Informational Channels of Financial Contagion

Additional material

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1 Model with a continuum of agents in each country

There are two countries in the economy, Country 1 and Country 2, and a continuum of agents (creditors) in each country indexed by $i_n \in [0,1]$, $n = 1,2$. There are two periods and agents related to country $n = 1,2$ are active only in period n. For simplicity, I assume that countries become active in the order of their nummeraire.

Both countries use standard debt contracts to finance their debt. These contracts specify an interim stage where agents can review their investment and decide whether to roll over their loan to maturity or to withdraw their funds prematurely. Creditors from country n have funds invested in country n. Even if the country is solvent, creditors might want to withdraw their funds at the interim stage if they fear that the country may default and not repay its debt, or if they fear that other creditors might withdraw. These fears are self-fulfilling since countries are more likely to default if more creditors withdraw.

Each country is potentially fragile to default. The state of fundamentals in each country is determined by a random variable $\theta_n \in \mathbb{R}$, $n = 1,2$, that is not known to creditors and determines the level of liquidity in Country n.

The two countries are linked through fundamentals, so θ_1 and θ_2 are correlated. A high level of fundamental co-movement between these economies would lead poor fundamentals in one country to imply bad states in the other one, which would increase the probability of a default in the second country, irrespective of the information available to creditors in the second country about the behavior of creditors in the first country. To model this fundamental link, I assume that the fundamentals in Country 1 are drawn from a normal distribution with mean μ and precision τ, i.e. $\theta_1 \sim N(\mu, \tau^{-1})$. Since events in Country 2 come after events in Country 1 have occurred, fundamentals in Country 2 depend on the realization of θ_1 by setting the realization of θ_1 to be the mean of the distribution from which θ_2 is drawn, i.e. $\theta_2|\theta_1 \sim N(\theta_1, \tau_s^{-1})$. The parameter τ_s illustrates the link between fundamentals. Even if it is not strictly a measure of correlation, it has the same

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interpretation since an increase in τ_s increases the probability that the realization of θ_2 is closer to θ_1. Keeping this clarification in mind, in the remaining of the paper I will refer to τ_s as an index of correlation between fundamentals.

1.1 Actions and payoffs

In each country, domestic creditors buy securities to finance the country's government debt. The setup in each individual country follows closely the setup of Morris and Shin (2004). The financing is undertaken via a standard debt contract that specifies two different face values, depending on the time of liquidation.[1] The face value of repayment at maturity is 1 and each creditor who rolls over her loan receives this amount if the country stays solvent. If the country defaults, then creditors who rolled over their investment get zero. At an interim stage, creditors have the opportunity to review their investment. If they choose to withdraw their funds prematurely they get the lower face value of early withdrawal $\lambda_n \in (0, 1)$.

Whether Country n honors its debt at maturity or defaults depends on two factors: the underlying state of the economy, θ_n, and the proportion of agents that withdraw, l_n. The outcome for Country n at maturity will be determined by comparing the realization of the state to the proportion of withdrawing creditors:

$$\text{Country } n = \begin{cases} \text{Stays solvent} & \text{if } l_n \leq \theta_n \\ \text{Defaults} & \text{if } l_n > \theta_n \end{cases}$$

In this sense, θ_n can be thought of as fundamentals that reflect the ability of the government to meet short-term claims from creditors, or an index of liquidity.

Therefore, the payoff of a creditor in Country n is given by:

	Solvency at maturity	Default
Roll over loan	1	0
Withdraw	λ_n	λ_n

If agents knew θ_n they would act as follows: If $\theta_n \geq 1$, it would be optimal to roll over their debt, irrespective of the actions of others (in this case, rolling over always yields the high face value $1 > \lambda_n$). If $\theta_n < 0$, it is optimal to withdraw the funds at the interim stage (in this case the country always defaults and rolling over the funds would lead to a payoff of $0 < \lambda_n$). For $\theta_n \in [0, 1)$ there is a coordination problem where the optimal action depends on the beliefs about the state θ_n and about the actions of the other creditors. However, in this model agents do not observe θ_n directly, but receive noisy private and public signals about it.

[1] Two different face values for short and long term debt are also studied in Szkup (2013). However, in that model there is no possibility for contagion and the face values are endogenously determined.

1.2 Information structure and equilibrium

Recall that fundamentals in the two countries are given by

$$\begin{aligned} \theta_1 &\sim N(\mu, \tau^{-1}) \\ \theta_2|\theta_1 &\sim N(\theta_1, \tau_s^{-1}) \end{aligned}$$

and this information is common knowledge to all agents in both countries.

1.2.1 Country 1

Besides holding prior beliefs, agents in Country 1 observe noisy private signals about their payoff-relevant state, θ_1, given by

$$x_1^i \sim N(\theta_1, \tau_1^{-1})$$

where x_1^i are *iid* across $i \in [0,1]$. Based on their prior beliefs and on their private signals, creditors in Country 1 update their beliefs so that

$$\theta_1|x_1^i \sim N\left(\frac{\tau\mu + \tau_{r_1}x_1^i}{\tau + \tau_{r_1}}, (\tau + \tau_{r_1})^{-1}\right)$$

Notice that the game in Country 1 corresponds to a standard static global game. We can interpret the prior distribution of θ_1 as a public signal that reflects the level of fundamentals in Country 1 in the previous period, which determines the expectations of agents. The precision of the prior τ thus reflects the stability of Country 1, in the sense that if the economy is stable (high τ), then fundamentals in Country 1 would have small variations across periods.

I solve the game in Country 1 using the usual techniques of global games (see Morris and Shin, 2003, Hellwig, 2002, or Morris and Shin, 2004, for details). I focus on monotone strategies to solve for equilibrium in both countries. Global games are characterized by a unique equilibrium in threshold strategies under mild conditions on the noise parameters. This threshold value corresponds to the marginal signal that makes agents in Country n indifferent between withdrawing their investment or rolling it over. So the action rule followed by investors in Country $n = 1, 2$ is given by:

$$a_n\left(x_n^i; \Omega_n\right) = \begin{cases} \text{Withdraw if } x_n^i < x_n^*\left(\Omega_n\right) \\ \text{Roll over if } x_n^i \geq x_n^*\left(\Omega_n\right) \end{cases}$$

Where Ω_n is the set of noise parameters that determine the equilibrium threshold in each country. For Country 1 $\Omega_1 = \{\tau, \tau_{r_1}\}$ and for Country 2 $\Omega_2 = \{\tau, \tau_{r_1}, \tau_\alpha, \tau_s, \tau_{r_2}\}$, which is explained in detail in the following subsection.

1.2.2 Country 2

In Country 2 the structure of signals is richer. Just like in Country 1, agents in Country 2 observe private signals about the state in their own country, θ_2, given by $x_2^i \sim N(\theta_2, \tau_{r_2}^{-1})$, where x_2^i are *iid*. In addition, agents in Country 2 observe a public signal about the proportion of agents in Country 1 that withdraw their money, which is given by

$$y|\theta_1 \sim N(\Phi^{-1}(l_1), \tau_\alpha^{-1})$$

where $l_1 = \Pr(x_1^i < x_1^*) = \Phi\left(\frac{x_1^* - \theta_1}{\tau_{r_1}^{-1/2}}\right)$ is the proportion of creditors in Country 1 that withdraw their funds.[2]

For agents in Country 2 the information updating process is less straight forward than for agents in Country 1. First notice that $y \sim N\left(\frac{x_1^* - \theta_1}{\tau_{r_1}^{-1/2}}, \tau_\alpha^{-1}\right)$, which is equivalent to $y = \frac{x_1^* - \theta_1}{\tau_{r_1}^{-1/2}} + \tau_\alpha^{-1/2}\xi_y$, where $\xi_y \sim N(0,1)$. Since agents in Country 2 care about θ_1 only because it is the mean of the distribution from which θ_2 is drawn, y can be reinterpreted as a public signal about θ_1, i.e. $\theta_1 = x_1^* - \tau_{r_1}^{-1/2} y + (\tau_{r_1}\tau_\alpha)^{-1/2}\xi_y$. Agents in Country 2 do not observe the realization of θ_1, but they know the setup of the game, so their prior belief about θ_1 is the same as that of agents in Country 1, $\theta_1 \sim N(\mu, \tau_{\theta 1}^{-1})$. Therefore, the posterior belief that agents in Country 2 hold about θ_1, given that they observe signal y is given by

$$\theta_1|y \sim N\left(\widehat{\theta}_1, (\tau + \widehat{\tau}_\alpha)^{-1}\right)$$

where $\widehat{\theta}_1 = \frac{\tau\mu + \widehat{\tau}_\alpha \widehat{y}}{\tau + \widehat{\tau}_\alpha}$, $\widehat{y} = x_1^* - \tau_{r_1}^{-1/2} y$ and $\widehat{\tau}_\alpha = \tau_{r_1}\tau_\alpha$.[3] This determines the beliefs of agents in Country 2 about the distribution from which θ_2 is drawn, since $\theta_2|\theta_1 \sim N(\theta_1, \tau_s^{-1})$. Call this the posterior distribution about θ_1.

Let $\theta_2 = \theta_1 + \tau_s^{-1/2}\zeta$, where $\zeta \sim N(0,1)$, and under the posterior distribution about θ_1, let $\theta_1 = \frac{\tau\mu + \widehat{\tau}_\alpha \widehat{y}}{\tau + \widehat{\tau}_\alpha} + (\tau + \widehat{\tau}_\alpha)^{-1/2}\,\widehat{\zeta}$, where $\widehat{\zeta} \sim N(0,1)$ and ζ and $\widehat{\zeta}$ are independent. Therefore,

$$\theta_2 = \frac{\tau\mu + \widehat{\tau}_\alpha \widehat{y}}{\tau + \widehat{\tau}_\alpha} + (\tau + \widehat{\tau}_\alpha)^{-1/2}\,\widehat{\zeta} + \tau_s^{-1/2}\zeta$$

By properties of the Normal distribution, linear combinations of independent Normal random variables follow a Normal distribution as well, so we can define $\theta_2|y \sim N\left(\frac{\tau\mu + \widehat{\tau}_\alpha \widehat{y}}{\tau + \widehat{\tau}_\alpha}, \tau_s^{-1} + (\tau + \widehat{\tau}_\alpha)^{-1}\right)$, or $\theta_2|y \sim N\left(\widehat{\theta}_1, \tau_s^{-1} + (\tau + \widehat{\tau}_\alpha)^{-1}\right)$. This is effectively the "updated" distribution that agents in Country 2 hold about their payoff relevant state θ_2.

Taking this into consideration, once agents in Country 2 observe their private signals about θ_2, $x_2^i \sim N(\theta_2, \tau_{r_2}^{-1})$, their posterior belief about θ_2 is given by

$$\theta_2|x_2^i, y \sim N\left(\widehat{x}_2, \left(\left(\tau_s^{-1} + (\tau + \widehat{\tau}_\alpha)^{-1}\right)^{-1} + \tau_{r_2}\right)^{-1}\right)$$

[2] Notice that this transformation assumes monotonic strategies from the part of agents in Country 1. Therefore, I restrict attention to this type of strategies. The transformation facilitates the analysis and follows Dasgupta (2007).

