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Bilateral Trade and Opportunism in a Matching Market

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Bilateral Trade and Opportunism in a Matching Market

Abstract

We develop a model of bilateral contracting in a dynamic market setting. Asset owners must be paired via a matching process in order to form productive relationships involving long−term investments and ongoing effort. Market frictions shape the owners' incentives to invest in the absence of complete contracts. We identify cases in which there exists an optimal positive level of market friction implementing first−best investment levels. We also endogenize the choice between integrated and nonintegrated organizational forms. Changes in structural variables can induce crashes by disrupting existing relationships.

1 Introduction[1]

The study of contracts and organizations as institutional mechanisms for mediating trade has yielded a wealth of important insights.[2] Attention has been largely restricted, however, to trading relationships among isolated groups of agents, leaving aside the larger market processes that bring agents together to form relationships in the first place. Our purpose here is to demonstrate that market forces operating outside of trading relationships may have a decisive impact on the nature of within-relationship contractual arrangements. Specifically, we propose a model of bilateral contracting in a dynamic market setting showing explicitly how markets can operate to stem opportunism in trading relationships, along with clarifying the trade-offs between market friction and trading efficiency, and between market and nonmarket trading institutions. Further, our analysis indicates how contractual institutions may in turn play a central role in determining the productivity and dynamic behavior of the aggregate economy.

We develop a model in which agents are asset owners who must be paired with other owners in order to form productive relationships. Market interaction among potential trading partners takes the form of a matching process, in which an unmatched owner of one asset encounters an owner of the other asset with a given probability in each period. Newly-matched pairs of owners make long-term investments that are specific to the matched assets, in that the investments are lost should the match ever break apart. Productive relationships continue over time and require that owners exert ongoing effort in order to maintain profitability. Neither long-term investment nor ongoing effort are directly contractible; incentives for high effort, in particular, are sustained by the owners' threat to sever the relationship and reenter the matching pools should either owner fail to exert high effort.

Many real contractual relationships fit with our theoretical model. In an employment relationship, a worker's personal assets (including his skills) are matched with the assets of a firm, and specific long-term investments (in human capital, for instance) are common. Trading relations between firms, such as the buyer and supplier of an intermediate good, also entail specific long-term investments (often in the design of the traded good or

[1] This paper appears, with minor modifcations, as it was written in 1995. At the end of the introduction are notes on the subsequent literature.

[2] This body of literature is broadly discussed by Williamson (1985) and Milgrom and Roberts (1992).

the location of assets). In environments where legal systems are weak (as studied by McMillan and Woodruff 1999a,b) or where there are limits on verifiability, the threat of severance can play an important role in contract enforcement. Many personal relationships also feature long-term investments and the threat of severance.

Our central result is that market interaction shapes the incentives for long-term investment, based on the fact that incentives for high investment depend on both the investment levels within the relationship and market frictions outside of the relationship. The continuing nature of the relationship therefore places an *effort constraint* on the long-term investments that varies with the level of market friction. We consider two cases. If higher investment serves to increase the returns to high versus low effort, then the trading partners must *overinvest* relative to unconstrained profit-maximizing levels in order to sustain effort incentives in the ongoing relationship. Importantly, a fall in market friction, as reflected by a rise in the probability of locating a new trading partner in the matching market, makes less onerous the threat of severing an existing trading relationship, and thereby tightens the effort constraint. Thus, investment levels must rise as market frictions fall.

As a consequence, lower levels of market friction can raise the value of trading relationships, by introducing an overinvestment constraint that counteracts the underinvestment incentive deriving from the noncontractibility of investment. If incentives to exert low effort are relatively strong, then there exists an optimal positive level of market friction at which relationships operate at first-best efficiency; thus, the market has the potential to counteract the adverse efficiency consequences of contractual incompleteness. As market friction falls further from this optimal level, the value of trading relationships also falls, and welfare may fall as well. A frictionless market can actually minimize welfare, by making necessary extremely high investment levels in order to sustain effort incentives.

We consider a second case, in which higher investment reduces the incentives for high effort, making the effort constraint harder to satisfy. In this case of underinvestment, a fall in market friction compounds the underinvestment incentive brought on by opportunism, and the value of relationships must decline. In this instance, relationships necessarily take on their minimum value in a frictionless market.

