire information. We are agnostic about the specific sources of the observed heterogeneity, such as intrinsic preferences for better information, but we investigate how this pattern of information choices arises in both environments.

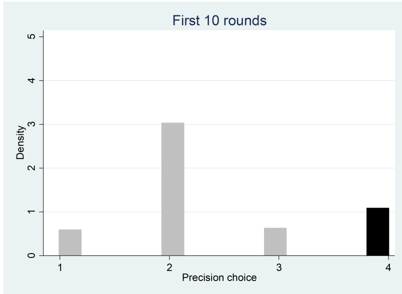
4.1.1 Learning Dynamics

To better understand information choices in both environments, we turn our attention to the evolution of precision choices throughout the 40 rounds of the experiment. Figure 3 presents similar histograms to Figure 1 for both environments in our baseline treatment, but for the first 10 rounds of the experiment.

As we can see in Figure 3, initial precision choices in the individual decision making environment are similarly spread out across all four precision levels, whereas in the strategic setup precision 2 is clearly favored. This difference in



(a) Individual



(b) Strategic

Figure 3: Precision choices in initial rounds, baseline treatment

initial choices suggests that strategic uncertainty might lead subjects to choose a higher precision for their private signal, in an effort to reduce the uncertainty they face in the coordination stage. Despite this difference in initial responses, subjects' behavior across these two environments converges once learning takes place, as illustrated by Figure 1 where precision levels 2 and 4 become clear absorbent states and the differences across environments disappear.[12] Therefore, it is not that strategic uncertainty has no effect on the choice of information, but rather that the learning dynamics of information choices lead the initial heterogeneity in both environments to converge to the same typology of information choices.

To understand how this shift takes place we turn to Figure 4, which presents, for each environment, the transition matrices of precision choices for rounds 1-10 and 11-40 separately. In each matrix the entry a_{ij} corresponds to the implied probability of a subject choosing precision level j in round $t+1$, given their choice of precision level i in round t, for $i, j \in [1, 4]$. The transition matrices for the first 10 rounds can help us understand the level of experimentation with different precisions in the early rounds of the experiment, whereas the matrices for rounds 11-40 help us understand how Precision levels 2 and 4 become absorbent states.

The transition matrices for the first ten rounds in Figure 4 show that subjects' precision choices are relatively stable, in the sense that the probability of a subject choosing the same precision in two consecutive rounds (main diagonal) is larger

[12]In particular, in the individual decision-making environment this convergence occurs by round 20. In the strategic environment convergence occurs by round 30, but after round 20 it is clear that precision levels 2 and 4 are absorbent states.

Rounds 1-10

Individual ($\alpha = 0$)

	P 1	P 2	P 3	P 4
P 1	0.73	**0.17**	0.09	*0.01*
P 2	0.13	0.64	0.15	*0.08*
P 3	0.11	**0.23**	0.43	**0.23**
P 4	0.02	0.06	0.08	0.84

Strategic ($\alpha = 0.25$)

	P 1	P 2	P 3	P 4
P 1	0.55	**0.32**	0.13	*0*
P 2	0.07	0.79	0.07	*0.07*
P 3	0.06	**0.34**	0.42	**0.18**
P 4	0.02	0.16	0.05	0.77

Rounds 11-40

Individual ($\alpha = 0$)

	P 1	P 2	P 3	P 4
P 1	0.85	**0.1**	0.03	0.02
P 2	0.06	0.85	0.04	0.05
P 3	0.02	**0.14**	0.67	**0.17**
P 4	0.01	0.03	0.01	0.95

Strategic ($\alpha = 0.25$)

	P 1	P 2	P 3	P 4
P 1	0.66	**0.31**	0	0.03
P 2	0.06	0.86	0.06	0.02
P 3	0	**0.18**	0.7	**0.12**
P 4	0	0.01	0.01	0.98

Figure 4: Transition matrices of precision choices

than the probability of choosing any other precision (off main-diagonal), for all precisions and environments. However, we see two important ways in which subjects transition across precisions that are present in both environments. The first one illustrates how subjects converge towards precision levels 2 and 4 and away from levels 1 and 3. The most likely movement from precision level 1 is towards level 2 and from level 3 is towards both levels 2 and 4 (numbers in bold). The second one shows that subjects who initially choose high precision levels (1 and 2) have an extremely low rate of experimentation with cheaper precisions: the numbers in italics show a negligible probability of choosing precision level 4 after choosing levels 1 or 2. This implies that those subjects who start the experiment choosing high precisions do not get to learn the benefits of cheaper, less precise private information.[13] We quantify the cost of this behavior later on when we

[13] The difficulties to engage in hypothetical thinking have been documented in the experimental literature (e.g., Esponda and Vespa (2014) and Martinez-Marquina et al. (2019)). For this reason, experimentation is especially important

analyze realized payoffs in the experiment.

In rounds 11-40, we can see in Figure 4 that all states are relatively absorbent in both environments (probability of choosing the same level of precision in two consecutive periods is at least 66%). Precision levels 2 and 4 are clearly absorbent as they have the largest probabilities of consecutive choices in both environments. Moreover, most of the movement off the main diagonals is directed from precision levels 1 or 3 towards precision levels 2 or 4 (numbers in bold), reinforcing the pattern of convergence that we observe in the first 10 rounds.

We will discuss the possible drivers of overacquisition of information to Section 4.3, after we analyze how subjects use information, as we need to understand first how subjects use information to be able discuss agents' incentives to overacquire information.

4.2 Use of information

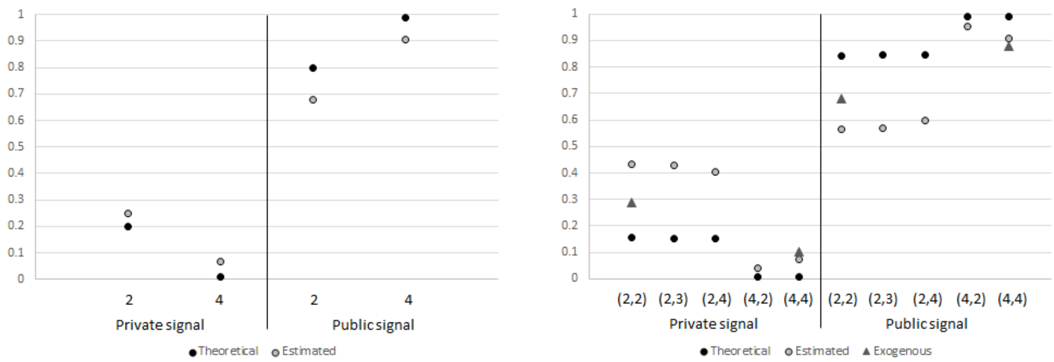
We now turn our attention to the analysis of how subjects use the different signals at their disposal. To do this, we estimate the weights subjects assign to private and public signals when choosing their actions in the coordination stage. Recall from Equation (5) that optimal actions are linear combinations of private and public signals. The theoretical weights given to these signals depend on the parameters of the model, in particular on noise parameters and on the coordination motive α. We run random-effects linear regressions for each treatment to estimate these weights, conditioning on individual precision choices and beliefs about the opponent's precision choice (for the strategic treatments), since when $\alpha > 0$ these beliefs affect the weights given to signals (see Equations (6)).[14]

We present the coefficients of these regressions for the individual and strategic baseline treatments in Figure 5 (panels (a) and (b), respectively). In each graph,

to learn the potential benefits of cheaper, less precise information. These benefits are large in our baseline treatments because the public signal is very informative about the state.

[14]We tested the assumption of linearity by running regressions with higher order terms, which were statistically insignificant in all specifications. Therefore, we only focus on linear specifications.

the vertical axis indicates the weights given to the different types of signals. The vertical line in the horizontal axis separates the weights estimated to private and public signals (left and right of the line, respectively). Each indicator on the horizontal axis corresponds to a specific precision choice (individual treatment) or to a combination of individual precision choice and belief about the opponent's precision choice (strategic treatment). Black dots indicate the theoretical weights predicted by the theory and gray dots correspond to the weights estimated with the data, for a given precision choice and beliefs about the other's precision choice. Given the results from Section 4.1, we focus on the weights corresponding to the two most popular precision choices, levels 4 (equilibrium) and 2 (overacquisition).



(a) Individual

(b) Strategic ($\alpha = 0.25$)

Figure 5: Weights given to signals baseline treatments.

The use of information in the individual decision making treatment is qualitatively similar to the Bayesian weights predicted by the theory. In particular, the weights to both private and public signals are not statistically different to the theoretical weights when subjects choose precision 2. For precision level 4 the estimated weights are statistically different from the theoretical weights to the 1% level of significance, but they are qualitatively very similar (0.068 vs 0.01 for the private signal and 0.905 vs 0.987 for the public signal).

We see a starkly different pattern in the use of information in the strategic setup. In all cases the estimated weights are different to the theoretical weights at the 1% level of significance. Since public information is very precise ($\sigma_y = 1$) the theory predicts most of the weight should be assigned towards it, but instead subjects overuse private information. The overuse of information is particularly pronounced for subjects who overacquire information (i.e., choose precision level 2), regardless of their beliefs about the precision choice of their opponent. Their actions suggest that these subjects give a weight of about 0.4 to their private signal, as opposed to the optimal theoretical weight once we adjust for more precise private information, which is less than 0.2.

Panel (b) of Figure 5 also includes estimated weights for the strategic treatment where the precision of private signals is exogenously determined (triangle markers), one where we exogenously set precision level 2 and one where we set precision level 4. This allows us to check whether the overuse of private information is due to a sunk-cost fallacy, where subjects who acquire more precise information might use it more in an effort to "make up" for the high cost paid for it. We can still see a significant overuse of private information for both precision levels. The magnitude of this bias, however, is decreased for subjects in the treatment with exogenously set precision 2, which can be interpreted as partial support for the sunk-cost fallacy, but the qualitative finding of the overuse bias is still present.

We investigate the robustness of the overuse of private information in the strategic setup by looking at the treatment with strong complementarities ($\alpha = 0.75$), where strategic motives are stronger and subjects care a lot about coordinating with others. Figure 6 finds similar qualitative patterns in the use of information as in our baseline with mild strategic complementarities, i.e., subjects overuse their private signal with respect to the theoretical prediction, whether information is endogenously or exogenously determined.

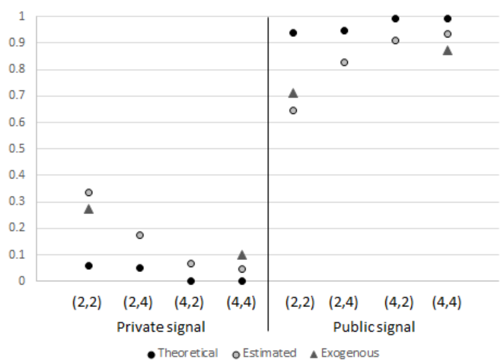


Figure 6: Weights given to signals, strong complementarities ($\alpha = 0.75$).

The strong overuse of private information that we identify for overacquirers in panel (b) of Figure 5 and Figure 6 implies that the weights given to private and public signals are "closer" to each other than what the theory suggests. It is important to establish that the overuse bias we identify is not just a product of subjects using a simple heuristic in the strategic environment where they give roughly the same weight ("50-50") to both signals.[15] Under this heuristic, we would see a similar pattern for weights, regardless of the precision of signals. To refute this explanation, we turn to our treatments with a low transparency of public information where $\sigma_y = 15$. Strategic uncertainty is still present in these treatments, but the main difference with the baseline cases ($\sigma_y = 1$) is that equilibrium predicts higher weights to private information, due to the low precision of the public signal. Figure 7 recreates Panel (b) of Figure 5 and Figure 6 for the case where $\sigma_y = 15$. We can see that, contrary to our baseline case, subjects clearly set weights far from the "50-50" midpoint. The "50-50" heuristic would imply that subjects would underuse private information to a large extent in these treatments as well, which is not what we observe. Therefore, we find no evidence of an overarching heuristic in the strategic environment where subjects give roughly the same weight to both signals.[16]

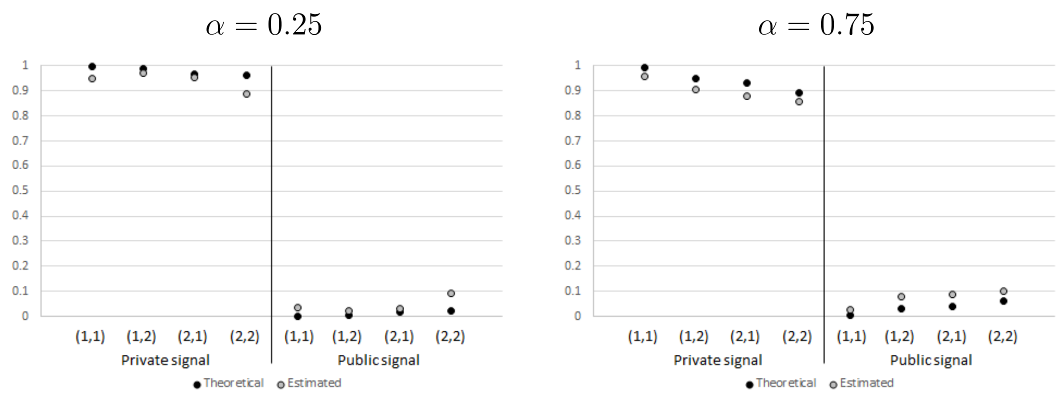


Figure 7: Weights given to signals, low transparency of public information

The fact that we do not observe subjects underusing private information when

[15] See Benjamin (2019) for a review of biases in probabilistic updating.