[3] Notice that $\frac{d\widehat{y}}{dy} < 0$, so that $\frac{d\widehat{\theta}_1}{dy} < 0$, which implies that when agents in Country 2 observe a signal that implies a high proportion of agents in Country 1 that have withdrawn their funds, they will update their beliefs about the state in Country 1 downwards.

where $\widehat{x}_2 = \frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}\widehat{\theta}_1+\tau_{r_2}x_2^i}{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}+\tau_{r_2}}$.

In this setup the outcome in Country 1 affects the beliefs of agents in Country 2 through two channels. One is through the signal about the proportion of agents that withdraw their funds in Country 1, y, which implies that, as agents observe a signal about a higher proportion of agents that withdraw in Country 1 (higher y), agents believe that fundamentals in Country 2 are weaker, because the states are correlated (the posterior belief about θ_2 decreases). This signal incorporates a component of social learning that is not present in the standard model of global games. Moreover, the precision of this signal, τ_α, plays an important role in determining the extent to which agents in Country 2 should take it into account when updating their beliefs. We can think of this precision τ_α as reflecting the accuracy (or quality) of information transmitted between Countries 1 and 2. Therefore, y and τ_α represent the social learning channel that, depending on the conditions in the economy, might exacerbate or dampen the beliefs that agents in Country 2 hold about the probability of default in Country 2, arising from the observation of the actions of creditors in Country 1. The parameter that ultimately determines how relevant it is for agents in Country 2 to pay attention to the information related to Country 1 (the prior beliefs about θ_1 and the signal about the behavior of agents in Country 1, y) is the level of correlation between fundamentals in the two countries, which is captured by τ_s.This parameter measures purely a fundamental link between countries. Notice that these two channels are informational channels, i.e. both fundamentals and social learning lead to contagion through the information that is revealed to agents. Moreover, from a theoretical perspective the social learning signal is equivalent to a public signal observed by agents about the realization of θ_1, and the fact that it is coming from the observation of behavior does not affect the way in which agents interpret it. However, the fact that this signal is coming from observed behavior plays an important role in determining outcomes in the experiment.

We can summarize the key variables for investigating the two channels of contagion as τ_s, which reflects fundamental ties and natural co-movement between countries, and $\{y, \tau_\alpha\}$, which illustrates the social learning channel characterized by noisy observations about the behavior of agents in the first country.

1.3 Equilibrium characterization

Since agents' payoffs do not depend directly on the actions that agents in the other country take (before or after), there are no strategic considerations across periods. Therefore the problem is simplified to a series of two static global games where the outcome in the first game affects the outcome in the second one. I solve the two subgames separately and then study the effects that the outcome in Country 1 has on the outcome in Country 2. The equilibrium thresholds $\{\widehat{x}_n^*, \theta_n^*\}$, $n = 1, 2$ are found by solving simultaneously a Critical Mass condition and a Payoff Indifference condition in each country. These conditions are derived below.

1.3.1 Country 1

Since this setup corresponds to a standard global game, it is easily established that there is a unique equilibrium in monotone strategies such that agents in Country 1 roll over their loan to maturity if they observe a signal higher than a threshold x_1^*, which depends on the parameters of the model. This threshold value corresponds to the marginal signal that makes agents in Country 1 indifferent between rolling over their loan and withdrawing their funds.

Define the value of the posterior mean for which creditors are indifferent between taking either action as

$$\widehat{x}_1^* = \frac{\tau\mu + \tau_{r_1} x_1^*}{\tau + \tau_{r_1}}$$

Or equivalently, if they observe the signal:

$$x_1^* = \frac{\tau + \tau_{r_1}}{\tau_{r_1}} \widehat{x}_1^* - \frac{\tau}{\tau_{r_1}} \mu \tag{1}$$

Critical Mass condition. The critical value of the fundamental at which Country 1 is indifferent between defaulting and honoring its debt is $\theta_1 = l_1$, where l_1 is the proportion of creditors who withdraw their funds in Country 1 as a result from the switching strategy around x_1^*. Let θ_1^* be the critical state at which this happens, i.e. $\theta_1^* = l_1$. The incidence of withdrawals is given by the mass of creditors that receive a signal below the threshold x_1^*, i.e. $l_1 = \Pr(x_1 < x_1^*) = \Phi\left(\sqrt{\tau_{r_1}}(x_1^* - \theta^*)\right)$. Since $\theta_1^* = l_1$, then the Critical Mass condition (CM) is given by:

$$\begin{aligned} \theta_1^* &= \Phi\left(\sqrt{\tau_{r_1}}(x_1^* - \theta_1^*)\right) && (2) \\ &= \Phi\left(\sqrt{\tau_{r_1}}\left(\frac{\tau_{\theta 1}}{\tau_{r_1}}(\widehat{x}_1^* - \mu) + (\widehat{x}_1^* - \theta_1^*)\right)\right) && (3) \end{aligned}$$

Payoff Indifference condition. At the switching point, a creditor is indifferent between rolling over her loan and withdrawing her funds. The payoff of early withdrawal is the low face value λ_1, and the expected payoff of rolling over the loan is equal to the probability that the country stays solvent (since this payoff is normalized to 1), which happens whenever $\theta_1 > \theta_1^*$. Since the conditional density over θ_1 has mean $\widehat{x}_1^*$ and precision $\tau + \tau_{r_1}$, the Payoff Indifference (PI) condition is given by:

$$\Pr\left(\theta_1 > \theta_1^* | x_1^i\right) = \lambda_1$$

which implies

$$\widehat{x}_1^* = \theta_1^* - \frac{\Phi^{-1}(1 - \lambda_1)}{\sqrt{\tau + \tau_{r_1}}} \tag{4}$$

1.3.2 Country 2

Just as for Country 1, agents in Country 2 will roll over their loan to maturity if they observe a signal higher than a threshold x_2^*, and withdraw otherwise.

The posterior value for which creditors are indifferent between withdrawing their money or rolling over the loan until maturity is given by:

$$\widehat{x}_2^* = \frac{\left(\tau_s^{-1} + (\tau + \widehat{\tau}_\alpha)^{-1}\right)^{-1} \widehat{\theta}_1 + \tau_{r_2} x_2^*}{\left(\tau_s^{-1} + (\tau + \widehat{\tau}_\alpha)^{-1}\right)^{-1} + \tau_{r_2}} \tag{5}$$

Or equivalently, if they observe the signal:

$$x_2^* = \frac{\left[\left(\tau_s^{-1} + (\tau + \widehat{\tau}_\alpha)^{-1}\right)^{-1} + \tau_{r_2}\right]}{\tau_{r_2}} \widehat{x}_2^* - \frac{\left(\tau_s^{-1} + (\tau + \widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\tau_{r_2}} \widehat{\theta}_1 \tag{6}$$

where $\widehat{\theta}_1 = \frac{\tau\mu + \widehat{\tau}_\alpha \widehat{y}}{\tau + \widehat{\tau}_\alpha}$, $\widehat{y} = x_1^* - \tau_{r_1}^{-1/2} y$, and $\widehat{\tau}_\alpha = \tau_{r_1}\tau_\alpha$.

Critical Mass condition. Just as in the case of Country 1, the critical value of fundamentals at which Country 2 is indifferent between being solvent and defaulting is when $\theta_2 = l_2$. Let θ_2^* be the critical state at which this happens. Since the mass of creditors that receive a signal below the threshold x_2^* is given by $l_2 = \Pr(x_2 < x_2^*) = \Phi\left(\sqrt{\tau_{r_2}}(x_2^* - \theta_2^*)\right)$, the Critical Mass condition for Country 2 (CM) is given by

$$\begin{aligned} \theta_2^* &= \Phi\left(\sqrt{\tau_{r_2}}(x_2^* - \theta_2^*)\right) \\ &= \Phi\left(\sqrt{\tau_{r_2}}\left(\frac{\left(\tau_s^{-1} + (\tau + \widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\tau_{r_2}}\left(\widehat{x}_2^* - \widehat{\theta}_1\right) + (\widehat{x}_2^* - \theta_2^*)\right)\right) \end{aligned} \tag{7}$$

Payoff Indifference condition. Since the payoff of early withdrawal is λ_2, and the expected payoff of rolling over is the probability that the country honors its debt, which happens whenever $\theta_2 > \theta_2^*$, the Payoff Indifference (PI) condition for Country 2 is given by:

$$1 - \Phi\left(\sqrt{\left(\tau_s^{-1} + (\tau + \widehat{\tau}_\alpha)^{-1}\right)^{-1} + \tau_{r_2}}\,(\theta_2^* - \widehat{x}_2^*)\right) = \lambda_2 \tag{8}$$

which implies

$$\theta_2^* - \widehat{x}_2^* = \frac{\Phi^{-1}(1 - \lambda_2)}{\sqrt{\left(\tau_s^{-1} + (\tau + \widehat{\tau}_\alpha)^{-1}\right)^{-1} + \tau_{r_2}}} \tag{9}$$

Definition 1 *A pure strategy Perfect Bayesian Nash Equilibrium of the game with two countries, $n = 1, 2$, is a decision rule $a_n(x_n^i; \Omega_n)$ such that:*

$$a_n\left(x_n^i; \Omega_n\right) = \begin{cases} \textit{Withdraw if } x_n^i < x_n^*\left(x_n^i; \Omega_n\right) \\ \textit{Roll over if } x_n^i \geq x_n^*\left(x_n^i; \Omega_n\right) \end{cases}$$

where

$$
\begin{aligned}
x_1^*\left(x_1^i;\Omega_1\right) &= \frac{\tau+\tau_{r_1}}{\tau_{r_1}}\theta_1^* - \frac{\tau}{\tau_{r_1}}\mu - \frac{\sqrt{\tau+\tau_{r_1}}}{\tau_{r_1}}\Phi^{-1}\left(1-\lambda_1\right) \\
x_2^*\left(x_2^i;\Omega_2\right) &= \frac{\left[\left(\tau_s^{-1}+\left(\tau+\widehat{\tau}_\alpha\right)^{-1}\right)^{-1}+\tau_{r_2}\right]}{\tau_{r_2}}\theta_2^* - \frac{\left(\tau_s^{-1}+\left(\tau+\widehat{\tau}_\alpha\right)^{-1}\right)^{-1}}{\tau_{r_2}}\widehat{\theta}_1 \\
&\quad - \frac{\sqrt{\left(\tau_s^{-1}+\left(\tau+\widehat{\tau}_\alpha\right)^{-1}\right)^{-1}+\tau_{r_2}}}{\tau_{r_2}}\Phi^{-1}\left(1-\lambda_2\right)
\end{aligned}
$$

and θ_n^* solve:

$$
\theta_1^* = \Phi\left(\sqrt{\tau_{r_1}}\left(\frac{\tau}{\tau_{r_1}}(\widehat{x}_1^*-\mu)+(\widehat{x}_1^*-\theta_1^*)\right)\right) \tag{10}
$$

$$
\theta_2^* = \Phi\left(\sqrt{\tau_{r_2}}\left(\frac{\left(\tau_s^{-1}+\left(\tau+\widehat{\tau}_\alpha\right)^{-1}\right)^{-1}}{\tau_{r_2}}\left(\widehat{x}_2^*-\widehat{\theta}_1\right)+(\widehat{x}_2^*-\theta_2^*)\right)\right) \tag{11}
$$

for $\widehat{x}_1^* = \frac{\tau\mu+\tau_{r_1}x_1^*}{\tau+\tau_{r_1}}$, $\widehat{x}_2^* = \frac{\left(\tau_s^{-1}+\left(\tau+\widehat{\tau}_\alpha\right)^{-1}\right)^{-1}\widehat{\theta}_1+\tau_{r_2}x_2^*}{\left(\tau_s^{-1}+\left(\tau+\widehat{\tau}_\alpha\right)^{-1}\right)^{-1}+\tau_{r_2}}$, $\widehat{\theta}_1 = \frac{\tau\mu+\widehat{\tau}_\alpha\widehat{y}}{\tau+\widehat{\tau}_\alpha}$, $\widehat{y} = x_1^* - \tau_{r_1}^{-1/2}y$, $\widehat{\tau}_\alpha = \tau_{r_1}\tau_\alpha$, $\Omega_1 = \{\tau,\tau_{r_1}\}$, *and* $\Omega_2 = \{\tau,\tau_{r_1},\tau_\alpha,\tau_s,\tau_{r_2}\}$.