We extend our model by incorporating the possibility that a newly-matched owner can sell his asset to the other owner, resulting in an integrated organizational form. This serves to endogenize the determination of organizational form as part of a market equilibrium. Integrated organization imposes a cost in terms of reduced investment in one of the assets, as pointed out by Grossman and Hart (1986); in our setting, however, integration allows high effort to be more easily sustained. This trade-off favors nonintegrated organization when market frictions are high and severing a relationship imposes high costs on owners. When market frictions are low, in contrast, the effort constraint might greatly reduce the value of nonintegrated relationships, and integration is then chosen in equilibrium. Thus, integrated relationships will tend to be associated with low levels of market friction.

Our analysis yields new insights into how shifts in underlying structural variables can produce abrupt "crashes" through their effect on incentives in preexisting relationships. A rise in the matching probability — reflecting an institutional liberalization, for example — will tighten the effort constraint on investment levels, thereby causing all preexisting relationships to violate the constraint; these relationships will therefore be severed, leading to a sharp drop in total production in the economy. Even when the reduction in friction leads to greater long-run welfare, the economy must pass through a costly transitional phase of rematching and reinvestment as new relationships are constituted.

This paper relates most closely to two strands of previous work. First, the incomplete-contracts literature studies institutional remedies to contractual incompleteness, building on the seminal contributions of Coase (1937), Klein, Crawford and Alchian (1978) and Williamson (1979). In addition, Grossman and Hart (1986), Hart and Moore (1990) and Bolton and Whinston (1993) consider one-shot trading models in which incentives for specific investment are constrained by the prospect of ex post opportunism, and in which asset ownership can partially compensate for contractual incompleteness. Halonen (1995) emphasizes how optimal ownership structure is sensitive to whether the specific investment is made repeatedly. Klein and Leffler (1981), in contrast, focus on the threat of terminating a repeated relationship as a substitute for direct contracts. Our analysis incorporates aspects from both these strands of the incomplete-contracts literature.

Second, there is a large literature that studies market interaction as a matching process. Most closely related to our work is Diamond and Maskin (1979,1981), Diamond (1990), and Kranton (1996a), who study various aspects of pairwise repeated trade when trading partners must meet in a matching market. Hosios (1990) offers a general model of long-term relationships that are formed on a matching market, where surplus is divided according to the agents' relative bargaining powers. Hosios examines various externalities arising from how individual decisions (for instance, about whether to consummate or sever relationships) alter the performance of the matching market. Also related is the efficiency wage literature (Shapiro and Stiglitz 1984), which shows that, when matching entails delay, opportunistic behavior can be deterred by the threat of severance. Our paper departs from these studies by examining how long-term specific investment and the choice of organizational form in bilateral relationships are influenced by market interaction. Our work is differentiated from papers such as Hosios (1990) in that we analyze how the matching market influences noncontractible investment within relationships, rather than look at externalities that operate through the matching process.

This paper remains essentially as we crafted it in 1995 and thus it predates a related, more recent, literature. Several recent papers have studied ways in which cooperation can be sustained in long-term relationships, in an environment in which matching is frictionless and agents are not able to observe each other's history from previous relationships. Opportunism can be deterred by the expectation of reduced value at the beginning of new relationships, in terms of wasteful gifts (Carmichael and MacLeod 1997) or gradualism (Datta 1996, Kranton 1996b). However, norms that reduce the value of individual relationships would not survive in an environment of active contracting and joint optimization. Frictions in the matching market, as we study here, can support cooperation between contract partners. Incomplete information may also provide the needed discipline as agents resort to starting small (Watson 1995, 1999, 2001, and Ghosh and Ray 1996) or costly signaling (Camerer 1988 and Watson 1995) at the beginning of new relationships.

Relative to the recent literature on contracting and matching, this paper was the first to analyze investment and contracting in long-term relationships that form through a frictional matching market. Work by McMillan and Woodruff (1999a,b) has provided empirical support for the phenomena that we model here. This paper also provides the foundation for our work on contractual fragility and compensation policies (Ramey and Watson 1997; den Haan, Ramey, and Watson 1999a,b, 2000). We encourage further research on institutions and ongoing contractual relationships, especially research that models renegotiation and institutions explicitly.

The sequel is organized as follows. Section 2 introduces our model, and Section 3 derives steady-state equilibria of the model. Our results are presented in Sections 4, 5 and 6, which consider the effect of market frictions on welfare, the choice of organizational form, and the effects of structural changes, respectively. Section 7 concludes.