[16] Note that in the treatments with $\sigma_y = 15$ we do not observe overuse of private information because the theoretical weights given to private information are large enough that there is barely any scope to overuse this signal.

$\sigma_y = 15$ also helps us to better characterize our bias in the use of information as true overuse, and not misuse, of private information. When public information is noisy ($\sigma_y = 15$), there is little competition between private and public information and it is optimal to predominantly rely on the private signal. However, when public information is precise, as in the baseline treatment, there is a tension between private and public information that gives rise to the overuse bias that we observe. From Figure 2, the lower precision of public information reduces the tensions associated to the acquisition of private information because subjects do not have a high-accuracy public signal anymore. This leads to a higher equilibrium precision (level 2, black bars in Figures 2) and we observe the same shift in the data, both for equilibrium and overacquisition of information. Absent this tension between transparency of public information and private information acquisition, we do not find evidence of qualitative biases in the use of information because, as shown in Figure 7, subjects use their information remarkably well in both environments.[17]

4.2.1 Overuse of Private Information in the Strategic Environment

To understand why subjects use information differently in the individual and strategic environments we turn our attention to the forces that are unique to strategic reasoning. When observing signals, subjects in the individual setup have to form beliefs only about the state, whereas in the strategic environments they have to form beliefs about the state, but also about the actions and beliefs of others. Our results in the individual setup suggest that subjects are relatively good at forming beliefs about the state, which is intuitive because they observe signals about the state and have to make inferences only about the state, which requires a relatively simple mapping. In contrast, in the strategic setup the mapping between signals and actions is more complex due to strategic uncertainty.

We refer to strategic uncertainty broadly as the uncertainty about the actions

[17]Notice that this tension has not been present in other papers that study these games experimentally since they focus on environments where there is little tension between private and public information because both signals have identical precision and subjects do not have the option to choose the precision of their private signal.

of the other player. In our experiment this is manifested in three ways. The first is captured by the theoretical model where, in equilibrium, players do not know the other player's signal, so they do not know their action, but they do know the mapping between signals and actions (strategy). The second manifestation of strategic uncertainty in our experiment, but not in the model, is that subjects do not know the strategy of the other player so they have to form beliefs about the mapping between signals and actions. Third, in our experiment strategic uncertainty can also manifest as uncertainty about the rationality of the other player.

To understand the overuse of private information in the strategic environment we focus on these 3 manifestations and run 2 additional treatments where we shut down the first and third manifestations, respectively. First, we explore whether uncertainty about the rationality of others is responsible for the overuse of private information. Second, we revisit the decomposition of Equation (10) in an effort to pin down the belief formation channel that could be behind this bias.

Uncertainty about the Rationality of Others Intuitively, if subjects are unsure about the rationality of their opponent they might be inclined to offset this added (strategic) uncertainty with a desire to purchase better private information because it is the only tool at their disposal to reduce uncertainty, and then using this information more than they should, leading to the overuse bias that we document. This force is clearly present only in the experiment, since the theory assumes rationality and common knowledge of rationality.

To explore this hypothesis, we run a treatment where subjects, instead of interacting with one another, interact with a computer that follows the unique equilibrium strategy. The experiment is identical to the baseline strategic treatment, except that in the instructions subjects are told that their opponent is a computer that chooses and uses information optimally. Subjects are not told what the actual computer's strategy is, in an effort to not influence their own strategy. Therefore, in this treatment we shut down only the uncertainty about the rationality of the opponent, while preserving the strategic uncertainty about the actions of their opponent, as in the model.

Figure 8 plots the estimated weights given to private and public signals in

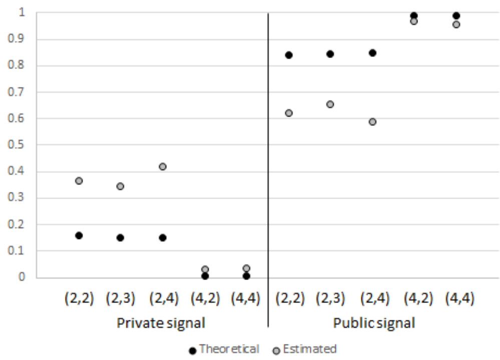


Figure 8: Weights given to private and public signals, computer opponent

this treatment, controlling for individual precision choices and beliefs about the computer's precision choice, just as in Figure 5. We see the same qualitative findings as in our baseline treatment, i.e., a clear overuse of private information by those subjects who overacquire information. In terms of information choices, Panel A of Figure 10 in the Appendix shows strikingly similar patterns in precision choices as in the individual and strategic baseline treatments (Figure 1).

We conclude that uncertainty about the rationality of others is not the driving force of the overuse of private information by overacquirers. Returning to the analogy of our beauty contest model as an approximation to decision making in financial markets, we could interpret these results as suggesting that the information processing bias we observe might not be due to the presence of noise traders, but rather to forces inherent to the uncertainty in the environment.

Decomposition of Beliefs in the Strategic Environment Our results suggest that the bias we identify in the game is related to the way individuals form beliefs about the beliefs and actions of others. To understand this further, we revisit the decomposition of the best-response functions of the strategic environment described in Section 2.5. From Equation (10), we know that there are three instances where subjects use their private and public signals when forming beliefs that can potentially lead to biases. The first one is when subject i uses signals x_i and y to form beliefs about the fundamental θ. Notice that there is no strategic component to this belief and that this is precisely the exercise that subjects in the individual decision environment perform. Our results suggest that subjects are relatively good at forming beliefs about θ (as illustrated in Panel (a) of Figure 5). This is consistent with the findings of Szkup and Trevino (2020) and Baeriswyl et al. (2021) who found that subjects form accurate beliefs about fundamentals in coordination games with incomplete information. Thus, we hypothesize that the overuse bias that we observe in the strategic environment is unlikely to come from a bias in the weights given to x_i and y when forming beliefs about the state, θ.

The two other instances where subjects have to form beliefs using private and public signals are unique to the strategic setup. Subject i uses signals x_i and y to form beliefs about the private signal observed by subject j, $E^i(x_j|x_i, y)$, and about the action taken by subject j, $E^i(a_j(x_j, y)|x_i, y)$. Equation (10) illustrates how we can decompose these two expectations into weighted sums of public and private signals, and these weights are potential sources of biases. In order to disentangle these two forces, we run an additional treatment where we shut down the first channel. That is, we provide subjects with $E^i(x_j|x_i, y)$. The experiment is identical to the baseline strategic treatment, except that subjects are provided with the Bayesian estimates of their opponent's private signal (referred to as the "best guess" in the instructions), which is based on the subject's own signals. The instructions are clear about this being a statistical guess, and not the actual signal observed by the other subject.

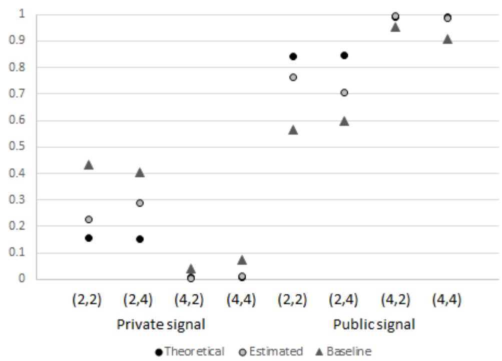


Figure 9: Weights given to signals, best guess of other's signal provided

Figure 9 plots the estimated weights given to private and public signals in this treatment, controlling for individual precision choices and beliefs about the other's precision choice, just as in Figure 5. We also include the weights estimated in the baseline treatment (triangle markers) for comparison. For subjects who choose the equilibrium precision (level 4), providing subjects with the best guess about the private signal of the other player removes any biases in the use of information. Subjects who choose the equilibrium precision use their information in a remarkably similar way to what the theory predicts. That is, we see a significant improvement in the use of information for those subjects who choose information optimally when we help them with the calculation about the information observed by their opponent (i.e., when we reduce their cognitive load).[18] Subjects who overacquire information still use their private signal more than what the theory suggests, but the magnitude of the overuse bias is significantly reduced with respect to the baseline case. The results from this treatment suggest that part of the overuse bias that we observe in the strategic setup is related to the complexity of forming beliefs about the information held by others that is only present under strategic uncertainty.

4.3 Mechanisms behind overacquisition of information

Similar to the discussion of possible mechanisms behind the overuse bias, we now discuss possible drivers of the overacquisition of information that we identify in our experiment. From Section 2.5, recall that using Equation (11) we identified three theoretical channels that can lead to deviations in information choices with respect to the equilibrium prediction. First, the overuse of information that we observe in the coordination game may encourage subjects to overacquire information. Second, this incentive can be further reinforced if agents expect other agents to overuse information. Third, subjects may have incorrect beliefs about the way their information choices improve their ability to predict θ and x_j. We investigate how these mechanisms affect the incentives to overacquire information, which we measure by the difference between the ex-ante expected utility from choosing the equilibrium precision (level 4) and deviating to precision level 2 unilaterally. The

[18]It is relevant to note that the equilibrium precision is the modal choice, just as in the baseline treatments, but the proportion of subjects who choose the equilibrium precision is higher in this treatment than in the baseline treatment. Panel B of Figure 10 in the Appendix shows the histogram of precision choices in this treatment.

details of these calculations can be found in Section B in the Appendix.

Our calculations suggest that overusing private information at the coordination stage and expecting that the other agent also overuses information do increase the incentives to overacquire information, but not enough to make that choice optimal. Therefore, these two channels cannot rationalize overacquiring information as an optimal choice for subjects in the presence of the overuse bias. Following the decomposition of Equation (11), we hypothesize that the third channel, i.e., an incorrect perception of the joint distribution of $\{x_i, x_j, \theta\}$ (in particular, an incorrect assessment of how a more precise private signal improves their ability to estimate θ and x_j) must play an important role in driving overacquisition. Panel B of Figure 10 in the Appendix, which shows the histogram of precision choices in the treatment where we provide subjects with the correct estimate of the other's signal provides indirect evidence in support of this channel. In that treatment we see a decrease in subjects' propensity to overacquire information, with respect to the baseline. One can interpret this reduction in overacquisition as being driven by partially correcting subjects' perceptions of the joint distribution of $\{\theta, x_i, x_j\}$ due to being given an estimate of x_j. Given the complexity of this channel, we are unable to investigate it deeper using our experiment. Understanding this bias further is an important avenue for future research.

4.4 Welfare

In this section, we analyze realized payoffs in the experiment and compare them to the payoffs that would result if players behaved according to the theoretical benchmark, given the realized states and signals in the experiment. We do this with the objective of quantifying the welfare effects of the two biases we identify. That is, we investigate what is the cost, in terms of foregone payoffs, of overacquiring information, for both the individual and strategic treatments, and the cost of overusing private information in the strategic treatment.

Table 3 reports the median payoffs for subjects in our baseline treatments as well as payoffs that subjects would have obtained had they followed optimal strategies, separated by precision choices that correspond to the equilibrium prediction and to the higher level of precision that represents overacquisition.[19] The second

[19]Theoretical payoffs are computed given the signals and fundamental state

and third column correspond to the median payoffs of the two stage game, whereas the fourth and fifth columns correspond to the payoffs only in the coordination stage, that is, we abstract from payoffs related to precision choices.

	Two-stage Model		Coordination Stage	
	Individual	*Strategic*	*Individual*	*Strategic*
Equilibrium σ	110.1	110.2	99.1	99.2
Overacquirers	105.3***	105.1***	99.3	99.1
Theoretical	110.4	110.8	99.4	99.8

Statistical difference wrt theoretical equilibrium payoffs at levels:*** 1%, ** 5%, * 10%.

Table 3: Realized payoffs for subjects who choose equilibrium precision and for overacquirers, baseline treatment

We see similar payoff patterns in the strategic and non-strategic environments. In particular, average payoffs of subjects who choose the equilibrium precision are not different from the theoretical benchmark in both environments. In terms of the two biases that we identify, we find that the overuse of private information has a negligible effect on payoffs, since there is little difference in the payoffs related to the coordination stage across precision choices (equilibrium vs. overacquirers). However, the overacquisition of information leads to significant welfare losses due to the higher cost of more precise information. These results are robust to treatment and parameter variations, as shown in Tables 4 and 5 in the Appendix. In particular, when the transparency of public information is low ($\sigma_y = 15$), subjects who overacquire information actually see payoff gains in the coordination stage, but these gains are not enough to offset the high cost paid for precision, which ultimately leads to overall payoff losses.