The following proposition presents the conditions to ensure a unique equilibrium in the model, which are analogous to those established in the global games literature (see Hellwig, 2002, and Morris and Shin, 2003).

Proposition 1 *Suppose that*

$$
\frac{\sqrt{\tau_{r_1}}}{\tau} > \frac{1}{\sqrt{2\pi}}
$$

and

$$
\frac{\sqrt{\tau_{r_2}}}{\left(\tau_s^{-1}+\left(\tau+\widehat{\tau}_\alpha\right)^{-1}\right)^{-1}} > \frac{1}{\sqrt{2\pi}}
$$

hold. Then there is a unique equilibrium of the game with two countries characterized by thresholds $\{x_1^*,\theta_1^*\}$ and $\{x_2^*,\theta_2^*\}$.

These conditions imply that in order to have a unique equilibrium, private signals have to be precise enough with respect to public information (see appendix for a proof). For Country 1 this means that private signals need to be precise enough with respect to the precision of the prior. The condition for Country 2 requires the precision of private signals, τ_{r_2}, to be higher than the precision of the public information that is composed by the strength of the fundamental link, τ_s, and by the information that agents in Country 2 possess about Country 1 (the precision of the prior about θ_1, τ, the precision of private signals in Country 1, τ_{r_1}, and the precision of the social learning signal, τ_α). This has an intuitive interpretation in terms of the model. For example, since the public signal y creates social learning, an increased precision of this signal might lead agents to rationally overreact to it and lead to multiplicity of equilibria. Therefore, in order to ensure uniqueness, we need all the components of the precision of the composed public signal to not be too high.

In a similar setup to the present paper, but where contagion is not a possibility and the analysis is equivalent to that of Country 1, Morris and Shin (2004) show that μ has important effects on the probability of default in Country 1. In particular, they show that θ_1^* is decreasing in the mean of the prior, μ. This means that a country is able to stay solvent for a wider range of fundamentals (lower θ_1^*) when creditors hold an optimistic prior about the state of the economy (higher μ).

1.3.3 Effect of introducing a signal about the behavior of agents in Country 1 on default in Country 2

The introduction of the public signal about the behavior of creditors in Country 1 captures the social learning channel of contagion and this will play an important role in the experimental results. However, before analyzing these behavioral results, we look at the effect that the introduction of this signal has on the probability of default in Country 2 from a theoretical point of view.

The noisy signal about the behavior of creditors in Country 1 determines the actions of creditors in Country 2 by affecting the posterior beliefs of agents. In general, the information structure in a global game gives rise to a unique equilibrium that is inefficient. Since θ_n^* determines the value of fundamentals for which country n defaults, as long as $\theta_n^* > 0$ there will be realizations of the fundamental where default occurs in cases where it could have been avoided. That is, when $\theta_n \in (0, \theta_n^*)$ default could in principle be avoided, but in equilibrium it occurs because creditors withdraw their funds due to self-fulfilling beliefs.

In this subsection I study the effect that the introduction of y, the signal about behavior of agents in Country 1, has on the range of fundamentals in Country 2 for which defaults are due to self-fulfilling beliefs. I compare the threshold level for fundamentals corresponding to this model, θ_2^*, where agents in Country 2 receive a social learning signal, to the threshold level that would arise if agents in Country 2 did not get any information about the actions of agents in Country 1. I refer to this threshold as $\widetilde{\theta}_2^*$. In particular, define $\widetilde{\theta}_2^*$ to be the threshold that would arise if the only information held by agents in Country 2 was be the public information composed by:

$$\begin{aligned} \theta_1 &\sim N(\mu, \tau^{-1}) \\ \theta_2 &\sim N(\theta_1, \tau_s^{-1}) \end{aligned}$$

And the private signals:

$$\widetilde{x}_2^i \sim N(\theta_2, \tau_{r_2}^{-1})$$

Using the same logic as before, I derive the PI and CM conditions to solve for the equilibrium thresholds $\widetilde{x}_2^*$ and $\widetilde{\theta}_2^*$. In equilibrium, $\widetilde{\theta}_2^*$ is defined by the following expression:

$$\widetilde{\theta}_2^* = \Phi\left(\left(\frac{\left(\tau_s^{-1}+\tau^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\left(\widetilde{\theta}_2^* - \mu - \frac{\sqrt{\left(\tau_s^{-1}+\tau^{-1}\right)^{-1}+\tau_{r_2}}}{\left(\tau_s^{-1}+\tau^{-1}\right)^{-1}}\Phi^{-1}\left(1-\lambda_2\right)\right)\right)\right) \tag{12}$$

Similar to the previous cases, in order to ensure a unique equilibrium we assume that $\frac{\sqrt{\tau_{r_2}}}{\left(\tau_s^{-1}+\tau^{-1}\right)^{-1}} > \frac{1}{\sqrt{2\pi}}$.

To understand the effect that the introduction of the signal about the proportion of withdrawing agents in Country 1, y, has on the probability of default in Country 2, we need to compare θ_2^* and $\widetilde{\theta}_2^*$. However, it is not possible to derive conclusive results for a wide range of parameters analytically, so I focus on results based on numerical simulations.[4] The effect of introducing signal y on the probability of default in Country 2 is found to depend heavily on prior beliefs. In particular, if agents have an optimistic prior (high μ), then in general $\theta_2^* > \widetilde{\theta}_2^*$, unless there is a very low realization of y, i.e. if agents have an optimistic prior about the state of the economy, introducing a noisy signal about the behavior of agents in Country 1 will increase the probability of default in Country 2, unless the realization of y is very low. This means that the introduction of this signal will in general make agents more hesitant to roll over and thus reduces the range of states for which Country 2 stays solvent.

On the other hand, if agents in Country 2 have pessimistic prior beliefs about the state of the economy, then $\theta_2^* < \widetilde{\theta}_2^*$, unless there is a very high realization of y. This means that when agents have a pessimistic prior, introducing a signal about the behavior of agents in Country 1 leads to a decrease on the probability of default in Country 2, unless they observe a very high realization of y. This means that the same signal realization can lead to more or less default, depending on the type of expectations held about the fundamental.

The strength of these results depends on the precision of y (τ_α) and on the correlation between states (τ_s).

As I will show in the next subsection, prior beliefs will also play an important role when analyzing comparative statics.

1.4 Comparative statics

We now turn our attention to understand how variations in the strength of the fundamental and social learning channels affect the probability of contagion across countries.[5] The two channels of contagion that have been outlined in the paper are related -albeit in different ways- to public information held by agents in Country 2. In this sense, we can refer to them as informational channels. The comparative statics with respect to the fundamental

[4] The algebraic expressions to study these results are not included in the appendix, but they are available from the author by request.

[5] In the first section of the appendix I study the effects that different parameters of the model have on the probability of default of each specific country. These parameters are the precision of private signals, τ_n, the mean of the prior in Country 1, μ_θ, the precision of the prior for Country 1, τ_{θ_1}, and the payoff of early withdrawal, λ_n, for $n = 1, 2$. These are basic comparative statics results that are usually performed for this type of models and that allow us to better understand the forces in the model.

channel characterize changes in the probability of default in Country 2 due to a change in the strength of the correlation between fundamentals, which is captured by τ_s. The comparative statics with respect to the social learning channel illustrate how the probability of default in Country 2 is affected when agents in Country 2 observe a signal about a higher proportion of agents that withdraw their funds in Country 1 (y), and by changes in the precision of this signal (τ_α). I focus on the effects on the probability of default in Country 2, measured by changes in θ_2^*. In particular, since default occurs for $\theta_2 < \theta_2^*$, an increase (decrease) in θ_2^* implies a larger (smaller) range of values of θ_2 for which Country 2 defaults. In this section I assume that the conditions for uniqueness of equilibrium hold. All proofs are relegated to the appendix.

The following remark presents the results for the fundamental channel of contagion.

Remark 1 *1. If the probability of default in Country* 2 *is low (low* θ_2^**) and agents have an optimistic prior about the state of the economy (high* $\widehat{\theta}_1$*), then a higher correlation between Country* 1 *and Country* 2 *(i.e. a higher precision* τ_s*) will further decrease the probability of default in Country* 2*.*

2. If the probability of default in Country 2 *is high (high* θ_2^**) and agents have a pessimistic prior about the state of the economy (low* $\widehat{\theta}_1$*), then a higher correlation between Country* 1 *and Country* 2 *(i.e. a higher precision* τ_s*) will increase the probability of default in Country* 2*.*

This result has a very intuitive interpretation. When agents have an optimistic prior about fundamentals in Country 2, they are optimistic about the realization of fundamentals in Country 1. Therefore, when agents in Country 2 hold an optimistic prior about the state in Country 2, a higher correlation between fundamentals, characterized by a higher τ_s, implies that agents in Country 2 assign a higher weight to these optimistic beliefs and this further decreases the probability of default in Country 2. This illustrates the positive effects of fundamental links in contagion. On the other hand, agents have a pessimistic prior about the state in Country 2 when they believe that the realized state in Country 1 was not good, so in this case a higher correlation between fundamentals in both countries will lead them to assign a higher weight to these pessimistic beliefs, which leads to an increase in the probability of default in Country 2. This illustrates the negative effects of increased fundamental links in the propagation of crises through contagion.

To analyze the social learning channel of contagion we look at the effect that the signal about the proportion of agents that withdraw their funds in Country 1, y, and its precision, τ_α, have on the probability of default in Country 2.