2 Model

2.1 Agents and Matching Market

The model consists of two types of agents, identified by their ownership of two types of indivisible assets. There is a continuum of each type of agents, with equal total masses of the two types. The two types of assets are referred to as asset 1 and asset 2. Periods are discrete and are numbered $t = 1, 2, ...$. Agents seek to maximize their expected discounted consumption streams, where δ gives the discount factor.

The matching market consists of equally-sized unmatched pools of owners of each type of asset. With probability λ, an unmatched owner of asset i encounters an unmatched owner of asset j at the start of a given period, and the two agents commence to forming a nonintegrated productive relationship, as described in the next two subsections. With probability $1-\lambda$, an unmatched owner is unsuccessful in forming a match in the current period, and he tries again in the following period.[3]

2.2 Nonintegrated Productive Relationships

At the start of their relationship, newly-matched owners of assets 1 and 2 simultaneously and independently choose long-term investments, denoted by α_1 and α_2, taken to be nonnegative real numbers. We assume that the investments are match-specific, in that they are valueless should the agents ever sever their trading relationship, and noncontractible, meaning that the agents are unable to commit ex ante to contractual arrangements that depend on the investment levels.

If a relationship continues to be productive, then the asset owners obtain returns at the end of each period. Prior to realizing these returns, however, the agents simultaneously choose whether to engage in high or low effort. Effort can be regarded as the intensity of an agent's personal contribution to

[3] Our analysis and results extend in a straightforward way to the case of differently-sized pools of asset owners.

the joint enterprise. Alternatively, "high effort" can be interpreted as effort contributing to the effective joint operation of the two assets, while "low effort" is effort that focuses closely on the owner's own asset at the expense of the other. Effort is also noncontractible, to the extent that the agents cannot commit to direct penalties for low effort. Importantly, however, if agents renege on their choice of effort, then the relationship becomes unproductive and the agents must reenter the matching market in search of new partners.[4]

If both agents choose high effort, then the relationship remains productive and the owner of asset i realizes the return $z_i(\alpha) > 0$, $i = 1, 2$, where $\alpha = (\alpha_1, \alpha_2)$. If one or both agents choose low effort, however, then the relationship ceases to be productive and is severed. Let $x_i(\alpha) \geq 0$ denote the liquidation value of the relationship, obtained at the end of the period, of an agent who chooses low effort when his partner has chosen high effort, and let $y_j(\alpha)$ denote the liquidation value of the agent choosing high effort. The liquidation value is zero if both agents choose low effort. We assume further that in each period, there is probability β that the relationship becomes obsolete for exogenous reasons; in this case, the returns $z_i(\alpha)$ are realized prior to liquidation. Once liquidation occurs, the agents must reenter the matching pool in the following period, and the investments made at the start of the severed relationship have no value in any future relationship.

The sum of returns from the two assets under high effort is given by $z(\alpha) = z_1(\alpha) + z_2(\alpha)$. Assume that z is a strictly concave function, strictly increasing in α_1 and α_2, with $z(0,0) \geq 0$ and $\partial z(0,0)/\partial\alpha_i - 2 > 0$ for $i = 1, 2$; the latter assumption assures that each manager is willing to invest at least a tiny amount in exchange for one-half the total returns for a single period. Further, $\partial^2 z/\partial\alpha_1\partial\alpha_2 > 0$ is assumed,[5] and also x_i and y_i are taken to be continuous functions. We assume $x_i(\alpha) + y_j(\alpha) \leq 0$, meaning that the relationship generates positive total returns if and only if both managers exert high effort. An individual owner, however, may gain from "cheating" on his partner if $z_i(\alpha) < x_i(\alpha)$; here it is in the private interest of the asset i owner to exert low effort, given high effort by the owner of asset j.

[4] An important feature of our model is that an agent choosing low effort in his current relationship retains the ability for form new relationships in the future, i.e. "reputation capital" is specific to particular relationships. This can be justified in terms of limited information flow in the market concerning an agent's past behavior or the particular reasons for prior relationship breakup.

[5] Thus, the investments are "strategic complements" in the sense of Bulow, Geanokoplos and Klemperer (1987).

2.3 Contract Negotiation

Agents are assumed to negotiate a long-term contract after they have chosen investments. This reflects opportunism to the extent that negotiation makes it possible for an agent to appropriate part of the returns from his partner's investment. We model contract negotiation simply by having the agents divide the discounted expected returns according to the Nash bargaining solution, subject to the constraint that each agent receive at least his value of returning to the matching pool.[6] The Nash solution is implemented by a long-term contract consisting of an upfront payment followed by a constant per-period transfer between the agents. The long-term contract is taken to be in force for the duration of the trading relationship.