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realization observed in the experiment, assuming that the other pair member also behaves optimally.

5 Discussion of Alternative Models

We have argued that the biases in the acquisition of information and its use are driven mostly by subjects' biased beliefs about other subjects' signals and strategies and by an incorrect assessment of how a more precise private signal improves their ability to estimate θ and x_j. However, one may wonder if there are alternative explanations for our findings. Below we argue that popular models of bounded rationality are unlikely to explain our results. All proofs and detailed descriptions of environments are relegated to the Online Appendix.

5.1 Level-k

Models with limited depth of strategic reasoning such as level-k and cognitive hierarchy models (see Nagel (1995) or Costa-Gomes and Crawford (2006)). In the context of beauty contest models to explain observed behavior in experiments (see, for example, Cornand and Heinemann (2014) or Shapiro et al. (2014)).

The main challenge when using Level-k or cognitive hierarchy models is choosing the appropriate rule for Level 0 ($L0$) types, that is, the non-strategic, anchoring types. In what follows we assume that agents of type $L0$ follow a linear strategy in the coordination stage given by $a = \pi z + (1 - \pi) x_i$, where π is an integrable random variable with support on $[0, 1]$. This specification is very flexible.[20] In all cases, we assume that $L0$ randomizes uniformly between all choices of information. Given that type $L0$ follows a linear strategy at the coordination stage, all types of level k (Lk), $k \geq 1$ also use linear strategies.

We first argue that the level-k model cannot explain overuse of information. Let τ_{Lk} be the precision of the private signal that type Lk chooses in the information acquisition stage. Given τ_{Lk}, let $\delta(\tau_{Lk})$ be the weight assigned to public information in the Bayesian posterior belief about θ. Finally, denote by γ_{Lk} the

[20] We follow Crawford and Iriberri (2007) by considering non-strategic L0 types and reserving more sophisticated thinking for higher level types. Our specification includes as special cases the specification where L0 ignores strategic considerations and simply minimizes the distance between their actions and fundamentals, as in Cornand and Heinemann (2014). It also includes "random types" that always assign weight of one-half to private and public signals, as in Shaprio et al. (2014).

weight that type Lk assigns to public information when choosing his optimal action in the coordination stage.

Lemma 3 *Consider $k \geq 1$. Then for any $\tau_{Lk} \geq 0$, we have $\gamma_{Lk} \geq \delta(\tau_{Lk})$.*

Applied to our particular experimental setup, Lemma 3 implies that level-k agents would never assign a weight on private information higher than 0.2 if they chose precision level 2, or 0.01 if they chose precision level 4. Since in our experiment we see subjects using weights much higher than that, we conclude that the level-k model cannot explain the overuse of private information that we document.

We next argue that the level-k model cannot explain the overacquisition of information we observe in our experiment.

Lemma 4 *Applied to our experimental setup, for all $\alpha \in (0,1)$ the Level-k model predicts that all types Lk, $k \geq 1$, choose to acquire precision level 4.*

Together, Lemmas 3 and 4 establish that despite the very flexible formulation of the level-k model that we consider, this model is unable to rationalize the biases in the choice and use of information that we identify in the experiment.

5.2 Anticipated Regret Minimization

It has been suggested that anticipated regret minimization can help explain both departures from equilibrium strategies (see Filiz-Ozbay and Ozbay (2007)) and excessive information acquisition (see Gretschko and Rajko (2015)) in auctions.

To model regret, we assume that agents can feel regret about mismatching the state and about miscoordinating with the other agent. The regret function is quadratic and $r_\theta, r_a \geq 0$ are the weights that an agent assigns to regret from mismatching the state and miscoordinating with the other, respectively. As above, let $\delta(\tau_i)$ denote the weight assigned to public information in the posterior belief about θ of a Bayesian player who observes a private signal with precision τ_i.

Lemma 5 *Let $\boldsymbol{\tau} = \{\tau_i, \tau_j\}$. For any $r_a, r_\theta \geq 0$ and any precision choices, we have $\gamma_i(\boldsymbol{\tau}) \geq \delta(\tau_i)$.*

Lemma 5 establishes that the weight on public information assigned by an agent that anticipates regret is always larger than $\delta(\tau_i)$. Since in our experiment, for each level of precision, we find that subjects assign weights to public signal much smaller than $\delta(\tau_i)$, we conclude that regret minimization cannot explain the observed overuse of information.

We also find that in our particular setup, regret minimization cannot induce information overacquisition unless the weight on anticipated regret from mismatching the action, r_a, is two orders of magnitude larger than the weight on actual payoff loss, α. Moreover, if regret minimization does lead to information overacquisition, then it induces the choice of the most precise information (see Figure 11 in the Online Appendix). Given this, it seems unlikely that anticipated regret minimization can be driving our findings.

5.3 Quantal Response Equilibrium

Quantal response equilibrium (QRE) is another popular model that has been successful in explaining deviations from equilibrium in many experimental settings (see e.g., McKelvey and Palfrey (1995) and Goree et al. (2016)). In the context of incomplete information games, QRE assumes that agents have correct beliefs about strategies used by other agents and they do not exhibit any biases in information processing. However, QRE relaxes the assumption that agents best respond to their beliefs by allowing them to make mistakes when choosing actions, with mistakes that are more costly being less likely to occur.

To investigate whether QRE can explain our experimental findings, we consider the popular symmetric logit QRE with logit parameter λ and apply it to the coordination stage (we focus only on the coordination stage for computational simplicity). Given the lack of closed-form solutions (a common feature of QRE), we compute the logit QRE of the coordination stage numerically for a wide range of λ's, where $\lambda = 0$ corresponds to random behavior, while $\lambda \to \infty$ corresponds to behavior that converges to Nash equilibrium.[21] For each λ, we simulate agents' behavior a large number of times using computed numerically QRE best-response

[21] QRE is typically used in the context of complete information, finite-action games and is not easily adaptable to environments with infinite action spaces and incomplete information. Therefore, to compute QRE we discretize our setup

functions and the signals observed in our experiment. Finally, using data from each simulation separately, we estimate the weight on private signals the same way as we did using our experimental data. Thus, for each λ we obtain a large number of estimated signal weights.

Figure 12 in the Online Appendix depicts the average weight on the private signal estimated using simulated data when both agents choose precision level 2 (overacquisition, left panel) and when both agents choose precision level 4 (equilibrium precision, right panel) in the information acquisition stage. The shaded areas represent two standard deviation bounds for estimated weights on private signals with 95% of estimated QRE weights on the private signal lying within these bounds.

Both panels show that average weights on the private signal implied by QRE are very close to the equilibrium weights. Moreover, for all values of λ and in all simulations, the estimated weight on private information lies below the weight estimated using our experimental data. Based on these results, we conclude that QRE is unlikely to explain our experimental results.

5.4 Overconfidence

Finally, we investigate whether our results can be explained by overconfidence which leads to subjects erroneously treating private signals as more precise than they objectively are. Formally, overconfidence would imply that each subject i acts as if the private signal he observes is distributed according to $N\left(\theta, (\xi_i \tau_i)^{-1}\right)$, where τ_i is subject i's precision choice at the information acquisition stage and $\xi_i \geq 1$ is the extent of overconfidence that subject i exhibits. Therefore, $\xi_i \tau_i$ is the perceived precision of the private signal by subject i.[22] This form of overconfidence (often referred to as overprecision) has been documented extensively in the literature (see Moore and Healy (2008) and the references therein) and has been studied in the context of financial markets (Odean (1998)), firm investment (Gervais et al.

considering fine but finite grids on both signals and actions. For more detail regarding our implementation of QRE, see Section B.3 of the Online Appendix.

[22] As in Odean (1998), we assume that subjects exhibit overconfidence only with respect to their own signal.

(2011)), or corporate culture (Bolton et al. (2013)).

Overconfidence can potentially rationalize the overuse of information. However, there are several issues with this explanation. First, the required level of overconfidence varies greatly across treatments and across precision choices. In particular, as Table 6 in the Appendix shows, the degree of overconfidence needed to rationalize the weights estimated with our data varies from 1.34 when $\alpha = 0$ and subjects choose precision level 2 (column P 2) to 22.99 when $\alpha = 0.75$ and subjects choose the equilibrium precision and believe that their pair member does the same (column P $\{4, 4\}$). Moreover, there is no level of overconfidence that can rationalize the estimated weight observed for subjects who choose precision level 2 and believe that their pair member chooses the same precision when $\alpha = 0.75$.

Furthermore, overconfidence is unable to explain overacquisition of information. In the Online Appendix we argue that when $\tau_y = 1$, regardless of the value of ξ, the theory predicts that an overconfident subject will always choose the equilibrium precision level 4. Taken together, these observations imply that overconfidence is unlikely to explain our results.

6 Conclusions

We have characterized empirically how people choose private information and use private and public information in an experiment where subjects face fundamental and strategic uncertainty. To our knowledge, this paper is the first attempt to broadly characterize biases in the choice and use of information that are driven by strategic uncertainty.

We use a simple 2-player beauty contest model with incomplete information to investigate how strategic interactions affect subjects' information choice and its use. We find that people choose information similarly in individual and strategic decision making environments once behavior has stabilized. In particular, in both environments the modal precision choice corresponds to the equilibrium prediction, but we also see sustained overacquisition of information across treatment conditions. We investigate learning dynamics and find that initial precision choices differ across environments. In the individual decision environment initial choices are close to uniform across precisions, whereas in the strategic case initial choices clearly favor overacquisition. Learning dynamics in both environments shift choices towards the equilibrium and second-to best precision (overacquisition), leading to indistinguishable distributions in later rounds.

Unlike information choices, we see significant differences in how people use information in individual and strategic environments. While subjects in the individual decision environment use information in a similar way to the Bayesian prediction, we see significant overuse of private information in the strategic case, especially for those subjects who overacquire information. This bias in the use of information is robust to treatment variations, such as changes in the transparency of public information or strength of coordination motives. We show that this bias is not due to uncertainty about the rationality of others and investigate mechanisms related to the belief formation process inherent to the strategic environment to explain our results.

Finally, we quantify the welfare effects of the two biases that we identify by comparing realized payoffs to theoretical benchmarks. We find that the bias in information use (overuse of private information) has a negligible effect on payoffs, while the overacquisition of information leads to significant welfare losses.

The biases we identify have implications for the large theoretical literature in coordination games that studies welfare effects that result from changes in the information structure when it is exogenously determined (Angeletos and Pavan (2007)) or when it is endogenous, as in our setup (Colombo et al. (2014)). On the one hand, we find empirical support for theoretical predictions such as the crowding out effects of private information acquisition that results from higher transparency of public information. On the other hand, we identify biases that affect these predictions and show that welfare effects due to these biases are largely driven by overacquiring private information.

In macroeconomic setups our results suggest that, for example, overacquisition and overuse of information could potentially dampen business cycle fluctuations (if applied in the context of Benhabib et al. (2016)) or decrease the destabilizing effect of information choices on the volatility of asset prices (if considered in the context of Bansal and Shaliastovich (2011)). Furthermore, these biases affect our understanding of the desirability of greater transparency in financial markets and the macroeconomy more broadly.

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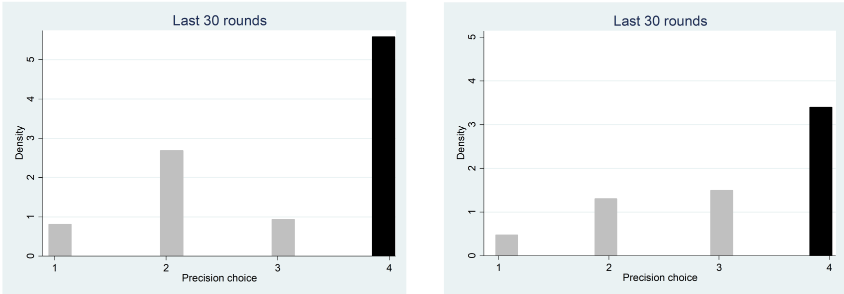
Appendix

A Empirical analysis



(A): Computer opponent (B): Best guess about x_j provided

Figure 10: Precision choices in robustness treatments.

	$\sigma_y = 1$, $\alpha = 0.25$	$\sigma_y = 1$, $\alpha = 0.75$	$\sigma_y = 15$, $\alpha = 0.25$
Equilibrium	110.72	110.86	102.49
Endogenous, σ_i^{eq}	110.16	110.07	100.59**
Exogenous, σ_i^{eq}	109.35	109.49	101.09
Endogenous, σ_i^{over}	105.05***	104.51***	98.76***
Exogenous, σ_i^{over}	105.27**	105.08**	-

Statistical difference with respect to efficient payoffs at levels:*** 1%, ** 5%, * 10%.