Remark 2 *A higher signal about the proportion of agents that withdraw their funds in Country* 1*,* y*, increases the probability of default in Country* 2*.*

This could be thought of as a first order effect of the social learning channel of contagion since it is related to changes in the magnitude of the signal about the actions of the agents in Country 1. To understand this point further, I investigate how this effect is determined by the precision of y, τ_α, by taking the second derivative $\frac{d^2\theta_2^*}{d\eta dy}$. However, due to the lack of an analytical characterization, I use numerical simulations to understand this result. Numerical simulations suggest that the effect of y on the probability of default in Country 2, characterized by θ_2^*, will be stronger as the precision of y, measured by τ_α, increases, for most parameter values. The only situation where the opposite effect is found is when μ is very high and y is even higher. This, however, is an unlikely scenario since, as we have established, a higher μ leads to a lower probability of agents in Country 1 withdrawing their money (a lower x_1^*). This, in turn, implies that agents in Country 2 will in general observe signals about the proportion of agents that withdraw their funds in Country 1 of lower magnitude (low realizations of y). However, there is a non-zero probability of this type of situation occurring (a high μ accompanied by an even higher y) since the support of the normal distribution of y is infinite. This could also happen, for example, if the variance of the distribution of y is very large so that the signal y is so noisy that even if the proportion of agents in Country 1 who withdraw is low, agents in Country 2 might observe a very high y.

Effect of an increase in τ_α on the probability of default in Country 2. To analyze the other path of the social learning channel of contagion, we take a step back to decompose the notion of optimistic (pessimistic) prior beliefs about the state of Country 2. On the one hand, τ_α, just like τ_s, is a component of the precision of the posterior or expected distribution of θ_2, denoted by $\left(\tau_s^{-1} + (\tau + \tau_{r_1}\tau_\alpha)^{-1}\right)^{-1}$. Therefore, just like τ_s, the effect arising from changes in τ_α on the probability of default, θ_2^*, will depend on whether beliefs about θ_2 are optimistic or pessimistic. However, the total effect of changes in τ_α on θ_2^* is more complex than that of τ_s, since a change in τ_α also affects the expected (or posterior) mean of the distribution of θ_2, whose mean, $\widehat{\theta}_1 = \frac{\tau\mu + \widehat{\tau}_\alpha \widehat{y}}{\tau + \widehat{\tau}_\alpha}$, determines whether beliefs are optimistic or pessimistic. This means that there are two effects that might go in different directions. The first effect makes agents put more weight on the mean of the prior by increasing the precision of the composed public signal and is called a "coordination effect", since it enhances coordination by aligning posterior beliefs across agents (this is the effect that is also common to the fundamental link through τ_s). I call the second an "information effect" since it changes the level of the expected or posterior mean of the distribution of θ_2, thus affecting the type of beliefs that agents hold. Therefore, an increase in the precision τ_α will, on the one hand, lead to a similar impact on θ_2^* as an increase in τ_s (it will either increase or decrease the probability of default depending on whether agents have a pessimistic or an optimistic prior about θ_2), but the final effect will actually depend on how τ_α affects this pessimism or optimism of agents through its impact on $\widehat{\theta}_1$. So variations in τ_α might actually change prior beliefs about θ_2 by changing whether agents are ex-ante optimistic or pessimistic, and depending on the outcome on these beliefs, we would have "new" prior beliefs about θ_2 that will determine the direction of the coordination effect. This implies that, in certain cases, an increase in τ_α might lead ex-ante beliefs to switch from optimism to pessimism (or vice versa), which would have very different implications on the probability of default in Country 2. In the first section of the appendix I derive the expression for the derivative of θ_2^* with respect to τ_α, however, it is not possible to draw intuitive conclusions from this expression. Numerical results indicate that if prior beliefs about θ_1 are pessimistic (low μ), then an increase in τ_α leads to a decrease in the probability of default in Country

2 if y is low, since an increase in the precision of a low y makes agents more optimistic, or to an increase in the probability of default in Country 2 if y is high, since an increase in the precision of a high y confirms the agents' pessimism. On the other hand, if agents have an optimistic prior about θ_1 (high μ) then an increase in τ_α leads to an increase in the probability of default in Country 2, since a positive proportion of withdrawals is always bad news, so an increase in the precision of this signal makes agents more pessimistic. As we can see, the information effect seems to be strong enough that, in some cases, it causes agents to switch from being optimistic to pessimistic (or vice versa). The precise magnitude of this effect depends on the parameters of the model.

2 Alternative estimation method

Below I reproduce the main regression table from the paper (Table 8) using a different estimation method. Instead of using a logistic specification, Table 1 presents the results of random-effects linear probability models for each treatment. The dependent variable is the probability of rolling over and the independent variables are the private signals x_2^i, the public signal about the number of agents that rolled over in Country 1, $y_{rollover}$, a dummy variable d_{prior} that takes a value of 0 for an induced optimistic prior and a value of 1 for an induced pessimistic prior; and two interacted terms of this dummy, one with the private signal (d_{prior}*x_2^i) and the other with the public signal (d_{prior}*y_{roll}) to account for any additional variation of the signals x_2^i and $y_{rollover}$, respectively, under a different prior. Location dummies are also included in all specifications to account for possible differences in behavior across the two locations where the experiment was run. The five specifications differ in the combination of the parameters (s, α) that define each treatment. In each of these specifications I pool the data from sessions where an optimistic and a pessimistic prior were induced. I test whether there is a significant difference in behavior under these priors by looking at the coefficient of the dummy d_{prior} and its interacted terms. The coefficients in bold indicate departures, in terms of significance, from the expected results stated in Hypothesis 1. These departures occurred either because coefficients that should be significant are not, or because coefficients that should not be significant are significant. Table 1 is equivalent to Table 8 from the paper. The coefficients in Table 1 should be compared to the coefficients in Table 12 in the paper, which reports the marginal effects of these regressions. In the linear probability models we can also identify an overreaction to social information in specifications 1-3, consistent with the social imitation bias. The lack of significance of the prior dummy in Specification 4 suggests base-rate neglect. However, the coefficient for the prior dummy in Specification 2 is slightly significant (at the 10% level), suggesting that maybe the prior was taken into account for some decisions. These results are consistent with the findings of the paper that characterize the overreaction bias as being more robust and persistent than the base-rate neglect bias, which disappears if subjects play the game for long periods of time or if we frame the social signal as a standard public signal.

	1	2	3	4	5
	$s=1/3$, $\alpha=1/3$	$s=3/4$, $\alpha=1/3$	$s=1/3$, $\alpha=3/4$	$s=3/4$, $\alpha=3/4$	$s=1$, $\alpha=1$
x_2^i	0.343***	0.336***	0.351***	0.257***	0.202***
	(0.038)	(0.04)	(0.037)	(0.034)	(0.032)
$y_{rollover}$	**0.058****	**0.043***	**0.09*****	0.151***	0.098**
	(0.029)	(0.025)	(0.025)	(0.039)	(0.04)
d_{prior}	-0.024	-0.167*	-0.101	**-0.14**	-0.46***
(0 opt, 1 pess)	(0.117)	(0.095)	(0.1)	(0.12)	(0.096)
$d_{prior}{*}x_2^i$	-0.03	**0.072**	0.013	0.097*	0.107***
	(0.054)	(0.05)	(0.047)	(0.052)	(0.04)
$d_{prior}{*}y_{roll}$	0.045	0.023	**0.072****	**-0.013**	0.146***
	(0.04)	(0.04)	(0.036)	(0.05)	(0.053)
$location$	0.086	0.096	-0.036	0.033	-0.331***
(0 NYU, 1 UCSD)	(0.118)	(0.12)	(0.098)	(0.103)	(0.114)
$x_2^i * location$	-0.006	-0.085	0.007	-0.023	0.065
	(0.056)	(0.053)	(0.045)	(0.049)	(0.046)
$y_{rollover} * location$	-0.029	-0.011	0.004	-0.006	0.112*
	(0.034)	(0.041)	(0.037)	(0.052)	(0.061)
$d_{prior} * location$	-0.034	0.001	-0.039	-0.066	0.359***
	(0.163)	(0.138)	(0.122)	(0.15)	(0.131)
$d_{prior}{*}x_2^i * location$	0.03	-0.013	-0.009	-0.018	-0.047
	(0.079)	(0.073)	(0.064)	(0.072)	(0.061)
$d_{prior}{*}y_{roll} * location$	-0.032	-0.016	0.01	0.013	-0.16**
	(0.059)	(0.063)	(0.061)	(0.069)	(0.077)
C	0.238***	0.305***	0.205**	0.284***	0.434***
	(0.076)	(0.087)	(0.08)	(0.085)	(0.077)
N	1600	1800	1858	1756	1836

Clustered (by subject) standard errors in parentheses
* significant at 10%; ** 5%; *** 1%

Table 1: Linear probability estimates of information taken into account for individual actions, by treatment

References

[1] Carlsson, H. and E. van Damme (1993) "Global games and equilibrium selection" *Econometrica* 61(5): 989–1018.

[2] Dasgupta, A. (2007) "Coordination and delay in global games" *Journal of Economic Theory*, 134(1): 195-225.

[3] Hellwig, C. (2002) "Public Information, Private Information, and the Multiplicity of Equilibria in Coordination Games" *Journal of Economic Theory*, 107(2): 191-222.

[4] Manz, M. (2010) "Information-based contagion and the implications for financial fragility" *European Economic Review*, 54(7): 900-910.

[5] Morris, S. and H. S. Shin (1998) "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks" *American Economic Review*, 88(3): 587–597.

[6] ——————— (2003) "Global Games: Theory and Applications" in *Advances in Economics and Econometrics* (Proceedings of the Eighth World Congress of the Econometric Society), M. Dewatripont, L. Hansen and S. Turnovsky, eds., Cambridge University Press.

[7] ——————— (2004) "Coordination Risk and the Price of Debt" *European Economic Review,* 48(1): 133-153.

[8] Szkup, M. (2013) "Optimal Debt Maturity Structure, Rollover Risk and Strategic Uncertainty", mimeo.0.