The timing of actions taken in trading relationships is illustrated in Figure 1, where the period in which matching occurs is taken to be period 0. Note that contract negotiation occurs in the period when the match is made, immediately after investments are chosen and prior to the choice of effort levels. Further, transfers under the long-term contract occur at the same time effort levels are chosen, so that an agent choosing low effort in a given period will continue to obtain his transfer in that period.

3 Steady-State Equilibrium

In this section we derive steady-state equilibria of the economy, having the properties that (i) the equilibrium investment profile (α_1, α_2) is the same for each newly-matched pair of owners; and (ii) the rate at which agents leave the matching pools via new matches equals the rate at which they enter due to obsolescence. In the first three subsections we consider equilibrium choices of investment and effort within a given trading relationship, holding fixed the behavior of agents outside of the relationship. These results will be used in the final subsection to construct market equilibria.

[6] In a noncooperative context, this corresponds to a setting in which the agents are able to voluntarily halt the negotiation process and return to the matching market, and there are no random forces that may disrupt negotiation. For further discussion see Binmore, Rubinstein, and Wolinsky (1986).

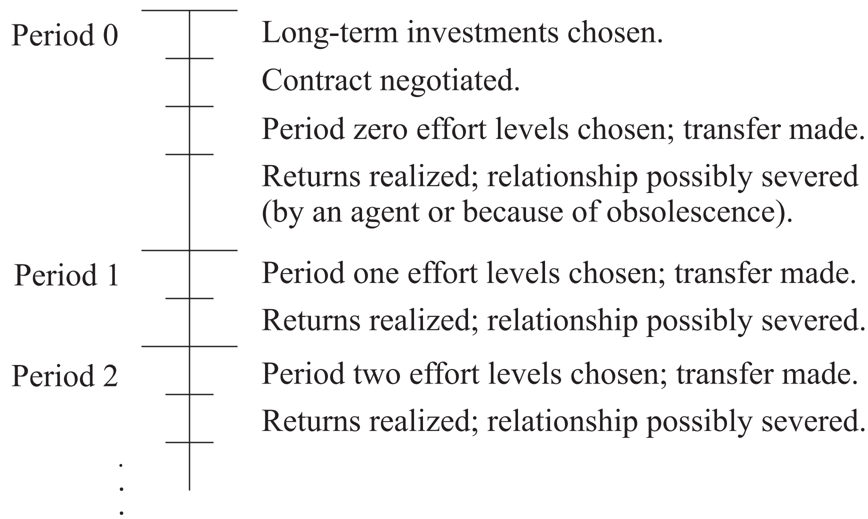


Figure 1: Timing of the trading relationship.

3.1 Effort Choice

Suppose that after making long-term investments in period 0, the newly-matched owners agree to a contract specifying a transfer m_i to the asset i owner in each of periods 1,2,..., as long as the relationship continues; note that $m_1 + m_2 = 0$ is implied. As long as both parties expend high effort in each period and do not choose to sever their relationship, the expected return to the owner of asset i is given by $[z_i(\alpha) + m_i]/\gamma + \delta\beta w_i/\gamma$, where $\gamma = 1 - \delta(1 - \beta)$ and w_i denotes the value to an owner of asset i of entering the asset i matching pool.

In an equilibrium where relationships have positive productivity, the owner of asset i must have no incentive to expend low effort, so the owner must face some punishment if low effort is chosen. Since low effort leads the relationship to become unproductive, the deviating asset i owner is punished by having to reenter the matching pool (there is no stronger punishment given our restrictions on allowable contracts, since the deviating owner can himself sever the relationship and reenter the matching pool). Thus, the deviator receives a payoff of $x_i(\alpha) + m_i + \delta w_i$ from the period when low effort is chosen. High effort can be sustained only if

$$[z_i(\alpha) + m_i]/\gamma + \delta\beta w_i/\gamma \geq x_i(\alpha) + m_i + \delta w_i$$

for $i = 1, 2$, which may be rewritten as

$$z_i(\alpha) - \gamma x_i(\alpha) \geq (1 - \beta)\delta(1 - \delta)w_i - \delta(1 - \beta)m_i. \tag{1}$$