Table 4: Median realized payoffs for subjects who choose equilibrium precision and for overacquirers, strategic treatments

B Incentives to Overacquire Information

We measure the incentives to overacquire information by looking at the difference in the ex-ante utility of an agent that chooses to overacquire information (precision level 2) and acquiring the equilibrium precision (level 4):

$$\begin{aligned} E\left[U\left(\tau_2\right)\right] - E\left[U\left(\tau_4\right)\right] &= -\left(1-\widehat{\gamma}_i\left(\tau_2\right)\right)^2 Var^i_{\tau_2}\left(\theta - x_i\right) - \widehat{\gamma}_i^2\left(\left(\tau_2\right)\right) Var^i_{\tau_2}\left(\theta - z\right) \\ &\quad + \left(1-\widehat{\gamma}_i\left(\tau_4\right)\right)^2 Var^i_{\tau_4}\left(\theta - x_i\right) + \widehat{\gamma}_i^2\left(\tau_4\right) Var^i_{\tau_4}\left(\theta - z\right) \\ &\quad + 2\alpha\widehat{\gamma}_j\left(\widehat{\gamma}_i\left(\tau_2\right) - \widehat{\gamma}_i\left(\tau_4\right)\right) - \left(C\left(\tau_2\right) - C\left(\tau_4\right)\right), \qquad (12) \end{aligned}$$

	$\sigma_y = 1$, $\alpha = 0.25$	$\sigma_y = 1$, $\alpha = 0.75$	$\sigma_y = 15$, $\alpha = 0.25$
Equilibrium	99.72	99.86	96.49
Endogenous, σ_i^{eq}	99.16	99.07	94.59
Exogenous, σ_i^{eq}	98.35	99.49	95.09
Endogenous, σ_i^{over}	99.05	98.51	**98.76**
Exogenous, σ_i^{over}	99.27	99.08	-

Statistical difference with respect to efficient payoffs at levels:*** 1%, ** 5%, * 10%.

Table 5: Median realized payoffs in coordination stage for subjects who choose equilibrium precision and for overacquirers, strategic treatments

where $Var^i_{\tau_i}(\cdot)$ is the variance conditional on agent i choosing precision τ_i.[23]

We consider first the effect of overusing information on agent i's incentives to overacquire information when agent i (i) expects the other agent to follow the equilibrium strategy and (ii) has the correct perception of the joint distribution of $\{x_i x_j, \theta, y\}$.[24] From our experiment, we know that subjects that overacquire information (and expect their partners to choose precision level 4) assign weight $\widehat{\gamma}_i(\tau_2, \tau_4) = 0.596$ to public information. While we do not observe directly the weight these subjects would use had they chosen precision level 4, we use the data from sessions with exogenous information to impute this weight indirectly.[25]

[23]To keep notation simple, we suppress the dependence of utility and weights on agent i's beliefs about agent j's choice of information and agent j's beliefs about agent i's choice of information. Throughout this section, we assume that player j chooses the equilibrium precision (i.e., precision level 4) and expects agent i to do the same.

[24]In terms of Equation (12) this implies that $\widehat{\gamma}_j = \gamma^*(\tau_4, \tau_4) \approx 0.9925$, $Var^i(\theta - x_i|\tau_4) = \tau_4^{-1}$, $Var^i(\theta - x_i|\tau_2) = \tau_2^{-1}$, and $Var^i(\theta - z|\tau_4) = \tau_z^{-1}$.

[25]To impute this weight, based on our experimental evidence, we assume that there are two types of agents: (1) those that choose the equilibrium precision level and (2) those that overacquire information. Based on subjects' information choices in our benchmark treatment, we compute the proportion of these types in our

The imputed weight is $\widehat{\gamma}_i(\tau_4, \tau_4) = 0.84$. Using these weights in Equation (11), we find that the expected utility from following the equilibrium prediction is still higher than from deviating to precision level 2 (i.e., $E[U(\tau_4)] - E[U(\tau_2)] > 0$), but the loss from the deviation decreases by 42.5% compared to the case where both agents behave optimally, both on and off the equilibrium path. This suggests that while the overuse of information by itself cannot explain overacquisition of information, it does increase the incentives to overacquire information.

We next consider how agents' beliefs about other agents' use of information affect the incentives to overacquire information. To do so, we continue assuming that agent i overuses information, but we also assume that he also expects the other agent to overuse information (while still believing that the other agent chooses precision 4 so that agent i expects $\widehat{\gamma}_j^i = 0.84$). Under these assumptions, the loss from deviation is decreased by 43%, compared to the case where both agents follow the equilibrium predictions. If, instead, we assume that agent i believes that the other agent completely neglects public information ($\widehat{\gamma}_j^i = 0$), the loss from deviating decreases by 45% compared to the equilibrium benchmark. Thus, we see that beliefs about other agent's use of information play only a minor role in determining agent i's incentives to overacquire information. This is intuitive since the other agent's use of information affects agent i's incentive to deviate only through the term $2\alpha\widehat{\gamma}_j(\widehat{\gamma}_i(\tau_4) - \widehat{\gamma}_i(\tau_2))$ (see Equation (12)). If α is low and agent i overuses information (so $\widehat{\gamma}_i$ are low), then the belief about $\widehat{\gamma}_j$ has little impact on $E[U(\tau_4)] - E[U(\tau_2)]$.

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benchmark treatment. Assuming that (1) the distribution of subjects' types is the same in the treatment with exogenous information and (2) both types of subjects use information in the same way as in the baseline treatment, we can interpret the estimated weight on the public signal in the treatment with exogenously provided signals with precision level 4 as the weighted average of the weight used by both types of agents. We can then obtain $\widehat{\gamma}_i(\tau_4, \tau_4)$ by simply computing the necessary weight that implies the average weight in the treatment with exogenously provided signals with precision level 4 to be 0.878.

Uncovering Biases in Information Choice and its Use: The Role of Strategic Uncertainty

(Online Appendix)

A Theoretical Model

We solve a general quadratic-Gaussian model with information acquisition, strategic complementarities, and two players. The results stated in the paper follow as simple corollaries of the general results established below. Our analysis complements the results of Colombo et al. (2014) who considered a general linear-Gaussian model with information acquisition and continuum of players and the results of Ui and Yoshizawa (2015) who considered a general quadratic-Gaussian model with finitely many players but with exogenous information structure.

We begin by describing the general quadratic-Gaussian model with information acquisition, of which our setup described in Section 2 is a special case, and characterize its equilibrium.

A.1 General Setup

The structure of the model is the same as of the setup described in Section 2 except that we allow for more general utility specification. The utility function in our general setup is given by

$$U\left(a_i, a_j, \theta\right) = \frac{1}{2}U_{aa}a_i^2 + U_{aa'}a_i a_j + U_{a\theta}a_i\theta + \frac{1}{2}U_{a'a'}a_j^2 + U_{a'\theta}a'\theta + \frac{1}{2}U_{\theta\theta}\theta^2 + Linear\ Terms, \quad (13)$$

where "linear terms" can be expressed as $\begin{pmatrix} v_a & v_{a'} & v_\theta \end{pmatrix} \times \begin{pmatrix} a_i & a_j & \theta \end{pmatrix}'$. Note that Equation (13) implies that U_{aa} is the second derivative of U with respect to own action, a_i, $U_{a'a'}$ is the second derivative of U with respect to action of the other player, a_j, and so on. The linear terms are grouped together as they play little role in the analysis. The information structure and the sequence of agents' is the same as in the setup described in Section 2.

We impose the standard regularity conditions on the utility function (see Angeletos and Pavan (2007) and Colombo et al. (2014)).

Assumption 2 $U_{aa} < 0$ *and* $-U_{aa'}/U_{aa} \in (0,1)$

Assumption 2 ensures that the best-response function are well defined, that the coordination stage has unique equilibrium, and that actions in the coordination stage are strategic complementarities.

It is easy to see that the game we consider in Section 2 is a special case of the general game described above with $U_{aa} = -2$, $U_{aa'} = 2\alpha$, and $U_{a'a'} = -2\alpha$ and $v_a = v_{a'} = v_\theta = 0$. Therefore, Assumption 2 is satisfied in the model of Section 2 as long as $\alpha \in (0,1)$.

A.2 Equilibrium under Complete Information

Consider the general model of Section A.1, but assume that both agents observe θ. The equilibrium under complete information is pair of action choices for player 1 and player 2, $\{a_1, a_2\}$, such that for each $i = 1, 2$, a_i solves

$$\frac{\partial}{\partial a_i} U(a_i, a_j, \theta) = 0, \ \ j \neq i \tag{14}$$

Using the fact the utility function is quadratic, it follows that the best-response function is given by

$$a_i(\theta, a_j) = -\frac{v_a}{U_{aa}} - \frac{U_{a\theta}}{U_{aa}}\theta - \frac{U_{aa'}}{U_{aa}} a_j \tag{15}$$

Following Angeletos and Pavan (2007) and Colombo et al. (2014), we refer to the slope of the best-response function as the equilibrium degree of coordination and denote it by α^*, where

$$\alpha^* \equiv \frac{\partial a_i}{\partial a_j} = -\frac{U_{aa'}}{U_{aa}} \tag{16}$$

Assumption 2 implies that $\alpha^* \in (0,1)$.[26] Solving simultaneously F.O.C.s of both agents we obtain the following result.

Lemma 6 *The unique equilibrium of the complete information model is the pair of action* $\{a_i, a_j\}$ *such that*

$$a_i = a_j = \kappa_0^* + \kappa_1^* \theta \tag{17}$$

[26] In our simple model of Section 2, $\alpha^* = -U_{aa'}/U_{aa} = \alpha$.

where

$$\kappa_0^* \equiv -\frac{v_a}{U_{aa}+U_{aa'}} \tag{18}$$

$$\kappa_1^* \equiv -\frac{U_{a\theta}}{U_{aa}+U_{aa'}} \tag{19}$$

A.3 Equilibrium of the Model with Incomplete Information

A.3.1 The Coordination Stage

Suppose that agents chose precisions $\{\tau_i, \tau_j\}$ in the information acquisition stage. In this section, we characterize agents' optimal actions in the coordination stage given that both agents have correct beliefs about each other precision choices. That is, we characterize the optimal play along a potential equilibrium path.

In the coordination stage, the problem of player i is given by

$$\max_{a_i} E\left[U\left(a_i, a_j, \theta\right)|x_i, y\right], \tag{20}$$

where τ_i and τ_j affect the expectations through the joint distribution of $\{\theta, y, x_i, x_j\}$. The first-order condition is associated with the above problem is given by

$$E\left[\frac{\partial}{\partial a_i} U\left(a_i, a_j, \theta\right)|x_i, y\right] = 0 \tag{21}$$

Since the utility function U is quadratic in its arguments, the above first-order equation can be simplified to the following expression for a_i:

$$a_i = E\left[\alpha^* a_j + (1-\alpha^*)\,\kappa^*(\theta)\right| x_i, y]$$

where, $\alpha^* \in (0,1)$ is the equilibrium degree of coordination (see Equation (16)) and $\kappa^*(\theta) = \kappa_0^* + \kappa_1^* \theta$ is the optimal action choice under the complete information.

A strategy is a function that maps signals $\{x_i, y\}$ into actions. Denote the strategy of player i by $a_i(x_i, y)$, $i = 1, 2$. The equilibrium strategies of player i and j have to satisfy simultaneously

$$a_i(x_i, y) = E\left[\alpha^* a_j(x_j, y) + (1-\alpha^*)\,\kappa(\theta)\right| x_i, y] \tag{22}$$

$$a_j(x_j, y) = E\left[\alpha^* a_i(x_i, y) + (1-\alpha^*)\,\kappa(\theta)\right| x_j, y] \tag{23}$$

Lemma 7 *Let $\boldsymbol{\tau} = \{\tau_i, \tau_j\}$. For each player i, $i = 1, 2$, the unique linear equilibrium strategy is*

$$a_i^*(x_i, y) = \kappa_0^* + \kappa_1^*\left(\beta_i^*(\boldsymbol{\tau})\, x_i + \gamma_i^*(\boldsymbol{\tau})\, z\right), \tag{24}$$

where

$$\beta_i^*(\boldsymbol{\tau}) = (1-\alpha^*)(1-\delta_i)\frac{1+\alpha^*(1-\delta_j)}{1-\alpha^{*2}(1-\delta_i)(1-\delta_j)} \tag{25}$$

and $\gamma_i^*(\boldsymbol{\tau}) = 1 - \beta_i^*(\boldsymbol{\tau})$.