3 Appendix

Proposition 1 *Suppose that*

$$\frac{\sqrt{\tau_{r_1}}}{\tau} > \frac{1}{\sqrt{2\pi}}$$

and

$$\frac{\sqrt{\tau_{r_2}}}{\left(\tau_s^{-1} + \left(\tau + \widehat{\tau}_\alpha\right)^{-1}\right)^{-1}} > \frac{1}{\sqrt{2\pi}}$$

hold. Then there is a unique equilibrium of the game with two countries characterized by thresholds $\{x_1^*, \theta_1^*\}$ *and* $\{x_2^*, \theta_2^*\}$*.*

Proof. I first focus on Country 1, and then in Country 2. For equilibrium, we need to solve simultaneously the Payoff Indifference and Critical Mass conditions from equations 2 and 4. In order to have a unique equilibrium, there needs to be a unique solution for $(\widehat{x}_1^*, \theta_1^*)$. Substituting $\widehat{x}_1^*$ in equation 2 and solving for θ_1^*:

$$\theta_1^* = \Phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu - \Phi^{-1}\left(1-\lambda_1\right)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right) \tag{13}$$

To ensure a unique solution for θ_1^*, the right hand side of equation 13 needs to have a slope smaller than one everywhere. As has been shown in the global games literature (see Hellwig, 2002, Morris and Shin, 2003), this is achieved by imposing certain restrictions on the noise parameters. In particular, the slope of the right hand side of equation 13 needs to be less than 1, i.e. $\frac{\tau}{\sqrt{\tau_{r_1}}}\phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu - \Phi^{-1}\left(1-\lambda_1\right)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right) < 1$. Since $\phi(\cdot) \leq \frac{1}{\sqrt{2\pi}}$ everywhere, then it is sufficient to impose that $\frac{\sqrt{\tau_{r_1}}}{\tau} > \frac{1}{\sqrt{2\pi}}$.

I now solve for equilibrium in Country 2. To solve for equilibrium, from equations 7 and 8 I solve for θ_2^* and $\widehat{x}_2^*$ simultaneously. Substituting equation **??** into the CM condition we get:

$$\theta_2^* = \Phi\left(\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}+\tau_{r_2}}}{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}\Phi^{-1}(1-\lambda_2)\right)\right) \tag{14}$$

In order to ensure a unique solution for θ_2^*, the right hand side of equation 14 needs to have a slope smaller than one everywhere. A sufficient condition for this to happen is to set

$$\begin{aligned}
&\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\times \\
&\phi\left(\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}+\tau_{r_2}}}{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}\Phi^{-1}(1-\lambda_2)\right)\right) \\
<\ & 1
\end{aligned}$$

Since $\phi(\cdot) \leq \frac{1}{\sqrt{2\pi}}$ then it is sufficient to impose that $\frac{\sqrt{\tau_{r_2}}}{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}} > \frac{1}{\sqrt{2\pi}}$. ■

3.0.1 Effect of introducing a signal about the behavior in Country 1 on default in Country 2

Define $\widetilde{\theta}_2^*$ to be the threshold that would arise if the only information held by agents in Country 2 was be the public information composed by:

$$\begin{aligned}
\theta_1 &\sim N(\mu, \tau^{-1}) \\
\theta_2 &\sim N(\theta_1, \tau_s^{-1})
\end{aligned}$$

And the private signals:

$$\widetilde{x}_2^i \sim N(\theta_2, \tau_{r_2}^{-1})$$

In this case, Bayesian updating would lead agents in Country 2 to believe

$$\theta_2|\widetilde{x}_2^i \sim N\left(\frac{\left(\tau_s^{-1}+\tau^{-1}\right)^{-1}\mu + \tau_{r_2}x_2^i}{\left(\tau_s^{-1}+\tau^{-1}\right)^{-1}+\tau_{r_2}}, \left(\left(\tau_s^{-1}+\tau^{-1}\right)^{-1}+\tau_{r_2}\right)^{-1}\right)$$

To find equilibrium, define the posterior value for which creditors are indifferent between withdrawing their money or rolling over the loan until maturity as:

$$\widetilde{\widetilde{x}}_2^* = \frac{\left(\tau_s^{-1}+\tau^{-1}\right)^{-1}\mu + \tau_{r_2}\widetilde{x}_2^*}{\left(\tau_s^{-1}+\tau^{-1}\right)^{-1}+\tau_{r_2}}$$

Or equivalently, if they observe the signal:

$$\widetilde{x}_2^* = \frac{\left[\left(\tau_s^{-1}+\tau^{-1}\right)^{-1}+\tau_{r_2}\right]}{\tau_{r_2}}\widetilde{\widetilde{x}}_2^* - \frac{\left(\tau_s^{-1}+\tau^{-1}\right)^{-1}\mu}{\tau_{r_2}}$$

The CM condition is then given by:

$$\widetilde{\theta}_2^* = \Phi\left(\sqrt{\tau_{r_2}}\left(\frac{\left(\tau_s^{-1}+\tau^{-1}\right)^{-1}}{\tau_{r_2}}\left(\widehat{\widetilde{x}}_2^* - \mu\right) + \left(\widehat{\widetilde{x}}_2^* - \widetilde{\theta}_2^*\right)\right)\right)$$

And the PI condition is:

$$1 - \Phi\left(\sqrt{\left(\tau_s^{-1}+\tau^{-1}\right)^{-1} + \tau_{r_2}}\left(\widetilde{\theta}_2^* - \widehat{\widetilde{x}}_2^*\right)\right) = \lambda_2$$

Putting the CM and PI conditions together and solving for $\widehat{\widetilde{x}}_2^*$ and $\widetilde{\theta}_2^*$ simultaneously to find equilibrium, we get equation 12 from the main text. Similar to the previous cases, in order to ensure a unique equilibrium we assume that $\frac{\sqrt{\tau_{r_2}}}{\left(\tau_s^{-1}+\tau^{-1}\right)^{-1}} > \frac{1}{\sqrt{2\pi}}$.

3.0.2 Comparative statics

This section presents a series of remarks about comparative statics that do not affect directly the strength of the two channels of contagion. These comparative statics correspond to the effect that the precision of private signals, τ_n, has on the probability of default in Country $n = 1, 2$, the effect that the mean and the variance of the prior about the state in Country 1, μ and τ respectively, have on the probability of default in Country 1, and the effect that the payoff of early withdrawals, λ_n, has on the probability of default in Country $n = 1, 2$.

For $n = 1, 2$, the following hold:

Remark A 1 *1. If the probability of default in Country n is low and agents have an optimistic prior about the state of the economy, then more precise private information, τ_n, will lead to a higher threshold x_n^* (i.e. to a higher incidence of withdrawal) and to an increase in the probability of default in Country n.*

2. If the probability of default in Country n is high and agents have a pessimistic prior about the state of the economy, then more precise private information, τ_n, will lead to a lower threshold x_n^* (i.e. to a lower probability of withdrawal) and to a decrease in the probability of default in Country n.

Proof. I first analyze the results for Country 1. Notice that

$$\frac{dx_1^*}{d\tau_{r_1}} = -\frac{\tau}{\tau_1^2}\theta_1^* + \frac{\tau}{\tau_1^2}\mu + \frac{\Phi^{-1}\left(1-\lambda_1\right)\left(\tau + \frac{1}{2}\tau_{r_1}\right)}{\tau_1^2\sqrt{\tau+\tau_{r_1}}}$$

So when $\theta_1^* < \mu + \frac{\Phi^{-1}(1-\lambda_1)\left(\tau+\frac{1}{2}\tau_{r_1}\right)}{\tau\sqrt{\tau+\tau_{r_1}}}$ i.e. when default is not very likely to occur and agents have an optimistic prior about the state of the economy, then a higher precision of the private signal will lead to a higher threshold x_1^*, and thus to a higher incidence of withdrawal. On the other hand, when $\theta_1^* > \mu + \frac{\Phi^{-1}(1-\lambda_1)\left(\tau+\frac{1}{2}\tau_{r_1}\right)}{\tau\sqrt{\tau+\tau_{r_1}}}$, i.e. when default is likely to occur and agents have a pessimistic prior about the state of the economy, then a higher precision of the private signal will lead to a lower threshold x_1^*, which effectively means a lower probability of withdrawal.

The effects of an increased precision of the private signal on the probability of default, θ_1^* are consistent with the previous result, since

$$\begin{aligned}
\frac{d\theta_1^*}{d\tau_{r_1}} &= \phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu - \Phi^{-1}\left(1-\lambda_1\right)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right)\times \\
&\left[-\frac{1}{2}\frac{\tau}{\left(\tau_{r_1}\right)^{3/2}}\left(\theta_1^* - \mu - \Phi^{-1}\left(1-\lambda_1\right)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right) - \frac{1}{2}\Phi^{-1}\left(1-\lambda_1\right)\frac{1}{\sqrt{\tau_{r_1}}\sqrt{\tau+\tau_{r_1}}} + \frac{\tau}{\sqrt{\tau_{r_1}}}\frac{d\theta_1^*}{d\tau_{r_1}}\right] \\
&= \frac{1}{2}\frac{-\phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu - \Phi^{-1}\left(1-\lambda_1\right)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right)\left[\frac{\tau}{\left(\tau_{r_1}\right)^{3/2}}\left(\theta_1^* - \mu - \Phi^{-1}\left(1-\lambda_1\right)\frac{\tau}{\tau\sqrt{\tau+\tau_{r_1}}}\right)\right]}{1-\phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu - \Phi^{-1}\left(1-\lambda_1\right)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right)\frac{\tau}{\sqrt{\tau_{r_1}}}}
\end{aligned}$$

To determine whether $\frac{d\theta_1^*}{d\tau_{r_1}}$ is positive or negative, we need to sign the term

$$\left[\frac{\tau}{\left(\tau_{r_1}\right)^{3/2}}\left(\theta_1^* - \mu - \Phi^{-1}\left(1-\lambda_1\right)\frac{\tau}{\tau\sqrt{\tau+\tau_{r_1}}}\right)\right]$$

If $\theta_1^* < \mu + \Phi^{-1}\left(1-\lambda_1\right)\frac{\tau}{\tau\sqrt{\tau+\tau_{r_1}}}$, then $\frac{d\theta_1^*}{d\tau_{r_1}} > 0$, i.e. if agents have an optimistic prior about the state of the economy and θ_1^* is low enough, i.e. default is not very likely to occur, then more precise private information will increase θ_1^*, which increases the probability of default

Alternatively, if $\theta_1^* > \mu + \Phi^{-1}\left(1-\lambda_1\right)\frac{\tau}{\tau\sqrt{\tau+\tau_{r_1}}}$, then $\frac{d\theta_1^*}{d\tau_{r_1}} < 0$, so that if agents have a pessimistic prior and θ_1^* is high enough (i.e. default is very likely to occur), then more precise information will decrease θ_1^*, thus decreasing the probability of a default. This means that when agents are pessimistic, having a more precise signal will lead them to put more weight on it, thus decreasing the probability of a default.