Letting $x(\alpha) = x_1(\alpha) + x_2(\alpha)$ and $w = w_1 + w_2$, we may sum equation (1) over $i = 1, 2$ to obtain the following necessary condition for high effort to be sustained:

$$s(\alpha) \equiv z(\alpha) - \gamma x(\alpha) \geq (1 - \beta)\delta(1 - \delta)w. \tag{2}$$

Further, (2) gives a sufficient condition for sustainability of high effort, since whenever (2) is satisfied, (1) can be made to hold for $i = 1, 2$ via an appropriate choice of m_i. We refer to (2) as the *effort constraint.*

3.2 Contract Negotiation

Consider next the contract negotiation that follows the owners' choices of long-term investment. If, at the beginning of period 0, the owners select α so that (2) fails to hold, then high effort cannot be sustained and the relationship collapses at the end of period 0; in this case, the relationship is severed immediately and the owner of asset i obtains δw_i in the continuation.

On the other hand, suppose that (2) is satisfied under the selected value of α. The owners can therefore agree on values m_1 and m_2 satisfying their individual effort constraints (1). The expected future returns of their relationship, discounted to period 0, are given by:

$$g(\alpha) = z(\alpha)/\gamma + \beta\delta w/\gamma.$$

This value does not reflect the cost of investment $\alpha_1 + \alpha_2$, since the latter cost is sunk at the time the contract is negotiated. It is not difficult to show that (2) implies $g(\alpha) \geq \delta w$, and thus the agents jointly prefer to maintain their relationship as opposed to returning to the matching market.

In addition to agreeing on m_1 and m_2, the agents negotiate an upfront payment to divide the value $g(\alpha)$. Our bargaining solution implies that the value is evenly split, subject to each owner receiving at least his outside option δw_i. That is, if $g(\alpha)/2 \geq \delta w_i$, $i = 1, 2$, then each owner receives $g(\alpha)/2$, while if $g(\alpha) < \delta w_i$ for some i, then the owner of asset i receives δw_i and the other owner receives $g(\alpha) - \delta w_i$. As we show below, in equilibrium the owners will have no incentive to make investments inducing $g(\alpha)/2 < \delta w_i$, allowing us to ignore the possibility of binding outside option constraints; thus, the utility of the asset i owner immediately following contract negotiation may be written:

$$\overline{u}_i(\alpha) = \begin{cases} g(\alpha)/2 & \text{if } s(\alpha) \geq (1-\beta)\delta(1-\delta)w, \\ \delta w_i & \text{if } s(\alpha) < (1-\beta)\delta(1-\delta)w. \end{cases} \tag{3}$$

3.3 Equilibrium Investment

Finally, consider the equilibrium investment choices of the trading partners under the negotiation outcomes given in (3). Since the asset owners choose their investments simultaneously and independently, an equilibrium $\overline{\alpha} = (\overline{\alpha}_1, \overline{\alpha}_2)$ has the property that, for $i = 1, 2$, $\overline{\alpha}_i$ solves the following problem:

$$\max_{\alpha_i \geq 0} \overline{u}_i(\alpha) - \alpha_i, \text{ given } \alpha_j = \overline{\alpha}_j. \tag{4}$$

We study the equilibrium investment choices by first examining the owners' reaction functions in the absence of the effort constraint built into the definition of $\overline{u}_i$. We then inquire as to how adding the effort constraint alters these reaction functions. Given our assumptions on the function z, it follows that g is strictly concave and strictly increasing in α_1 and α_2, and $g(\alpha)/2-\alpha_i$ is uniquely maximized in α_i by the positive and strictly-increasing function $\alpha_i^R(\alpha_j)$, serving as the *unconstrained reaction function* of owner i. Figure 2 illustrates the two unconstrained reaction functions, where the unique intersection $\alpha^N = (\alpha_1^N, \alpha_2^N)$ gives the *unconstrained Nash equilibrium* investment choices in the absence of the effort constraint.

The figure also depicts a few other objects that will be used in the subsequent analysis. The *efficient point* $\alpha^E = (\alpha_1^E, \alpha_2^E)$ indicates the investment levels at which the joint returns are maximized, while the shaded region P represents the set of α such that $z(\alpha)/2\gamma - \alpha_i \geq 0$ for $i = 1, 2$, i.e., when $w = 0$, nonnegative returns are obtained by both owners if and only if $\alpha \in P$. The set P will be useful later in establishing the existence of a positive-investment market equilibrium. Further, we assume that there exists a point $\alpha^F = (\alpha_1^F, \alpha_2^F)$ at which $z(\alpha^F)/2\gamma - \alpha_i^F = 0$ for both i, so that each owner obtains zero returns when $w = 0$.