Proof. We guess that player i and j use the following linear strategies

$$\begin{aligned} a_i^*(x_i, y) &= \zeta_i^*(\boldsymbol{\tau}) + \beta_i^*(\boldsymbol{\tau}) x_i + \gamma_i^*(\boldsymbol{\tau}) z && (26) \\ a_j^*(x_j, y) &= \zeta_j^*(\boldsymbol{\tau}) + \beta_j^*(\boldsymbol{\tau}) x_j + \gamma_j^*(\boldsymbol{\tau}) z && (27) \end{aligned}$$

respectively. Combining the guess (26) with Equation (22) and using the observation that $E[\theta|x_i, y] = (1-\delta_i) x_i + \delta_i z$, where $\delta_i = \tau_z/(\tau_i + \tau_z)$, we obtain

$$\begin{aligned} a_i^*(x_i, y) &= \left[\alpha^* \zeta_j^*(\boldsymbol{\tau}) + (1-\alpha^*)\kappa_0^*\right] + (1-\delta_i)\left(\alpha^* \beta_j^*(\boldsymbol{\tau}) + (1-\alpha^*)\kappa_1^*\right) x_i \\ &\quad + \left(\alpha^* \beta_j^*(\boldsymbol{\tau})\delta_i + \alpha^* \gamma_j^*(\boldsymbol{\tau}) + (1-\alpha^*)\kappa_1^* \delta_i\right) z && (28) \end{aligned}$$

Following the same steps we get

$$\begin{aligned} a_j^*(x_j, y) &= \left[\alpha^* \zeta_i^*(\boldsymbol{\tau}) + (1-\alpha^*)\kappa_0^*\right] + (1-\delta_j)\left(\alpha^* \beta_i^*(\boldsymbol{\tau}) + (1-\alpha^*)\kappa_1^*\right) x_j \\ &\quad + \left(\alpha^* \beta_i^*(\boldsymbol{\tau})\delta_j + \alpha^* \gamma_i^*(\boldsymbol{\tau}) + (1-\alpha^*)\kappa_1^* \delta_j\right) z && (29) \end{aligned}$$

Comparing the coefficients in Equations (26) and (27) with those in (28) and (29), we see that the constants ζ_i^* and ζ_j^* have to satisfy

$$\begin{aligned} \zeta_i^*(\boldsymbol{\tau}) &= \left[\alpha^* \zeta_j^*(\boldsymbol{\tau}) + (1-\alpha^*)\kappa_0^*\right] \\ \zeta_j^*(\boldsymbol{\tau}) &= \left[\alpha^* \zeta_i^*(\boldsymbol{\tau}) + (1-\alpha^*)\kappa_0^*\right] \end{aligned}$$

From these equations we obtain that $\zeta_i^*(\boldsymbol{\tau}) = \zeta_j^*(\boldsymbol{\tau}) = \kappa_0^*$.

Next, comparing coefficients on x_i and x_j in Equations (26) and (27) with those in (28) and (29), we obtain a system of linear equations that $\{\zeta_i^*(\boldsymbol{\tau}), \beta_i^*(\boldsymbol{\tau}), \gamma_i^*(\boldsymbol{\tau})\}$ and $\{\zeta_j^*(\boldsymbol{\tau}), \beta_j^*(\boldsymbol{\tau}), \gamma_j^*(\boldsymbol{\tau})\}$ have to satisfy. Solving this system of equations for these unknowns yields the desired result. ■

A.3.2 The Coordination Stage after an Undetected Deviation

Suppose now that in a candidate equilibrium agents were to choose precisions $\{\tau_i, \tau_j\}$. Furthermore, suppose that agent i deviated from this prescribed behavior and chose instead precision, $\widehat{\tau}$. To keep notation simple let $\widehat{\boldsymbol{\tau}} = \{\widehat{\tau}, \{\tau_i, \tau_j\}\}$, so that $\widehat{\boldsymbol{\tau}}$ is a vector that contains precision choice to which player i deviated and the precision choices $\{\tau_i, \tau_j\}$, which agents were supposed to make in the information acquisition stage. Note that since precision choices are made privately, agent j is unaware of this deviation and behaves as if the agent i chose τ_i. On the other hand, agent i has correct believes about precision choice of agent j.

Given that agent j believes that agent i chose the prescribed equilibrium precision, agent j finds it optimal to follows a linear strategy $a_j^*(x_j, y) = \kappa_0^* + \kappa_1^* \left(\beta_j^*(\boldsymbol{\tau}) x_j + \gamma_j^*(\boldsymbol{\tau}) z\right)$ as characterized in Section A.3.1. Agent i's optimal action choice is the best-response to this strategy employed by agent j. That is, agent i solves

$$\max_{a_i} E\left[U\left(a_i, a_j^*, \theta\right)|x_i, y\right],$$

where his precision choice is $\widehat{\tau}$ instead of τ_i. The following result follows immediately from solving the above maximization problem.

Lemma 8 *Suppose that agents were expected to choose precisions $\{\tau_i, \tau_j\}$. Suppose further that agent i instead chose precision $\widehat{\tau}$. Then, in the coordination stage, player i's optimal strategy is given by*

$$\widehat{a}_i(x_i, y) = \kappa_0^* + \kappa_1^* \left(\widehat{\beta}_i(\widehat{\boldsymbol{\tau}}) x_i + \widehat{\gamma}_i(\widehat{\boldsymbol{\tau}}) z\right) \tag{30}$$

where, $\widehat{\boldsymbol{\tau}} = \{\widehat{\tau}, \{\tau_i, \tau_j\}\}$,

$$\widehat{\beta}_i(\widehat{\boldsymbol{\tau}}) = (1-\alpha^*)\left(1-\widehat{\delta}\right)\frac{1+\alpha^*(1-\delta_j)}{1-\alpha^{*2}(1-\delta_i)(1-\delta_j)}, \tag{31}$$

$\widehat{\delta} = \tau_z/(\tau_z + \widehat{\tau})$, *and* $\widehat{\gamma}_i(\widehat{\boldsymbol{\tau}}) = 1 - \widehat{\beta}_i(\widehat{\boldsymbol{\tau}})$.

A.3.3 The Information Acquisition Stage

We now consider agents optimal precision choices. A pair of precision choices $\{\tau_i, \tau_j\}$ constitutes an equilibrium if and only if neither agent has incentives to unilaterally deviate from it. Thus, our first goal is to determine agents' optimal unilateral deviations in the information acquisition stage from a candidate equilibrium precision choices.

Suppose that in a candidate equilibrium, agents are expected to choose precisions, $\{\tau_i, \tau_j\}$. Consider agent i who is deciding whether to unilaterally deviate from the prescribed precision choice. That is, agent i is choosing an optimal precision choice in response to agent j (i) choosing precision τ_j and (ii) acting in the coordination stage according to the belief that precision levels chosen in the first-stage were $\{\tau_i, \tau_j\}$. Agent i's problem is then given by

$$\max_{\widehat{\tau}} E\left[U\left(\widehat{a}_i^*, a_j^*, \theta\right)\right] - C\left(\widehat{\tau}\right) \tag{32}$$

where the expectations are taken over $\{\theta, y, x_1, x_2\}$, $C\left(\widehat{\tau}\right)$ is the cost of purchasing precision $\widehat{\tau}$, $\widehat{a}_i^*$ is agent i's optimal strategy in the coordination stage following an undetected deviation (see Lemma 8), and a_j^* is agent j's optimal strategy in coordination stage given that agent j believes that both agents will make prescribed precision choices.

Lemma 9 *Let $\widehat{\tau}(\boldsymbol{\tau})$ denote agent i's optimal deviation from any prescribed precision choice $\boldsymbol{\tau} = \{\tau_i, \tau_j\}$. Then for any $\boldsymbol{\tau} \in \mathbb{R}_+^2$, $\widehat{\tau}(\boldsymbol{\tau})$ is defined as the unique solution to*

$$F\left(\widehat{\boldsymbol{\tau}}\right) \equiv \frac{|U_{aa}|\left(\kappa_1^* \widehat{\beta}_i^*\left(\widehat{\boldsymbol{\tau}}\right)\right)^2}{2\widehat{\tau}^2} - C'\left(\tau_i\right) = 0, \tag{33}$$

where $\widehat{\boldsymbol{\tau}} = \{\widehat{\tau}, \{\tau_i, \tau_j\}\}$.

Proof. The first-order condition associated with agent i's problem stated in (32) is given by

$$\begin{aligned} &\int_{(\theta,y)} \int_{\{x_i,x_j\}} U\left(\widehat{a}_i^*, a_j^*, \theta\right) \frac{\partial p\left(x_i|\theta, y\right)}{\partial \tau_i} p\left(x_j|\theta, y\right) dx_i dx_j dP\left(\theta, y\right) \\ &+ \int_{(\theta,y)} \int_{\{x_i,x_j\}} U_k\left(\widehat{a}_i^*, a_j^*, \theta\right) \frac{\partial \widehat{a}_i^*}{\partial \tau_i} dP\left(x_i, x_j|\theta, y\right) dP\left(\theta, y\right) - C'\left(\tau_i\right) = 0 \end{aligned} \tag{34}$$

Note that the second integral in Equation (34) can be written as

$$\int_{(x_i,y)} \frac{\partial \widehat{a}_i^*}{\partial \tau_i} \left\{ \int_{\{\theta,x_j\}} U_k\left(\widehat{a}_i^*, a_j^*, \theta\right) \frac{\partial \widehat{a}_i^*}{\partial \tau_i} dP\left(\theta, x_j|x_i, y\right) \right\} dP\left(x_i, y\right) = 0,$$

where we used the observation that the inner integral in the above expression corresponds to agent i's F.O.C. at the coordination stage following undetected deviation. Next, we use integration by parts and simplify the first integral in Equation (34). Following these steps we find that the above F.O.C. can be written as

$$\frac{|U_{aa}|\left(\kappa_1^* \widehat{\beta}_i^*\left(\widehat{\boldsymbol{\tau}}\right)\right)^2}{2\widehat{\tau}^2} - C'\left(\widehat{\tau}\right) = 0, \tag{35}$$

where $\widehat{\beta}_i^*$ is defined in Equation (31) and is a function of $\widehat{\boldsymbol{\tau}} = \{\widehat{\tau}, \{\tau_i, \tau_j\}\}$.

Let $F(\widehat{\boldsymbol{\tau}})$ denote the LHS of Equation (35). It is straightforward to see that $\partial F(\widehat{\boldsymbol{\tau}})/\partial\widehat{\tau} < 0$, $F(\widehat{\boldsymbol{\tau}}) > 0$ at $\widehat{\tau} = 0$, and $\lim_{\widehat{\tau}\to\infty} F(\widehat{\boldsymbol{\tau}}) = -\infty$. Therefore, Equation (35) has always a unique interior solution implying that $\widehat{\tau}(\boldsymbol{\tau})$ is well defined. ■

Lemma 10 $\widehat{\tau}(\boldsymbol{\tau})$ *is increasing in both* τ_i *and* τ_j.

Proof. Using the definition of $\widehat{\beta}_i^*$ (see Equation (31)) is is straightforward to see that $\widehat{\beta}_i^*/\partial\tau_j > 0$ and $\partial\widehat{\beta}_i^*/\partial\tau_i > 0$ implying that $\partial F(\widehat{\boldsymbol{\tau}})/\partial\tau_j > 0$ and $\partial F(\widehat{\boldsymbol{\tau}})/\partial\tau_i > 0$. Since, $\partial F(\widehat{\boldsymbol{\tau}})/\partial\widehat{\tau} < 0$, by the implicit function theorem applied to Equation (33), we conclude that $\widehat{\tau}(\boldsymbol{\tau})$ is increasing in both τ_i and τ_j. ■

Before proceeding further, note that if $\widehat{\boldsymbol{\tau}} = \{\tau_i, \{\tau_i, \tau_j\}\}$ then $\widehat{\beta}_i^*(\widehat{\boldsymbol{\tau}}) = \beta_i^*(\boldsymbol{\tau})$, where $\boldsymbol{\tau} = \{\tau_i, \tau_j\}$. Therefore, if $\boldsymbol{\tau}^* = \left\{\tau_i^*, \tau_j^*\right\}$ is a vector of equilibrium precision choices, then $\boldsymbol{\tau}^*$ has to satisfy

$$\frac{|U_{aa}|\left(\kappa_1^*\beta_i^*(\boldsymbol{\tau}^*)\right)^2}{2\tau_i^{*2}} - C'(\tau_i^*) = 0 \tag{36}$$

for each $i \in \{1,2\}$, $j \neq i$.

Using the above observations, we now show that our two-stage game has a unique equilibrium in which both agents choose the same precision level.

Proposition 1 *The unique equilibrium precision choice for agent* i, $i = 1,2$, *is* τ^*, *where* τ^* *is the unique solution to*

$$\tau^* = \sqrt{\frac{1}{2}\frac{|U_{aa}|\kappa_1^{*2}}{C'(\tau^*)}} - \frac{1}{1-\alpha^*}\tau_z \tag{37}$$

Proof. We first show that there exists a unique symmetric equilibrium. Let $\tau_i = \tau_j = \tau^*$ and $\boldsymbol{\tau}^* = \{\tau^*, \tau^*\}$. Then

$$\beta^*(\boldsymbol{\tau}^*) \equiv \frac{(1-\delta^*)(1-\alpha^*)}{1-\alpha^*(1-\delta^*)} \tag{38}$$

and $\delta^* = \tau_z/(\tau^* + \tau_z)$. Hence, the symmetric equilibrium precision choice has to satisfy

$$\frac{|U_{aa}|\left(\kappa_1^*\beta^*(\boldsymbol{\tau}^*)\right)^2}{2\tau^{*2}} - C'(\tau^*) = 0 \tag{39}$$

Using the definition of $\beta^*(\boldsymbol{\tau}^*)$ in the above equation and simplifying the resulting expression we obtain

$$\tau^* = \sqrt{\frac{|U_{aa}|\,\kappa_1^{*2}}{2C'(\tau^*)}} - \frac{\tau_z}{1-\alpha^*}, \tag{40}$$

It is straightforward to see that Equation (40) has a unique solution, which implies that our model features a unique symmetric equilibrium.