I perform the same analysis for Country 2. From equations **??** and **??** we can write x_2^* as

$$x_2^* = \frac{\left(\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}\right)}{\tau_{r_2}}\theta_2^* - \frac{\widehat{\widehat{\tau}}_\alpha}{\tau_{r_2}}\widehat{\theta}_1 - \frac{\Phi^{-1}\left(1-\lambda_2\right)\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}{\tau_{r_2}}$$

where $\widehat{\widehat{\tau}}_\alpha = \left(\tau_s^{-1} + \left(\tau + \widehat{\tau}_\alpha\right)^{-1}\right)^{-1}$. Therefore,

$$
\begin{aligned}
\frac{dx_2^*}{d\tau_{r_2}} &= \frac{\tau_{r_2}-\widehat{\widehat{\tau}}_\alpha-\tau_{r_2}}{\tau_{r_2}^2}\theta_2^*+\frac{\widehat{\widehat{\tau}}_\alpha}{\tau_{r_2}^2}\widehat{\theta}_1-\frac{\frac{1}{2}\tau_{r_2}\left(\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}\right)^{-1/2}-\sqrt{\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}}}{\tau_{r_2}^2}\Phi^{-1}\left(1-\lambda_2\right)\\
&= -\frac{\widehat{\widehat{\tau}}_\alpha}{\tau_{r_2}^2}\theta_2^*+\frac{\widehat{\widehat{\tau}}_\alpha}{\tau_{r_2}^2}\widehat{\theta}_1+\frac{\Phi^{-1}\left(1-\lambda_2\right)\left(\frac{1}{2}\tau_{r_2}+\widehat{\widehat{\tau}}_\alpha\right)}{\tau_{r_2}^2\sqrt{\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}}}
\end{aligned}
$$

So when $\theta_2^*<\widehat{\theta}_1+\frac{\Phi^{-1}(1-\lambda_2)\left(\frac{1}{2}\tau_{r_2}+\widehat{\widehat{\tau}}_\alpha\right)}{\widehat{\widehat{\tau}}_\alpha\sqrt{\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}}}$ i.e. when default is not very likely to occur and agents are ex-ante optimistic about the state of the economy, then a higher precision of the private signal will lead to a higher threshold x_1^*, and thus to a higher incidence of withdrawal. On the other hand, when $\theta_2^*>\widehat{\theta}_1+\frac{\Phi^{-1}(1-\lambda_2)\left(\frac{1}{2}\tau_{r_2}+\widehat{\widehat{\tau}}_\alpha\right)}{\widehat{\widehat{\tau}}_\alpha\sqrt{\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}}}$, i.e. when default is likely to occur and agents are ex-ante pessimistic about the state of the economy, then a higher precision of the private signal will lead to a lower threshold x_2^*, which effectively means a lower probability of withdrawal.

Similarly, the effect on the probability of default in Country 2 given an increase in the precision of private signals τ_{r_2} is the following:

$$
\begin{aligned}
\frac{d\theta_2^*}{d\tau_{r_2}} &= \phi\left(\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\theta_2^*-\widehat{\theta}_1-\frac{\sqrt{\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}\left(1-\lambda_2\right)\right)\right)\times\\
&\quad\left[-\frac{1}{2}\frac{\widehat{\widehat{\tau}}_\alpha}{\tau_{r_2}^{3/2}}\left(\theta_2^*-\widehat{\theta}_1-\frac{\sqrt{\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}\left(1-\lambda_2\right)\right)+\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\frac{d\theta_2^*}{d\tau_{r_2}}-\frac{1}{2}\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\frac{\left(\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}\right)^{-1/2}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}\left(1-\lambda_2\right)\right]\\
&= \frac{1}{2}\frac{-\phi\left(\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\theta_2^*-\widehat{\theta}_1-\frac{\sqrt{\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}\left(1-\lambda_2\right)\right)\right)\left[\frac{\widehat{\widehat{\tau}}_\alpha}{\tau_{r_2}^{3/2}}\left(\theta_2^*-\widehat{\theta}_1-\frac{\widehat{\widehat{\tau}}_\alpha}{\widehat{\widehat{\tau}}_\alpha\sqrt{\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}}}\Phi^{-1}\left(1-\lambda_2\right)\right)\right]}{1-\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\phi\left(\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\theta_2^*-\widehat{\theta}_1-\frac{\sqrt{\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}\left(1-\lambda_2\right)\right)\right)}
\end{aligned}
$$

where $\widehat{\widehat{\tau}}_\alpha=\left(\tau_s^{-1}+\left(\tau+\widehat{\tau}_\alpha\right)^{-1}\right)^{-1}$.

To determine whether $\frac{d\theta_2^*}{d\tau_{r_2}}$ is positive or negative, we need to sign the term

$$
\left[\frac{\widehat{\widehat{\tau}}_\alpha}{\tau_{r_2}^{3/2}}\left(\theta_2^*-\widehat{\theta}_1-\frac{\widehat{\widehat{\tau}}_\alpha}{\widehat{\widehat{\tau}}_\alpha\sqrt{\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}}}\Phi^{-1}\left(1-\lambda_2\right)\right)\right]
$$

If $\theta_2^*<\widehat{\theta}_1+\frac{\widehat{\widehat{\tau}}_\alpha}{\widehat{\widehat{\tau}}_\alpha\sqrt{\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}}}\Phi^{-1}\left(1-\lambda_2\right)$, then $\frac{d\theta_1^*}{d\tau_{r_1}}>0$, i.e. if default is not likely to occur (i.e. θ_2^* is low enough) and agents' public signals make them are optimistic about the state of the economy, then more precise private information will increase θ_1^*, which increases the probability of default.

Alternatively, if $\theta_2^* > \widehat{\theta}_1 + \frac{\widehat{\widehat{\tau}}_\alpha}{\widehat{\widehat{\tau}}_\alpha \sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}} \Phi^{-1}(1-\lambda_2)$, then $\frac{d\theta_1^*}{d\tau_{r_1}} < 0$, so that if default is very likely to occur (i.e. θ_2^* is high enough) and agents are pessimistic (low $\widehat{\theta}_1$), then more precise information will lead agents to assign a higher weight on their private information, thus giving a lower weight on their initial pessimistic beliefs about the state, which decreases the probability of a default by decreasing θ_2^*. The intuition for this result is the following. Creditors use both private and public information to assess whether they should withdraw their funds or roll over their loans. In order to roll over their loans, they need to make sure that fundamentals are in a good state and that other agents will not withdraw their funds. Thus, in intermediate states, a creditor wants to coordinate her action with the others to either roll over their debt and avoid a default, or to withdraw her funds early and provoke the country to default. Private signals have a direct incentive on the coordination effect, so the higher the precision of the private signal, τ_n, the more likely it is for creditors to coordinate because their information sets will be more aligned. In addition, a higher precision of the private signal increases the weight that creditors assign to it, thus decreasing the weight given to public information. Therefore, when creditors have an optimistic prior about the state of the economy and believe that default is not very likely to occur, creditors refrain from withdrawing their funds because they know that the probability of default is small. However, an increase in the precision of their private signal will lead them to put less weight on their prior belief that the state is good, thus increasing the individual probability of withdrawal (by increasing their threshold x_n^*), which also increases the probability of default with respect to the case of a lower precision of private signals. This means that when agents have an optimistic prior, a higher precision of private information might lead them to withdraw their funds more often with respect to what they would have done if they had just followed their initial optimistic beliefs. A similar logic applies to the case where agents have a pessimistic prior about the state of the economy and believe that the probability of a default is high. These results are consistent with those presented by Metz (2002) in a similar setup. ■

Remark A 2 *In Country* 1*, the public signal* μ *decreases the probability of a default.*

Proof.

$$\frac{d\theta_1^*}{d\mu} = \frac{\tau}{\sqrt{\tau_{r_1}}}\phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu + \Phi^{-1}(\lambda_1)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right)\left[\frac{d\theta_1^*}{d\mu} - 1\right]$$

$$\frac{d\theta_1^*}{d\mu} = -\frac{\frac{\tau}{\sqrt{\tau_{r_1}}}\phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu + \Phi^{-1}(\lambda_1)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right)}{1 - \frac{\tau}{\sqrt{\tau_{r_1}}}\phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu + \Phi^{-1}(\lambda_1)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right)} < 0$$

The higher the mean of the prior μ (or the public signal), the more optimistic creditors are about the state of the economy. A higher μ decreases θ_1^*, which implies that the range of values of θ_1 for which the country stays solvent increases (i.e. default occurs for $\theta < \theta_1^*$, so if θ_1^* decreases, then default is less likely to occur). ■

Remark A 3 *1. If the probability of default in Country* 1 *is low and agents have an optimistic prior about the state of the economy, then a higher transparency of public information,*

τ, will further decrease the probability of default in Country 1.

2. If the probability of default in Country 1 is high and agents have a pessimistic prior about the state of the economy, then a higher transparency of public information, τ, will further increase the probability of default in Country 1.

Proof.

$$\begin{aligned}
\frac{d\theta_1^*}{d\tau} &= \phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu - \Phi^{-1}(1-\lambda_1)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right) \times \\
&\left[\begin{array}{c} \frac{1}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu - \Phi^{-1}(1-\lambda_1)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right) \\ -\frac{\tau}{\sqrt{\tau_{r_1}}}\Phi^{-1}(1-\lambda_1)\left[\frac{1}{2}\frac{1}{\sqrt{\tau+\tau_{r_1}}\tau} - \frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right] + \frac{\tau}{\sqrt{\tau_{r_1}}}\frac{d\theta_1^*}{d\tau_{r_1}} \end{array}\right] \\
&= \frac{\phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu - \Phi^{-1}(1-\lambda_1)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right) \times \left[\frac{1}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu - \frac{1}{2}\Phi^{-1}(1-\lambda_1)\frac{1}{\sqrt{\tau+\tau_{r_1}}}\right)\right]}{1 - \frac{\tau}{\sqrt{\tau_{r_1}}}\phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu - \Phi^{-1}(1-\lambda_1)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right)}
\end{aligned}$$

In order to determine how the probability of default is affected by changes in the precision of the public signal, we need to determine the sign of $\left(\theta_1^* - \mu - \frac{1}{2}\Phi^{-1}(1-\lambda_1)\frac{1}{\sqrt{\tau+\tau_{r_1}}}\right)$.

In particular, if $\theta_1^* < \mu + \frac{1}{2}\Phi^{-1}(1-\lambda_1)\frac{1}{\sqrt{\tau+\tau_{r_1}}}$, then $\frac{d\theta_1^*}{\tau} < 0$, which implies that when agents have an optimistic prior and the probability of default is small, then a higher transparency of the public signal will reinforce these optimistic beliefs and lead to an even lower probability of default. On the other hand, if $\theta_1^* > \mu + \frac{1}{2}\Phi^{-1}(1-\lambda_1)\frac{1}{\sqrt{\tau+\tau_{r_1}}}$, then creditors are ex-ante pessimistic about the state of the economy and believe that the probability of default is large, so a higher precision of the public signal will exacerbate this pessimism and lead to an even higher probability of default.