We now distinguish between two cases.

1. Overinvestment Case. $s(\alpha)$ is a strictly increasing function of α_1 and α_2. In this instance, the effort constraint (2) is satisfied if and only if α_1 and α_2 are sufficiently large. This possibility is considered in Figure 3. The curve labeled w gives the set of α that satisfy $s(\alpha) = (1-\beta)\delta(1-\delta)w$, so that (2) holds if and only if α lies *above* the curve. For this value of w, the unconstrained Nash equilibrium α^N satisfies the effort constraint, and so it will determine the investment choices.

For the larger value $w' > w$, however, (2) is satisfied only by the smaller set of α that lie above the curve w'. In this case, the actual reaction functions determined by (4) will coincide with the curve w' at points where the unconstrained reaction functions lie below the curve, as long as an owner would not prefer to induce a violation of the effort constraint by choosing $\alpha_i = 0$; in the following subsection we show that owners will prefer not to do the latter. Thus, the reaction functions intersect at every point of the segment AB, and any point along this segment gives (constrained) Nash equilibrium choices of investment levels. Observe that higher values of w lead the segment AB to shift upward and to the right in a continuous manner.

With this, we may define the Nash equilibrium investment choices for given w, written $\overline{\alpha}(w)$, as follows. As long as $s(\alpha^N) \geq (1-\beta)\delta(1-\delta)w$, we may set $\overline{\alpha}(w) = \alpha^N$, since the unconstrained Nash equilibrium satisfies the effort constraint. For $s(\alpha^N) < (1-\beta)\delta(1-\delta)w$, we select the element of the

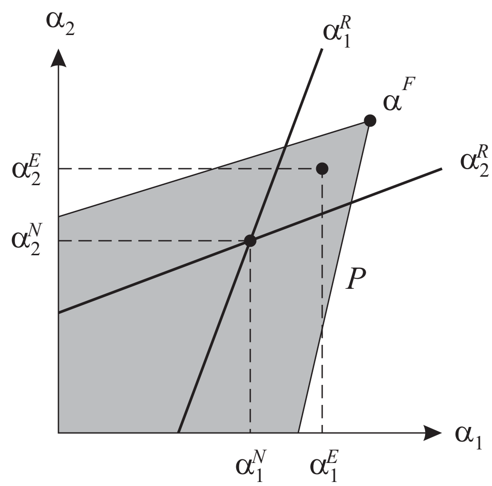


Figure 2: Reaction function and the positive return region.

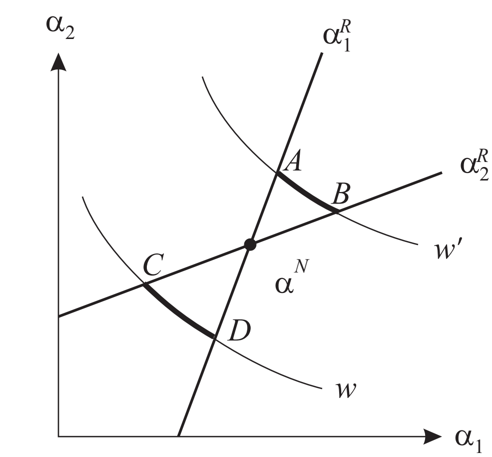


Figure 3: Nash equilibrium investment subject to effort constraint.

segment AB that maximizes the sum of net returns, subject to the condition that the selection lie in the set P; if no element of AB lies in P, then the owners choose zero investment and sever the relationship immediately. Thus, $\overline{\alpha}(w)$ is given by:

$$\overline{\alpha}(w) = \begin{cases} \alpha^N, & \text{if } s(\alpha^N) \geq (1-\beta)\delta(1-\delta)w, \\ \arg\max_{\alpha \in AB \cap P}\{g(\alpha) - \alpha_1 - \alpha_2\}, & \text{if } s(\alpha^N) < (1-\beta)\delta(1-\delta)w \\ & \quad \text{and } AB \cap P \neq \emptyset, \\ (0,0), & \text{if } s(\alpha^N) < (1-\beta)\delta(1-\delta)w \\ & \quad \text{and } AB \cap P = \emptyset. \end{cases} \tag{5}$$

The value of joint returns under this solution may be written:

$$v(w) = g(\overline{\alpha}(w)) - \overline{\alpha}_1(w) - \overline{\alpha}_2(w). \tag{6}$$

Clearly, $v(w)$ is a continuous function of w.