To show that there exist no asymmetric equilibria we use iterative deletion of strictly dominated strategies. Let, $\overline{\tau}$ be the unique solution to

$$\lim_{\tau_i,\tau_j\to\infty} F(\widehat{\boldsymbol{\tau}}) = 0$$

Since $\widehat{\tau}(\boldsymbol{\tau})$ is increasing in τ_i and τ_j (Lemma 10), we know that for all τ_i,$\tau_j \in \mathbb{R}_+$

$$\widehat{\tau}(\boldsymbol{\tau}) < \overline{\tau}$$

Since agents are symmetric, it follows then no agent will ever choose precision larger than $\overline{\tau}$. Let $\overline{\tau}_0 = \overline{\tau}$ and suppose that agents are prescribed to choose $\overline{\boldsymbol{\tau}}_0 = \{\overline{\tau}_0, \overline{\tau}_0\}$. Since $\widehat{\tau}(\boldsymbol{\tau})$ is increasing in both τ_i and τ_j, it follows that $\overline{\tau}_1 = \widehat{\tau}(\overline{\boldsymbol{\tau}}_0) < \overline{\tau}_0$. Since agents are symmetric, it follows that no agent will find it ever optimal to choose precision larger than $\overline{\tau}_1$. Iterating in this fashion, we obtain a decreasing sequence $\{\overline{\tau}_k\}_{k=0}^{\infty}$ bounded from below by 0. Therefore, this sequence converges and we denote its limit by $\overline{\tau}_\infty$. Note that $\overline{\tau}_\infty$ has to satisfy

$$\frac{|U_{aa}|\left(\kappa_1^*\beta^*(\overline{\boldsymbol{\tau}}_\infty)\right)^2}{2\overline{\tau}_\infty^2} - C'(\overline{\tau}_\infty) = 0, \tag{41}$$

where $\overline{\boldsymbol{\tau}}_\infty = \{\overline{\tau}_\infty, \overline{\tau}_\infty\}$, as otherwise we would be able to iterate further.

We then follow an analogous approach from "below" starting with $\underline{\tau}_0 = 0$ and $\underline{\boldsymbol{\tau}}_0 = \{\underline{\tau}_0, \underline{\tau}_0\}$. Following this approach we obtain an increasing sequence $\{\underline{\tau}_k\}_{k=0}^{\infty}$ bounded from above by $\overline{\tau}_\infty$. Therefore, this sequence converges and we denote its limit by $\underline{\tau}_\infty$. Note that $\overline{\tau}_\infty$ has to satisfy

$$\frac{|U_{aa}|\left(\kappa_1^*\beta^*(\underline{\boldsymbol{\tau}}_\infty)\right)^2}{2\underline{\tau}_\infty^2} - C'(\underline{\tau}_\infty) = 0, \tag{42}$$

where $\underline{\boldsymbol{\tau}}_\infty = \{\underline{\tau}_\infty, \underline{\tau}_\infty\}$. Comparing Equations (41) and (42) we see that $\overline{\tau}_\infty$ and $\underline{\tau}_\infty$ satisfy the same equation which, as we argued above, has a unique solution. Therefore, $\underline{\tau}_\infty = \overline{\tau}_\infty = \tau^*$, where τ^* is the unique symmetric equilibrium choice. It follows that there are no asymmetric equilibria. ■

Proposition 1 implies that in the unique equilibrium agents choose the same precision, τ^*, and follow symmetric strategies in the coordination game. Therefore, in what follows we drop subscript i (j) when referring to agent i's (agent j's) choices.

Lemma 11 *Consider the equilibrium precision choice τ^*.*

1. *The equilibrium precision choice τ^* is decreasing in the precision of public information τ_z, that is $\partial\tau^*/\partial\tau_z < 0$.*
2. *The equilibrium precision choice τ^* is decreasing in the degree of strategic complementarities α, that is $\partial\tau^*/\partial\alpha < 0$.*

Proof. Immediate from the Equation (37). ■

A.4 Derivation of Equation (11)

In this section, we derive Equation (11) and discuss the assumptions regarding agent i's perception of the relations between signals and signals that one needs to impose to arrive at this equation.

Recall that the ex-ante utility of agent i is given by

$$E\left[U\left(\tau_i\right)\right] = -E\left[\left(1-\alpha\right)\left(a_i-\theta\right)^2 + \alpha\left(a_i-a_j\right)^2\right],$$

Assume that agent i uses a linear strategy given by $a_i = \widehat{\gamma}_i z + (1-\widehat{\gamma}_i)\,x_i$, where $\widehat{\gamma}_i \in [0,1]$ is the weight (potentially non-optimal) that player i assigns to public signal. Furthermore, assume that player i believes that player j also follows a linear strategy given by $a_j = \widehat{\gamma}_j z + \left(1-\widehat{\gamma}_j\right) x_j$, where $\widehat{\gamma}_j \in [0,1]$ is the weight that player i believes that player j assigns to public signal. We can now wrote ex-ante utility as

$$\begin{aligned} E\left[U\left(\tau_i\right)\right] \quad = \quad & -E\left[\left(1-\alpha\right)\left(\widehat{\gamma}_i\left(z-\theta\right)+\left(1-\widehat{\gamma}_i\right)\left(x_i-\theta\right)\right)^2\right. \\ & \left.+\,\alpha\left(\widehat{\gamma}_i\left(z-\theta\right)+\left(1-\widehat{\gamma}_i\right)\left(x_i-\theta\right)-\widehat{\gamma}_j\left(z-\theta\right)+\left(1-\widehat{\gamma}_j\right)\left(x_j-\theta\right)\right)^2\right] \end{aligned}$$

Up to this point, we only imposed linearity of strategies. If we further assume that agents understand that signals are unbiased (which we emphasized to subjects in instructions), that is, $E[x_i - \theta] = E[x_j - \theta] = E[z - \theta] = 0$, then we can simplify ex-ante utility to

$$
\begin{aligned}
E[U(\tau_i)] \quad = \quad & -\widehat{\gamma}_i Var^i(z-\theta) - (1-\widehat{\gamma}_i) Var^i(x_i - \theta) - 2\widehat{\gamma}_i (1-\widehat{\gamma}_i) Cov^i(z-\theta, x_i - \theta) \quad (43) \\
& -\alpha \left\{ \widehat{\gamma}_j Var^i(z-\theta) - \left(1-\widehat{\gamma}_j\right) Var^i(x_j-\theta) - 2\widehat{\gamma}_j \left(1-\widehat{\gamma}_j\right) Cov^i(z-\theta, x_j-\theta) \right\} \\
& +\alpha \left\{ 2\widehat{\gamma}_i\widehat{\gamma}_j Var^i(z-\theta) + 2\widehat{\gamma}_i \left(1-\widehat{\gamma}_j\right) Cov^i(z-\theta, x_j-\theta) \right. \\
& \left. + 2(1-\widehat{\gamma}_i)\widehat{\gamma}_j Cov^i(x_i-\theta, z-\theta) + 2(1-\widehat{\gamma}_i)\left(1-\widehat{\gamma}_j\right) Cov^i(x_i-\theta, x_j-\theta) \right\},
\end{aligned}
$$

where $Var^i(\cdot)$ and $Cov^i(\cdot)$ capture agent i's subjective beliefs about of the variances and covariances of relevant random variables. If we further assume that agents understand that

$$E[(z-\theta)(x_i-\theta)] = E[(z-\theta)(x_j-\theta)] = E[(x_i-\theta)(x_j-\theta)] = 0$$

then Equation (43) can be further simplified to

$$
\begin{aligned}
E[U(\tau_i)] \quad = \quad & -\widehat{\gamma}_i Var^i(z-\theta) - (1-\widehat{\gamma}_i) Var^i(x_i-\theta) \quad (44) \\
& -\alpha \left\{ \widehat{\gamma}_j Var^i(z-\theta) - \left(1-\widehat{\gamma}_j\right) Var^i(x_j-\theta) - 2\widehat{\gamma}_i\widehat{\gamma}_j Var^i(z-\theta) \right\},
\end{aligned}
$$

which is Equation (11) in the text.

A.5 Comparison with Morris and Shin (2002)

The model used in the experiment is a natural simplification of the framework introduced by Morris and Shin (2002). Nevertheless, this simplification does have implications for some predictions of the model. In particular, in the unique equilibrium of our model, agents underuse public information while in Morris and Shin (2002) they overuse it. The reason for this difference is easiest to highlight in the context of a model with a continuum of players.

Recall that in Morris and Shin (2002), agents' utility function is given by

$$U^{MS}(\mathbf{a}, \theta) = -(1-\alpha)(a_i - \theta)^2 - \alpha\left(L_i - \bar{L}\right), \quad (45)$$

where $L_i = \int_0^1 (a_i - a_j)^2 dj$ is the average mean-squared difference between agent i and other agents' actions, and $\bar{L} = \int_0^1 L_j dj$ is the average of the mean squared differences across all agents. Thus, in Morris and Shin (2002) agents care about how far their mean-squared difference is from the average mean-squared difference.

Since the average $L_i - \bar{L}$ across agents is zero, from a social perspective one should not care about the coordination motive. However, from the individual perspective, coordination motive matters and, thus, from the social perspective agents use public information too much. Therefore, equilibrium degree of coordination exceeds the efficient degree of coordination.

In contrast, suppose that the utility function was given by

$$U(\mathbf{a}, \theta) = -(1-\alpha)(a_i - \theta)^2 - \alpha L_i \tag{46}$$

In this case, miscoordination of action is also important from social perspective. However, agents do not take into account the effect that their actions have on the ability of other agents to match their actions. As such, they underuse information. In our 2-player model, utility function is given by $U(\mathbf{a}, \theta) = -(1-\alpha)(a_i - \theta)^2 - \alpha(a_i - a_j)^2$ and, thus, it captures inefficiencies arising from the utility function in Equation (46) rather than the one in Equation (45).

B Alternative Models

B.1 Level-k model

Proof of Lemma 3. Consider the problem of type Lk player at the coordination stage who chose precision τ_{Lk} and who believes that type $Lk-1$ player assigns the weight γ_{Lk-1} to public information, where γ_{Lk-1} is an integrable random variable (independent of x_j and θ) with mean $\overline{\gamma}_{Lk-1} \in [0,1]$.[27] Then, type Lk player chooses action a_i to solve

$$\min_{a_i} E\left[-(1-\alpha)(a_i - \theta)^2 - \alpha(a_i - a_j)^2 |x_i, y\right]$$

where $a_j = \gamma_{Lk-1} z + \left(1 - \gamma_{Lk-1}\right) x_j$ and the expectations are taken over $\left\{\theta, x_j, \gamma_{Lk-1}\right\}$. Taking F.O.C. and rearranging, we obtain

$$a_i = E\left[(1-\alpha)\theta + \alpha\left(\gamma_{Lk-1} z + \left(1 - \gamma_{Lk-1}\right) x_j\right) |x_i, y\right]$$

[27] The randomness in γ_{LK-1} captures the possibility that level $Lk-1$ agent may randomize his precision choices. From type Lk's perspective randomization over precision choice translates then into randomness in γ_{LK-1}.

Since γ_{Lk-1} is independent of x_j and θ, and since $E[x_j|x_i, y] = \delta(\tau_{Lk}) z + (1-\delta(\tau_{Lk})) x_i$ we have

$$\begin{aligned} a_i &= \left[(1-\alpha)\delta(\tau_{Lk}) + \alpha\overline{\gamma}_{Lk-1} + \alpha\delta(\tau_{Lk})\left(1-\overline{\gamma}_{Lk-1}\right)\right] z \\ &\quad + \left[(1-\alpha)\theta(1-\delta(\tau_{Lk})) + \alpha\left(1-\overline{\gamma}_{Lk-1}\right)(1-\delta(\tau_{Lk}))\right] x_i \end{aligned}$$

Therefore, the weight that player of type Lk assigns to public signal is given by

$$\gamma_{Lk} = \delta(\tau_{Lk}) + \alpha\overline{\gamma}_{Lk-1}(1-\delta(\tau_{Lk}))$$

It follows that $\gamma_{Lk} \geq \delta(\tau_{Lk})$ with equality if and only if $\overline{\gamma}_{Lk-1} = 0$. ■

Lemma 3 does not depend on the specific assumptions made in our experimental setup. In contrast, the proof of Lemma 4 makes use of specific assumptions and parameter values chosen in our experimental setup.