The intuition behind this result is analogous to the one above for the case on an increase in the precision of private signals. If agents have an optimistic prior and the probability of default is small, then an increase in the precision of the public signal will further decrease the probability of default. In contrast to the private signal, the public signal only contains information about the fundamental and is included in every agent's information set. Thus, when the precision of the public signal increases, agents will assign a higher weight to the public signal, which would reinforce their initial optimistic beliefs, thus making them less likely to withdraw their funds, which would in turn reduce the likelihood of a default. On the other hand, if creditors have a pessimistic prior and the probability of default is high, a higher precision of the public signal will exacerbate this pessimism and lead agents to give a higher weight to it, thus increasing the incidence of withdrawals and the probability of a default, since agents believe that the state is probably not good and that the proportion of withdrawals required to default is small. This result is consistent with Morris and Shin (2002) and Metz (2002), who highlight that more transparency of public information does not necessarily lead to higher welfare since in some cases it might increase the probability of a default. ■

Remark A 4 *The probability of a default in Country $n = 1, 2$ increases with an increase in λ_n.*

Proof. Notice that

$$\begin{aligned}
\frac{d\theta_1^*}{d\lambda_1} &= \phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu - \Phi^{-1}(1-\lambda_1)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right)\left[-\sqrt{\frac{\tau+\tau_{r_1}}{\tau_{r_1}}}\frac{d\Phi^{-1}(1-\lambda_1)}{d\lambda_1} + \frac{\tau}{\sqrt{\tau_{r_1}}}\frac{d\theta_1^*}{d\lambda_1}\right] \\
&= \frac{-\phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu - \Phi^{-1}(1-\lambda_1)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right)\sqrt{\frac{\tau+\tau_{r_1}}{\tau_{r_1}}}\frac{d\Phi^{-1}(1-\lambda_1)}{d\lambda_1}}{1 - \frac{\tau}{\sqrt{\tau_{r_1}}}\phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu - \Phi^{-1}(1-\lambda_1)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right)} > 0
\end{aligned}$$

Since $\frac{d\Phi^{-1}(1-\lambda_1)}{d\lambda_1} < 0$ and $\frac{\tau}{\sqrt{\tau_{r_1}}}\phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu + \Phi^{-1}(\lambda_1)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right) < 1$, by the uniqueness condition. Likewise, for Country 2

$$\begin{aligned}
\frac{d\theta_2^*}{d\lambda_2} &= \frac{-\phi\left(\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}+\tau_{r_2}}}{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}\Phi^{-1}(1-\lambda_2)\right)\right)\times \left[\frac{\sqrt{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}+\tau_{r_2}}}{\sqrt{\tau_{r_2}}}\frac{d\Phi^{-1}(1-\lambda_2)}{d\lambda_2}\right]}{1-\left[\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\times \phi\left(\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}+\tau_{r_2}}}{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}\Phi^{-1}(1-\lambda_2)\right)\right)\right]} \\
&> 0
\end{aligned}$$

Since $\frac{d\Phi^{-1}(1-\lambda_2)}{d\lambda_2} < 0$ and $\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\phi(\cdot) < 1$, by the uniqueness condition. This means that, in each individual country, as the payoff from early withdrawal increases, the incentives to withdraw funds, and thus provoke a default, increase. ■

3.0.3 Comparative statics about the channels of contagion: proofs

The following lemma will be useful to prove some comparative statics results about the channels of contagion.

Lemma A 1 *1. If the probability of default in Country 2 is low and agents have an optimistic prior about the state of the economy, then a higher transparency of public information, measured by the precision of the composed public signal $\widehat{\widehat{\tau}}_\alpha$, will further decrease the probability of default in Country 2.*

2. If the probability of default in Country 2 is high and agents have a pessimistic prior about the state of the economy, then a higher transparency of public information, measured by the precision of the composed public signal $\widehat{\widehat{\tau}}_\alpha$, will further increase the probability of default in Country 2.

Proof. Recall from section 2 that all public information held by agents in Country 2 can be summarized by

$$\theta_2 | y \sim N\left(\frac{\tau\mu + \widehat{\tau}_\alpha \widehat{y}}{\tau + \widehat{\tau}_\alpha}, \tau_s^{-1} + (\tau + \widehat{\tau}_\alpha)^{-1}\right)$$

where $\widehat{y} = x_1^* - \tau_{r_1}^{-1/2} y$ and $\widehat{\tau}_\alpha = \tau_{r_1}\tau_\alpha$. For simplicity, let $\widehat{\widehat{\tau}}_\alpha = \left(\tau_s^{-1} + (\tau + \widehat{\tau}_\alpha)^{-1}\right)^{-1}$ be the precision of the composed public information held by agents in Country 2. What we are interested in is the effect of some of the components of the term $\widehat{\widehat{\tau}}_\alpha$ on the probability of default in Country 2, in particular I will focus on the effect of the correlation between fundamentals in countries 1 and 2, measured by the precision of θ_2, τ_s, and on the effect of the precision of the public signal about the proportion of agents that withdraw their funds in Country 1 (τ_α). In order to study those effects we first explore the effect that $\widehat{\widehat{\tau}}_\alpha$ has on the probability of default in Country 2.

$$\begin{aligned}
\frac{d\theta_2^*}{d\widehat{\widehat{\tau}}_\alpha} &= \phi\left(\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}(1-\lambda_2)\right)\right) \times \\
&\quad \left[\begin{array}{c} \frac{1}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}(1-\lambda_2)\right) \\ -\Phi^{-1}(1-\lambda_2)\left(\frac{\frac{1}{2}\widehat{\widehat{\tau}}_\alpha\left(\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}\right)^{-1/2} - \sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}{\left(\widehat{\widehat{\tau}}_\alpha\right)^2}\right) + \frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\frac{d\theta_2^*}{d\widehat{\widehat{\tau}}_\alpha} \end{array}\right] \\
&= \frac{\begin{array}{c}\phi\left(\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}(1-\lambda_2)\right)\right) \times \\ \left[\frac{1}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{1}{2}\frac{\Phi^{-1}(1-\lambda_2)}{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}\right)\right]\end{array}}{1 - \frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\phi\left(\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}(1-\lambda_2)\right)\right)}
\end{aligned}$$

In order to determine how the probability of default in Country 2 is affected by changes in the precision of the aggregate public signal, we need to determine the sign of the term $\left(\theta_2^* - \widehat{\theta}_1 - \frac{1}{2}\frac{\Phi^{-1}(1-\lambda_2)}{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}\right)$.

If $\theta_2^* < \widehat{\theta}_1 + \frac{1}{2}\frac{\Phi^{-1}(1-\lambda_2)}{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}$, then $\frac{d\theta_2^*}{d\widehat{\widehat{\tau}}_\alpha} < 0$, which implies that when agents have an optimistic prior about the state of the economy in Country 2 and the probability of default is low, then an increase in the precision of the public signal will further decrease the probability of default since agents set a higher weight on the public information, which makes them feel even more optimistic about the economy, and thus less likely to withdraw their funds, thus reducing the likelihood of a default.

On the other hand, if $\theta_2^* > \widehat{\theta}_1 + \frac{1}{2}\frac{\Phi^{-1}(1-\lambda_2)}{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}$, then creditors believe that the probability of default is high and have a pessimistic prior about the state of the economy, so a higher precision of the public signal will lead to an even higher probability of default in Country 2.An increase in the precision of the public information will exacerbate this pessimism and lead agents to put more weight on the public signal, which would eventually lead to an even higher probability of default in Country 2. Just as in the case of Country 1, more precise public information does not necessarily lead to a lower probability of default. ■

Remark 1 *1. If the probability of default in Country 2 is low and agents are ex-ante optimistic about the state of the economy, then a higher correlation between Country 1 and Country 2 (i.e. a higher precision τ_s) will further decrease the probability of default in Country 2.*

2. If the probability of default in Country 2 is high and agents are ex-ante pessimistic about the state of the economy, then a higher correlation between Country 1 and Country 2 (i.e. a higher precision τ_s) will increase the probability of default in Country 2.

Proof. From lemma $A1$ we know that

$$\frac{d\theta_2^*}{d\widehat{\widehat{\tau}}_\alpha} = \frac{\phi\left(\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}(1-\lambda_2)\right)\right)\left[\frac{1}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{1}{2}\frac{\Phi^{-1}(1-\lambda_2)}{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}\right)\right]}{1 - \frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\phi\left(\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}(1-\lambda_2)\right)\right)}$$

And notice that

$$\frac{d\widehat{\widehat{\tau}}_\alpha}{d\tau_s} = \left(\tau_s^{-1} + (\tau + \tau_{r_1}\tau_\alpha)^{-1}\right)^{-2}\tau_s^{-2} > 0$$

Where $\widehat{\widehat{\tau}}_\alpha = \left(\tau_s^{-1} + (\tau + \tau_{r_1}\tau_\alpha)^{-1}\right)^{-1}$ is the precision of the composed public signal held by agents in Country 2. We now simply apply the chain rule to find that

$$\begin{aligned}
\frac{d\theta_2^*}{d\tau_s} &= \frac{d\theta_2^*}{d\widehat{\widehat{\tau}}_\alpha}\cdot\frac{d\widehat{\widehat{\tau}}_\alpha}{d\tau_s} \\
&= \frac{\begin{array}{c}\phi\left(\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}(1-\lambda_2)\right)\right)\times \\ \left[\frac{1}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{1}{2}\frac{\Phi^{-1}(1-\lambda_2)}{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}\right)\right]\end{array}}{\begin{array}{c}\left[1 - \frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\phi\left(\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}(1-\lambda_2)\right)\right)\right]\times \\ \left(\tau_s^{-1} + (\tau + \tau_{r_1}\tau_\alpha)^{-1}\right)^2\tau_{\theta_2}^2\end{array}}
\end{aligned}$$

The sign of $\frac{d\theta_2^*}{d\tau_s}$ depends on the sign of the term $\left(\theta_2^* - \widehat{\theta}_1 - \frac{1}{2}\frac{\Phi^{-1}(1-\lambda_2)}{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}\right)$. In particular, if $\theta_2^* < \widehat{\theta}_1 + \frac{1}{2}\frac{\Phi^{-1}(1-\lambda_2)}{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}$, then $\frac{d\theta_2^*}{d\tau_s} < 0$, i.e. if the probability of default is low and agents have an optimistic prior about fundamentals, then a higher correlation between countries 1 and 2 will further decrease the probability of default. On the other hand, if $\theta_2^* > \widehat{\theta}_1 + \frac{1}{2}\frac{\Phi^{-1}(1-\lambda_2)}{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}$, then creditors believe that the probability of default is high and are ex-ante pessimistic about the state of the economy. A similar logic applies as in the previous case, so a higher correlation between the two countries will exacerbate this pessimism by leading agents to give a higher weight to the aggregate public signal, thus increasing the incidence of withdrawals and the probability of a default, since agents know that the state is probably not good and that the proportion of withdrawals required to default is small. ■

Remark 2 *A higher signal about the proportion of agents that withdraw their funds in Country 1, y, increases the probability of default in Country 2.*

Proof. To prove this result I first analyze the effect that an increase in the posterior mean $\widehat{\theta}_1$ has on the probability of default in Country 2 and then we apply the chain rule to isolate the effect of the signal about the proportion of agents that withdraw their funds in Country 1, y.