2. Underinvestment Case. $s(\alpha)$ is a strictly decreasing function of α_1 and α_2. Here the effort constraint (2) is satisfied if and only if α_1 and α_2 are sufficiently small. In Figure 3, points *below* the curve w' now give the set of α that satisfy (2) for the value w', and in this case α^N would give the Nash equilibrium investment choices. For the *higher* value $w > w'$, the curve labelled w gives the upper bound of the points that satisfy (2), and any point on the segment CD gives a Nash equilibrium. Note that as w rises, the segment CD shifts continuously downward and to the left. We may define the equilibrium investment levels $\overline{\alpha}(w)$ and joint returns $v(w)$ just as in (5) and (6), except that AB in (5) is replaced by CD, and further we may ignore the constraint $\alpha \in P$ since the segment CD must lie entirely in region P.

3.4 Market Equilibrium

We now derive steady-state matching equilibria incorporating the within-period investment equilibria obtained in the preceding subsections; we will refer to these as *market equilibria*. Letting θ denote the proportion of each asset-owner population that is currently matched in a given period, it follows that the steady-state hypothesis implies:

$$\theta = \theta(1-\beta) + (1-\theta+\theta\beta)\lambda. \tag{7}$$

The first term on the right-hand side of (7) gives the proportion of agents matched in the previous period whose relationships do not become obsolete, while the second term gives the proportion of unmatched agents (including those who entered the matching pool at the end of the previous period) who successfully make matches at the start of the period. Solving (7) gives the steady-state proportion of matched agents:

$$\theta^* = \frac{\lambda}{\beta + (1-\beta)\lambda}. \tag{8}$$

It remains to calculate the equilibrium value of entering the matching pools. The expected payoff w_i obtained by an asset i owner who is in the matching pool at the start of a period must satisfy:

$$w_i = \lambda v_i(w) + (1-\lambda)\delta w_i, \tag{9}$$

where $v_i(w)$ gives the owner's payoff in the investment equilibrium:

$$v_i(w) \equiv g(\overline{\alpha}(w))/2 - \overline{\alpha}_i(w).$$

Summing (9) over i and solving for $w = w_1 + w_2$ yields:

$$w = \overline{\lambda} v(w), \tag{10}$$

where $\overline{\lambda} = \lambda/(1-(1-\lambda)\delta)$. The values θ^* and w^*, together with the corresponding investment choices $\alpha^* = \overline{\alpha}(w^*)$, constitute a market equilibrium if w^* satisfies (10) and the individual owners' values $w_i^* = \overline{\lambda} v_i(w^*)$, $i = 1, 2$, are both nonnegative.

Existence of positive-investment market equilibria turns on whether it is possible to satisfy the effort constraint (2) at a solution to (10). In the overinvestment case, a sufficient condition for possible satisfaction of the effort constraint can be obtained by restricting payoffs at the point α^F, as the following proposition demonstrates.

Proposition 1. Suppose the overinvestment case holds. If $s(\alpha^F) \geq 0$, then there exists a market equilibrium in which $\alpha^* > 0$.

Proof. Given in the Appendix.

The condition $s(\alpha^F) \geq 0$ assures that there are points in region P at which the effort constraint can be satisfied for $w \geq 0$. The proof then locates a point w^* that satisfies (10). As long as $s(\alpha^F) > 0$, the equilibrium necessarily has $w^* > 0$. When $s(\alpha^F) < 0$, in contrast, the effort constraint must be violated at every $\alpha \in P$, so that satisfying the constraint would require at least one owner to accept a negative payoff. In this case, it is not possible to sustain productive trading relationships in a market equilibrium.

The next proposition shows that, in the underinvestment case, satisfaction of the effort constraint requires that payoffs be restricted at the point $\alpha = (0, 0)$.

Proposition 2. Suppose the underinvestment case holds. If $z(0, 0) > x(0, 0) \geq 0$, then there exists a market equilibrium in which $\alpha^* > 0$.

Proof. Given in the Appendix.