Proof of Lemma 4. Let p_n be the probability that agent of type $Lk-1$ chooses precision level n, $n \in \{1,2,3,4\}$, $\gamma_{Lk-1}(\tau_n)$ be the weight he assigns to public information if he chose precision level n, and $\overline{\gamma}_{Lk-1} = \sum_{n=1}^{4} p_n \gamma_{Lk-1}(\tau_n)$. Denote by τ_m precision level chosen by type Lk agent. From Lemma 3 we know that if agent of type Lk chose precision τ_n then his optimal action at the coordination stage is

$$a^*_{Lk}(\tau_m) = \gamma_{Lk}(\tau_m) z + (1-\gamma_{Lk}(\tau_m)) x_i,$$

where $\gamma_{Lk}(\tau_m) = \delta(\tau_m) + \alpha\overline{\gamma}_{Lk-1}(1-\delta(\tau_m))$ and $\delta(\tau_n) = \tau_z/(\tau_z+\tau_m)$. Given these observations and suppressing the dependence of weights on precision choices for notational convenience, the expected utility of agent of type Lk who chooses precision τ_m at the information stage is given by

$$U_{Lk}(\tau_k) = -\left[\gamma_{Lk}^2\tau_z^{-1} + (1-\gamma_{Lk})^2\tau_k^{-1}\right] - 2\alpha\sum_{n=1}^{4} p_n\gamma_{Lk}\gamma_{Lk-1}\tau_z^{-1} - \Delta,$$

where

$$\Delta \equiv \alpha\sum_{n=1}^{4} p_n\left[\gamma_{Lk-1}^2\tau_z^{-1} + \left(1-\gamma_{Lk-1}\right)^2\tau_n^{-1}\right]$$

captures the terms that do not depend type Lk agent's precision choice.

We now show that $U_{Lk}(\tau_4) - U_{Lk}(\tau_m) > 0$ for $m \in \{1,2,3\}$. First, we note that

$$\gamma_{Lk}^2(\tau_4)\tau_z^{-1} + (1-\gamma_{Lk}(\tau_4))^2\tau_4^{-1} < 1$$

since by Lemma 3 we have $\gamma_{Lk}(\tau_4) \geq \delta(\tau_4) \equiv \tau_z/(\tau_z + \tau_4)$ and $\gamma_{Lk}^2(\tau_4)\tau_z^{-1} + (1-\gamma_{Lk}(\tau_4))^2\tau_4^{-1}$ is a convex in γ_{Lk} and achieves its minimum at $\gamma_{Lk} = \delta(\tau_4)$. Next, we note that given that $\tau_z = 1$, we have

$$\gamma_{Lk}^2(\tau_m)\tau_z^{-1} + (1-\gamma_{Lk}(\tau_m))^2\tau_k^{-1} > \delta^2(\tau_m)$$

Therefore, for any $m \in \{1,2,3\}$, we have

$$\begin{aligned} U_{Lk}(\tau_4) - U_{Lk}(\tau_m) &> C(\tau_m) - C(\tau_4) - \left(1-\delta^2(\tau_m)\right) \\ &\quad +2\alpha\sum_{n=1}^{4} p_n\left[\gamma_{Lk}(\tau_4) - \gamma_{Lk}(\tau_m)\right]\gamma_{Lk-1}\tau_z^{-1} \\ &> C(\tau_k) - C(\tau_4) - 1 \end{aligned}$$

where the last inequality follows by observing that $\gamma_{Lk}(\tau_4) - \gamma_{Lk}(\tau_m) > 0$ and $\delta^2(\tau_m) > 0$. Since $C(\tau_k) - C(\tau_4) > 1$ for all $m \in \{1,2,3\}$ (see Table 1), we conclude that for all $m \in \{1,2,3\}$ we have $U_{Lk}(\tau_4) - U_{Lk}(\tau_m) > 0$. Since the above argument does not depend on the level of reasoning (except that agent must best respond to the agent of lower level of reasoning), we conclude that all types of level k, $k \geq 1$, find it optimal to choose precision 4. ■

B.2 Anticipated regret

We denote an agent's regret by R, where $R : \mathbb{R}^2 \to \mathbb{R}$ and R maps $(\theta - a_i)$ and $(a_j - \alpha_i)$ into a real number. We assume that R is a quadratic function given by

$$R(\theta - a_i, a_j - \alpha_i) = -r_\theta(\theta - a_i)^2 - r_a(a_j - \alpha_i)^2, \tag{47}$$

where $r_a, r_\theta \geq 0$, so that in the model with regret, agents maximize utility

$$U^R = U + R,$$

where U is the underlying utility function (Equation (1)). This choice of regret function is natural given our payoff function. Moreover, R has also an intuitive property that agent i feels only a small amount of regret if his losses from mismatching θ and a_j are small, but strongly regrets his choices when his losses are large.[28]

[28]Filiz-Ozbay and Ozbay (2007) and Gretschko and Rajko (2015) consider piecewise-linear regret functions. However, since in our setting choosing a_i smaller

Note that if r_θ is large relative to r_a then agents regret more mismatching the state. This should make them use public information less in equilibrium than in the benchmark model since in this case they value less the coordinating effect of the public signal. Similarly, high values of r_θ and r_a may increase the value of information since agents have stronger incentives to avoid large losses.

Proof of Lemma 5. We have

$$U^R(a_i, a_j, \theta) = 100 - (1 - \alpha + r_\theta)(\theta - a_i)^2 - (\alpha + r_a)(a_j - \alpha_i)^2$$

Therefore, the above utility function fits into the general setup described in Section A.1 of this appendix, with $U_{aa} = 2(1 + r_\theta + r_a)$, $U_{aa'} = 2(\alpha + r_a)$, $U_{a\theta} = 2(1 - \alpha + r_\theta)$, $U_{a'a'=}(\alpha + r_a)$, $U_{a'\theta} = 0$, and $U_{\theta\theta} = 0$. Therefore, from Lemma 7 in this appendix, we know that the equilibrium weight on public information is given by

$$\gamma^* = \frac{\delta_i + \alpha^* \delta_j (1 - \delta_i)}{1 - \alpha^{*2}(1 - \delta_i)(1 - \delta_j)},$$

where $\alpha^* = -U_{aa'}/U_{aa} = \frac{\alpha + r_a}{1 + r_\theta + r_a} \in (0, 1)$. Since $\gamma^* = \delta_i$ if $\alpha^* = 0$ and $\partial\gamma^*/\partial\alpha^* > 0$ if follows that

$$\gamma^* \geq \delta_i$$

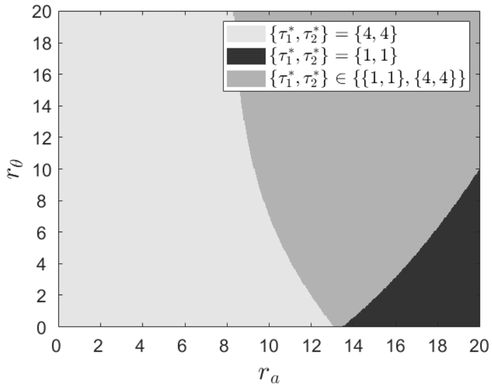
Finally, from the expression for α^*, we see that this bound is achieved only as $r_\theta \to \infty$. ■

B.3 Quantal Response Equilibrium

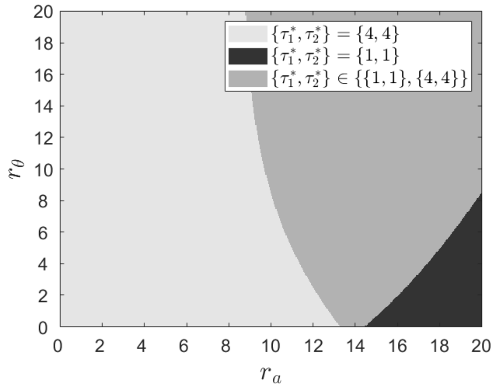
B.3.1 Overview

In this section, we describe in detail how we implemented quantal response equilibrium in our setting. Note that QRE is typically applied in the context of finite-action models with complete information (see Goree et al. (2016) and references therein). Even in those relatively simple settings, QRE often has to be solved numerically. In contrast, our model features a continuum of actions and incomplete information. With a continuum of actions the so-called statistical reaction functions (which play the role of best-response functions in QRE) may not be well-defined. In addition, due to incomplete information, even if statistical

or larger than θ or a_j leads to symmetrical utility losses, we choose a symmetric regret function.



(a) $\alpha = 0.25$



(b) $\alpha = 075$

Figure 11: Equilibrium precision choices for different combinations of r_θ and r_a

reaction functions are well-defined they will be complex functionals that map any pair of signals $\{x_i, y\} \in \mathbb{R}^2$ into a probability distribution over $\mathbb{R}$. This makes implementation of QRE in our setting challenging.

We focus on the so-called logistic QRE (see McKelvey and Palfrey (1995)) where agents' choices are subject to errors that follow type I extreme value distribution. To compute logistic QRE, we discretize the model, that is we consider an approximation to our model where actions and signals take values on fine but finite grids. Discretizing action space allows us to circumvent the issue that statistical response functions may be not well-defined. Discretizing signal space implies that statistical response functions will take the form of matrices rather than functions. Finally, to further decrease computational difficulty, we focus on the coordination stage assuming that both agents chose the same precision level and they have correct beliefs about the precision choice of the other agents.[29]

Computing QRE Denote the common private precision choice by τ_x. All other parameters of the model are the same as in the experiment. The algorithm to solve for QRE consists of two parts: discretization of the model and computation of statistical response functions in the discretized model.

[29]Our implementation of QRE allows us to extend it to the case of asymmetric precision choices but at a cost of an additional computational burden.

Discretization of the model To discretize the model perform the following steps.

1. Fix a realization of public signal, y (and, hence a realization of z).
2. Restrict action choices to belong to interval $[-100, 100]$ and choose an equally spaced grid on this interval consisting of n_A points. Denote by $a(i)$ the i^{th} element of the gird over possible actions.
3. Restrict private signals to belong to interval $[-100, 100]$ and choose an equally spaced grid on this interval consisting of n_x points.[30] Denote by $x(i)$ the i^{th} element of the gird over possible signals.
4. Let $P_x(m,n) = \Pr(x_j = x(m) | x_i = x(n), z)$ so that P_x is a matrix whose $\{m,n\}^{th}$ element is the probability that agent i who observed private signal $x(n)$ assigns to agent j observing private signal $x(m)$. To compute P_x we adapt Tauchen method for discretizing AR(1) processes (Tauchen (1986)) to our setting as follows.
   (a) Recall that $x_j | x_i \sim N\left(\delta z + (1-\delta)x_i, (\tau_x + \tau_z)^{-1} + \tau_x^{-1}\right)$, where $\delta = \frac{\tau_z}{\tau_z + \tau_x}$.
   (b) Denote by $x(1)$ is the smallest signal on the grid over signals. Then, we set
   $$P_x(1,n) = \Phi\left(\frac{x(1) - \delta z + (1-\delta)x(n)}{\sqrt{(\tau_x + \tau_z)^{-1} + \tau_x^{-1}}}\right)$$
   for each $n \in \{1, ..., n_x\}$.
   (c) Denote by $x(n_x)$ is the largest signal on the grid over signals. Then, we set
   $$P_x(n_x,n) = 1 - \Phi\left(\frac{x(n_x) - \delta z + (1-\delta)x(n)}{\sqrt{(\tau_x + \tau_z)^{-1} + \tau_x^{-1}}}\right)$$

[30] In our experimental data all signals belong to the interval $[-60, 60]$. Thus, the chosen interval includes all observed signals. Moreover, given the parameters used in experiment, conditional on observing the highest (lowest) signal, the probability attached by agent i to either θ or x_j exceeding 100 (or being lower than -100) is negligible.

for each $n \in \{1, ..., n_x\}$.

(d) For all $m \in \{2, ..., n_x - 1\}$ and all $n \in \{1, ..., n_x\}$ we set

$$P_x(m,n) = \Phi\left(\frac{x(m) - \delta z + (1-\delta)x(n)}{\sqrt{(\tau_x + \tau_z)^{-1} + \tau_x^{-1}}}\right) - \Phi\left(\frac{x(m-1) - \delta z + (1-\delta)x(n)}{\sqrt{(\tau_x + \tau_z)^{-1} + \tau_x^{-1}}}\right)$$

This concludes the discretization step. At this point we have a grid over actions, a grid over signal, and a matrix that captures each agents' beliefs about the private signal observed by the other agent. In our numerical analysis we set $n_a = n_x = 1001$ so that P_x is a 1001-by-1001 matrix.

Computing QRE We next describe how we compute logistic QRE in our discretized model. For a given z, our goal here is to find matrix P_a where $\{m,n\}^{th}$ element of this matrix, is the probability that each agent i plays action $a(m)$ conditional on observing private signal $x(n)$ given that he believes that the other agents chooses his actions according to P_a.

1. Fix $\lambda \geq 0$, the logistic parameter.

2. Keep z fixed at the same value as in the discretization step.

3. Guess P_a, the matrix of probabilities with which agents play actions on the grid for each private signal on the grid.