$$\begin{aligned}
\frac{d\theta_2^*}{d\widehat{\theta}_1} &= \phi\left(\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\left(\theta_2^*-\widehat{\theta}_1-\frac{\sqrt{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}+\tau_{r_2}}}{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}\Phi^{-1}(1-\lambda_2)\right)\right)\times \\
&\quad \left[\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\frac{d\theta_2^*}{d\widehat{\theta}_1}-\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\right] \\
\frac{d\theta_2^*}{d\widehat{\theta}_1} &= \frac{-\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\phi\left(\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\left(\theta_2^*-\widehat{\theta}_1-\frac{\sqrt{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}+\tau_{r_2}}}{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}\Phi^{-1}(1-\lambda_2)\right)\right)}{1-\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\phi\left(\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\left(\theta_2^*-\widehat{\theta}_1-\frac{\sqrt{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}+\tau_{r_2}}}{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}\Phi^{-1}(1-\lambda_2)\right)\right)} \\
&< 0
\end{aligned}$$

Therefore, a higher expected or posterior mean will lead to a lower probability of default, i.e. the higher the posterior mean $\widehat{\theta}_1$, the more optimistic creditors are about the state of the economy in Country 2. To analyze the effect on θ_2^* of the signal about the proportion of agents that withdraw their funds in Country 1, notice that

$$\widehat{\theta}_1=\frac{\tau\mu+\widehat{\tau}_\alpha\widehat{y}}{\tau+\widehat{\tau}_\alpha}=\frac{\tau\mu+\tau_{r_1}\tau_\alpha\left(x_1^*-\tau_{r_1}^{-1/2}y\right)}{\tau+\tau_{r_1}\tau_\alpha}$$

So that

$$\frac{d\widehat{\theta}_1}{dy}=\frac{-\eta\tau_{r_1}^{1/2}}{\tau+\tau_{r_1}\tau_\alpha}<0$$

By the chain rule, we can establish that

$$\frac{d\theta_2^*}{dy}=\frac{d\theta_2^*}{d\widehat{\theta}_1}\cdot\frac{d\widehat{\theta}_1}{dy}>0$$

■

Effect of an increase in τ_α on the probability of default in Country 2. A change in τ_α affects both the posterior mean, $\widehat{\theta}_1$, and the precision of the composed public signal through $\widehat{\widehat{\tau}}_\alpha=\left(\tau_s^{-1}+(\tau+\tau_{r_1}\tau_\alpha)^{-1}\right)^{-1}$. This leads to a "coordination" effect which makes agents put more weight on the posterior mean and to an "information effect" which changes the level of this mean. I derive some expressions to investigate the overall effect, however, it is not possible to fully characterize it analytically.

Recall that $\widehat{\theta}_1 = \frac{\tau\mu + \tau_{r_1}\tau_\alpha \widehat{y}}{\tau + \tau_{r_1}\tau_\alpha}$ and $\widehat{\widehat{\tau}}_\alpha = \left(\tau_s^{-1} + (\tau + \tau_{r_1}\tau_\alpha)^{-1}\right)^{-1}$.

We first look at the effect that the precision of the public signal, $\widehat{\widehat{\tau}}_\alpha$, has on the probability of default in Country 2 (coordination effect, without decomposing it):

$$\frac{d\theta_2^*}{d\widehat{\widehat{\tau}}_\alpha} = \frac{\phi\left(\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}(1-\lambda_2)\right)\right)\left[\frac{1}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{1}{2}\frac{\Phi^{-1}(1-\lambda_2)}{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}\right)\right]}{1 - \frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\phi\left(\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}(1-\lambda_2)\right)\right)}$$
$$\begin{cases} > 0 \text{ if } \theta_2^* > \widehat{\theta}_1 + \frac{1}{2}\frac{\Phi^{-1}(1-\lambda_2)}{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}} \\ < 0 \text{ if } \theta_2^* < \widehat{\theta}_1 + \frac{1}{2}\frac{\Phi^{-1}(1-\lambda_2)}{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}} \end{cases}$$

Notice that the precision of the public signal $\widehat{\widehat{\tau}}_\alpha$ is increasing in τ_α:

$$\begin{aligned} \frac{d\widehat{\widehat{\tau}}_\alpha}{d\eta} &= \left(\tau_s^{-1} + (\tau + \tau_{r_1}\tau_\alpha)^{-1}\right)^{-2}(\tau + \tau_{r_1}\tau_\alpha)^{-2}\tau_{r_1} \\ &> 0 \end{aligned}$$

Now we look at the effect of the posterior mean $\widehat{\theta}_1$ on the probability of default in Country 2 (information effect, without decomposing it):

$$\begin{aligned} \frac{d\theta_2^*}{d\widehat{\theta}_1} &= \frac{-\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\phi\left(\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}+\tau_{r_2}}}{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}\Phi^{-1}(1-\lambda_2)\right)\right)}{1 - \frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\phi\left(\frac{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}+\tau_{r_2}}}{\left(\tau_s^{-1}+(\tau+\widehat{\tau}_\alpha)^{-1}\right)^{-1}}\Phi^{-1}(1-\lambda_2)\right)\right)} \\ &< 0 \end{aligned}$$

which is unambiguously negative. Now we look at how τ_α affects the posterior mean of the distribution about θ_2:

$$\begin{aligned} \frac{d\widehat{\theta}_1}{d\eta} &= \frac{\tau_{r_1}\widehat{y}(\tau + \tau_{r_1}\tau_\alpha) - \tau_{r_1}(\tau\mu + \tau_{r_1}\tau_\alpha\widehat{y})}{(\tau + \tau_{r_1}\tau_\alpha)^2} \\ \frac{d\widehat{\theta}_1}{d\eta} &= \frac{\tau_{r_1}\tau(\widehat{y} - \mu)}{(\tau + \tau_{r_1}\tau_\alpha)^2}\begin{cases} > 0 \text{ if } x_1^* > \mu + \tau_{r_1}^{-1/2}y \\ < 0 \text{ if } x_1^* < \mu + \tau_{r_1}^{-1/2}y \end{cases} \end{aligned} \tag{15}$$

Since $\widehat{y} = x_1^* - \tau_{r_1}^{-1/2}y$. The effect of the precision of the signal about the proportion of withdrawing agents in Country 1 on the posterior mean $\widehat{\theta}_1$ depends on the relative magnitudes of the equilibrium threshold used by creditors in Country 1, the prior beliefs of agents in Country 1 (measured by the mean of the prior μ), and the signal about the proportion of agents that withdraw their funds in Country 1, y. We take one step back and analyze the effect of the mean of the prior μ on the optimal threshold for agents in Country 1, x_1^*. Recall that $x_1^* = \frac{(\tau+\tau_{r_1})}{\tau_{r_1}}\theta_1^* - \frac{\Phi^{-1}(1-\lambda_1)(\tau+\tau_{r_1})}{\tau_{r_1}\sqrt{\tau+\tau_{r_1}}} - \frac{\tau}{\tau_{r_1}}\mu.$

$$
\begin{aligned}
\frac{dx_1^*}{d\mu} &= \frac{(\tau+\tau_{r_1})}{\tau_{r_1}}\frac{d\theta_1^*}{d\mu} - \frac{\tau}{\tau_{r_1}} \\
&= -\frac{(\tau+\tau_{r_1})}{\tau_{r_1}} \frac{\frac{\tau}{\sqrt{\tau_{r_1}}}\phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu + \Phi^{-1}(\lambda_1)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right)}{1-\frac{\tau}{\sqrt{\tau_{r_1}}}\phi\left(\frac{\tau}{\sqrt{\tau_{r_1}}}\left(\theta_1^* - \mu + \Phi^{-1}(\lambda_1)\frac{\sqrt{\tau+\tau_{r_1}}}{\tau}\right)\right)} - \frac{\tau}{\tau_{r_1}} < 0
\end{aligned}
$$

So an increase in the mean of the prior μ decreases thresholds. On the other hand, when creditors in Country 1 set a low threshold they withdraw their funds for a smaller range of signals, which leads creditors in Country 2 to observe signals about a lower proportion of agents that withdraw their funds in Country 1, y. This implies that a high μ is associated with a low x_1^*, which leads to a low y, and a low μ is associated with a high x_1^*, which leads to a high y. However, notice that y enters condition 15 multiplied by the standard deviation of private signals in Country 1, $\tau_{r_1}^{-1/2}$, which we assume to be low enough (high τ_{r_1}) for the uniqueness condition.

Now we characterize the effect of a change in the precision of the public signal about the proportion of agents that withdraw in Country 1 on the probability of default in Country 2.

$$
\begin{aligned}
\frac{d\theta_2^*}{d\eta} &= \phi\left(\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}(1-\lambda_2)\right)\right) \times \\
&\left[\begin{array}{c} \frac{1}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}(1-\lambda_2)\right)\cdot\frac{d\widehat{\widehat{\tau}}_\alpha}{d\eta} + \\ \frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\frac{d\theta_2^*}{d\eta} - \frac{d\widehat{\theta}_1}{d\eta} - \frac{\frac{1}{2}\left(\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}\right)^{-1/2}\widehat{\widehat{\tau}}_\alpha - \sqrt{\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}}}{\left(\widehat{\widehat{\tau}}_\alpha\right)^2}\cdot\frac{d\widehat{\widehat{\tau}}_\alpha}{d\eta}\Phi^{-1}(1-\lambda_2)\right)\end{array}\right] \\
&= \frac{\begin{array}{c}\frac{\tau_{r_1}}{\sqrt{\tau_{r_2}}}\phi\left(\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}(1-\lambda_2)\right)\right)\times \\ \left[\left(\theta_2^* - \widehat{\theta}_1 - \frac{1}{2}\left(\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}\right)^{-1/2}\Phi^{-1}(1-\lambda_2)\right)\left(\left(\widehat{\widehat{\tau}}_\alpha\right)^2(\tau+\tau_{r_1}\tau_\alpha)^{-2}\right) - \frac{\widehat{\widehat{\tau}}_\alpha\tau(\widehat{y}-\mu)}{(\tau+\tau_{r_1}\tau_\alpha)^2}\right]\end{array}}{1-\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\phi\left(\frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{\sqrt{\widehat{\widehat{\tau}}_\alpha+\tau_{r_2}}}{\widehat{\widehat{\tau}}_\alpha}\Phi^{-1}(1-\lambda_2)\right)\right)}
\end{aligned}
$$

Proof. The sign of this derivative will depend on the sign of the term

$$
\left[\frac{1}{\sqrt{\tau_{r_2}}}\left(\theta_2^* - \widehat{\theta}_1 - \frac{1}{2}\left(\widehat{\widehat{\tau}}_\alpha + \tau_{r_2}\right)^{-1/2}\Phi^{-1}(1-\lambda_2)\right)\left(\left(\widehat{\widehat{\tau}}_\alpha\right)^2(\tau+\tau_{r_1}\tau_\alpha)^{-2}\right) - \frac{\widehat{\widehat{\tau}}_\alpha}{\sqrt{\tau_{r_2}}}\frac{\tau(\widehat{y}-\mu)}{(\tau+\tau_{r_1}\tau_\alpha)^2}\right]
$$

which illustrates the two effects that we have described, i.e. the coordination effect through the first term and the information effect through the term through the second term. As is clear from the expression above, it is not possible to sign this term for all parameter values, which is why in the body of the paper I present results based on numerical simulations. ■