4. Compute the probability that agent i who observed private signal $x(n)$ assigns to player j taking action $a(l)$, and denote this probability by $P_{a_j|x}(l,n)$. This probability is given by

$$P_{a_j|x}(l,n) = \Pr(a_j = a(l) | x_i = x(n), z) = \sum_{m=1}^{n_a} P_a(l,m) P_x(m,n)$$

   Repeat this for each action $a(l)$ and each private signal $x(n)$ on the grids to obtain $P_{a_j|x}$.

5. Let $\overline{U}$ be a matrix whose $\{k,n\}^{th}$ element is agent's expected utility from taking action k conditional on observing signals $x(n)$ and z and given $P_{a_j|x}$.

Compute $\overline{U}(k,n)$, agent's expected utility from taking action $a(k)$ conditional on observing signal $x(n)$, according to

$$\overline{U}(k,n) = -(1-\alpha) E\left[(\theta - a(k))^2\right] - \alpha \sum_{l=1}^{n_a} P_{a_j}(l,n)(a(l) - a(k))^2,$$

which can be simplified to

$$\begin{aligned} \overline{U}(k,n) &= -(1-\alpha)(\tau_x + \tau_z)^{-1} + (\delta_z z + (1-\delta) x_i - a(k))^2 \\ &\quad -\alpha \sum_{l=1}^{n_a} P_{a_j}(l,n)(a(l) - a(k))^2 \end{aligned}$$

Repeat this for each action and each private signal on the grid.

6. Let $\widehat{P}_a$ be a matrix whose $\{k,n\}^{th}$ element is the probability with which an agent chooses action $a(k)$ conditional on observing private signal $x(n)$. To compute $\widehat{P}_a(k,n)$ apply the logistic choice function so that

$$\widehat{P}_a(k,n) = \frac{e^{\lambda \overline{U}(k,n)}}{\sum_{\ell=1}^{n_a} e^{\lambda \overline{U}(\ell,n)}}$$

Compute the above probability for each action and each signal on the grid.

7. Set $P_a = \widehat{P}_a$ and repeat Steps $1-5$

8. Iterate till $\max\left(\left|P_a - \widehat{P}_a\right|\right) < \varepsilon$, where the max operator is taken over all elements of $\left|P_a - \widehat{P}_a\right|$ matrix and where ε is the chosen tolerance level.

The above procedure computes the statistical response functions, P_a, that constitutes a QRE. In our implementation of the above algorithm, we set $\varepsilon = 10^{-7}$ and set initial guess to a uniform distribution over all actions for each possible value of private signal.

B.3.2 Simulate Data

Having solved numerically for QRE, the final step is to simulate agents behavior using signals observed in the experimental data. That is, our goal is to obtain simulated actions for each pair of private and public signals observed in our experiment. In what follows we consider only the observations in which subject chose precision k and believed that the pair member also chose precision level $k \in \{2,4\}$, as these were the most commonly chosen precision levels.

Let S_k be a matrix such that its first column contains to all private signals observed by subjects who chose precision k and believed that other subject also chose precision k while the second columns contains corresponding public signals that were observed by these subjects. Thus, n^{th} row of S_k is the n^{th} pair of signals $\{x_i, y\}$ observed by a subject who chose precision k and believed that other agent also chose precision k in the experimental data.

To simulate actions follow the steps outlined below.

1. Fix the logistic parameter $\lambda \geq 0$.

2. Consider the first pair $\{x_i, y\}$ that belong to $\mathcal{S}_k$.

3. Compute z and solve for P_a associated with z using the algorithm described above.

4. Find a signal on the gridpoint that is closest to x_i. Let n denote the index of that gridpoint.

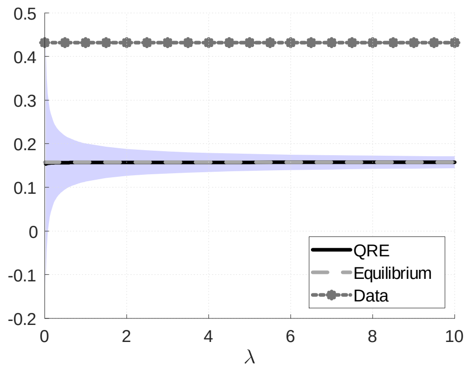
5. Use a random number generator to draw a random number, call it ξ, from a continuous uniform distribution on $[0, 1]$ and find the smallest index k such that
$$\xi < \sum_{l=1}^{k} P_a(l, n)$$
Then set $a(k)$ as the simulated action of an agent who observed signal $\{x_i, y\}$. Store this action as the first entry in a column vector A_k.

6. Continue in this fashion for each pair of signals that belong to S_k.

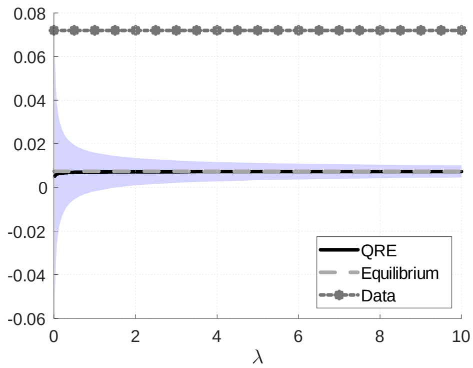
7. Regress A_k on S_k to estimate the implied weights by agents to private and public signals.

8. Repeat this N times, $N \in \mathbb{N}$.

In our simulations we set $N = 1000$ and repeat the above simulation for 60 different values of λ in the interval $[0, 10]$. Thus, we obtain 1000 estimates of weights on private and public signals implied by logistic QRE with a given parameter λ. Figure 12 is generated using this data.



(a) Precision level 2



(b) Precision level 4

Figure 12: Comparison of estimated weights using QRE for different values of λ with the theoretical and estimated weights.

B.4 Overconfidence

We model overconfidence as in Odean (1998). That is, we assume that each agent i, $i \in \{1, 2\}$, believes erroneously that the acquired precision of his signal is $\xi_i \tau_i$, with $\xi_i \geq 1$. When $\xi_i = 1$ then agent does not exhibit overconfidence and perceives precision of his signal correctly. In addition, we assume that agents perceive precision of other agents correctly.[31]

Lemma B.1 *Suppose that agent i chose precision τ_i and believes that subject j chose precision τ_j. Then for any $\widehat{\beta} \in \left[\beta^*, \overline{\beta}\right]$, where $\overline{\beta} = (1-\alpha)\frac{1+\alpha(1-\delta_j)}{1-\alpha^2(1-\delta_j)}$, there exists ξ_i such that optimal weight assigned by agent i to private signal is $\widehat{\beta}$.*

Proof. Following the same steps as in the proof of Lemma 7 one can show that the optimal weight assigned by the agent i to private signal when his overconfidence

[31] Odean (1998), motivated by the behavioral literature, assumes that agent i actually underestimates the precision of others' signal. This allows the model to capture another type of overconfidence, referred to as overplacement (overconfidence about one's performance relative to others). This additional source of overconfidence would only strengthen the case against overconfidence as a driver of our results as it would lead to shrinking of set of weight on private signal that overconfidence could justify.

level is ξ is given by

$$\beta^*(\xi_i) = (1-\alpha)(1-\delta_i(\xi)) \frac{1+\alpha(1-\delta_j)}{1-\alpha^2(1-\delta_i(\xi))(1-\delta_j)},$$

where $\delta_i(\xi) = \tau_z/(\tau_z + \xi\tau_i)$ and $\delta_j = \tau_z/(\tau_z+\tau_j)$. Note that if $\xi_i = 1$ then the above weight corresponds to the optimal weight in the baseline model. Furthermore, we have

$$\frac{\partial\beta^*(\xi_i)}{\partial\xi} = (1-\alpha)\frac{1+\alpha(1-\delta_j)}{[1-\alpha^2(1-\delta_i(\xi))(1-\delta_j)]^2}\frac{\tau_z\tau_i}{(\tau_i+\tau_z)} > 0$$

Finally,

$$\lim_{\xi_i\to\infty}\beta^*(\xi_i) = (1-\alpha)\frac{1+\alpha(1-\delta_j)}{1-\alpha^2(1-\delta_j)}$$

Setting $\overline{\beta} = (1-\alpha)\frac{1+\alpha(1-\delta_j)}{1-\alpha^2(1-\delta_j)}$ establishes the claim. ■

Constructing Table 6 To compute levels of overconfidence needed to rationalize the estimated weights using our experimental data we first check whether the estimated weight is smaller than $\overline{\beta}$. If $\alpha = 0.75$ and agent i chose precision level 2 and believes that agent j also chose precision 2 then the estimated weight exceeds $\overline{\beta}$ and, hence, there exists no value of $\xi_i \geq 1$ that can rationalize estimated weight. In all other cases, the level of overconfidence needed to rationalize particular estimated weight can be found by solving

$$\beta^*(\xi_i) = \widehat{\beta}$$

By Lemma B.1 this equation has unique solution. Table 6 reports ξ_i that solves the above equations across treatments and precision choices.

Overacquisition The next Lemma considers our specific experimental setting with public signal having standard deviation of $\sigma_y = 1$ and agents facing only four precision choices as specified in Table 1.

Lemma B.2 *Suppose that $\alpha = 0$. Then for any $\xi_i \geq 1$, agent i finds it optimal to acquire the lowest level precision.*

Proof. Using the properties of Gaussian distribution, it is straightforward to show that the perceived ex-ante utility of an overconfident agent as a function of precision choice τ_i is given by

$$\mathbb{E}[U(\tau_i)] = -(1-\delta_i(\xi))^2(\xi\tau_i)^{-1} - \delta_i^2(\xi)\tau_z^{-1} - C(\tau_i),$$

where $\delta_i = \tau_z / (\tau_z + \xi\tau_i)$ is the weight assigned by an overconfident agent to public signal in his posterior about θ belief. Simplifying the above expression we obtain

$$\mathbb{E}[U(\tau_i)] = -\frac{1}{\tau_z + \xi\tau_i} - C(\tau_i),$$

It follows that for all $\xi \geq 1$ and all τ_i and $\tau_{i'}$ such that $\tau_i < \tau_{i'}$ we have

$$\mathbb{E}[U(\tau_i)] - \mathbb{E}[U(\tau_{i'})] > -\frac{1}{\tau_z + \xi\tau_i} + [C(\tau_{i'}) - C(\tau_i)] \geq -\frac{1}{\tau_z + \tau_i} + [C(\tau_{i'}) - C(\tau_i)],$$

where the last inequality follows from the fact that $\xi \geq 1$.

Given the parameters of our model chosen for the experiment, we have $\frac{1}{\tau_z + \xi\tau_4} > 0.987$ and $C(\tau_3) - C(\tau_4) = 1.5$, $C(\tau_2) - C(\tau_4) = 5$, and $C(\tau_1) - C(\tau_4) = 11$. Thus, it follows that $\mathbb{E}[U(\tau_4)] > \mathbb{E}[U(\tau_i)]$ for all $i \in \{1, 2, 3\}$. That is precision choice four, τ_4, leads to the highest ex-ante utility for any level of overconfidence. ■

To investigate whether overconfidence can explain information overacquisition when $\alpha > 0$ we solve our model numerically. In particular, we consider a grid over possible overconfidence levels of both agents. For each agent we then choose level of overconfidence from this grid and solve the model with overconfident agents numerically. Figure 13 plots equilibrium precision choices for each of these simulations, where each equilibrium precision choice pair is denoted with different color.[32] As can be readily observed, for all pairs of $\{\xi_i, \xi_j\}$ in equilibrium agents choose to acquire signals of the lowest precision (precision level 4). Thus, Figure 13 indicates that overconfidence cannot explain overacquisition of information.

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[32] While the legend only shows symmetric equilibrium, we also checked for non-symmetric equilibria, too. We found no asymmetric equilibria for any values of ξ_i and ξ_j we considered.

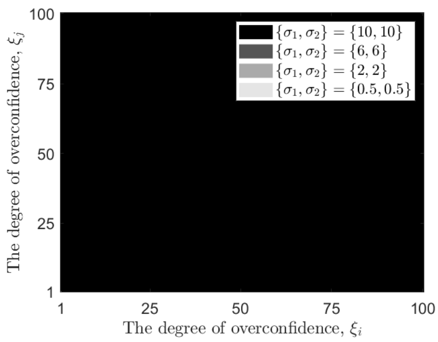

Figure 13: Equilibrium precision choices as players' degree of overconfidence varies between 1 and 100.

$\alpha = 0$		$\alpha = 0.25$		$\alpha = 0.75$	
P 2	P 4	P $\{2, 2\}$	P $\{4, 4\}$	P $\{2, 2\}$	P $\{4, 4\}$
1.34	7.32	4.80	10.62	N/A	22.99

Table 6: The degree of overconfidence implied by estimated weights across treatments for different precision choices. P k refers to precision choice by an agent when $\alpha = 0$; P $\{k, k\}$ refers to the individual precision choice and the belief about opponent's precision choice when $\alpha > 0$.

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