The condition $z(0, 0) > x(0, 0) \geq 0$, which implies $s(0, 0) > 0$, is sufficiently strong to ensure satisfaction of the effort constraint when the relationship involves zero investment and rematching is immediate upon severing the relationship; i.e., $\lambda = 1$. If this condition does not hold, then either the effort constraint must fail for every α, or else we must have $v(w) > w$ for all $w \geq 0$ associated with positive investment, in which case (10) cannot be satisfied for λ close to unity.[7]

4 Market Friction and Welfare

In this section we consider the effect of market friction, as reflected by the matching probability λ, on the value of trading relationships in the market equilibrium. Further, we focus on the case of β very close to zero, which implies that exogenous obsolescence is very unlikely and nearly all agents are matched at any point in time; thus, the agents' total payoffs in any period are nearly proportional to the steady-state value of productive trading relationships, which is $v(w^*)$. This value may be regarded as a direct measure of social welfare to the extent that agents derive their utility from consuming the output of productive relationships.

4.1 Value of Trading Relationships

To study how λ affects the equilibrium value of trading relationships, we first consider the form of the function $v(w)$, which gives the Nash equilibrium value of trading relationships for given w; market equilibria are analyzed in

[7]Other consitions may be imposed to guarantee the existence of a positive-investment market equilibrium. For example, in the underinvestment case, if $z(0, 0) > \gamma x(0, 0)$, then there is a number $\widehat{\lambda} > 0$ such that existence is assured for all $\lambda < \widehat{\lambda}$.

the following subsection. Given our focus on the small β case, the function $v(w)$ will be related to the values arising in the unconstrained Nash and efficient investment outcomes for $\beta = 0$, which are denoted by v^N and v^E, respectively:

$$v^N = \frac{z(\alpha^N)}{1-\delta} - \alpha_1^N - \alpha_2^N, \quad v^E = \frac{z(\alpha^E)}{1-\delta} - \alpha_1^E - \alpha_2^E.$$

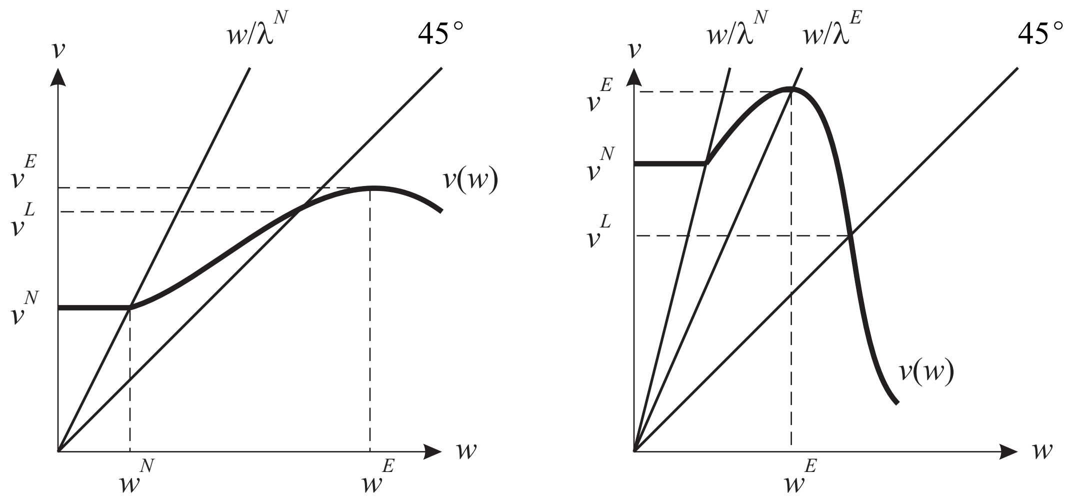
Note that v^N and v^E are strictly positive as a consequence of the fact that α^N and α^E lie in region P, and $v^E \geq v(w)$ for all w as a consequence of the efficiency of α^E. Further, for this section we assume $s(\alpha^N) > 0$, meaning that the unconstrained Nash equilibrium in investment choices satisfies the effort constraint (2) when $w = 0$. It follows that $v(w) = v^N$ when w is close to zero.

It may also happen that $v(w) = v^E$ for some value of w. In the overinvestment case, we have $s(\alpha^E) > s(\alpha^N)$, making it possible for the efficient outcome α^E to satisfy the effort constraint. In fact, for the effort constraint to be binding at $\alpha = \alpha^E$, it must be true that w takes on the value w^E defined by:

$$s(\alpha^E) = \delta(1-\delta)w^E. \tag{11}$$

Comparing (11) and (2), it may be seen that under (11), the effort constraint holds with equality and $\alpha = \alpha^E$ and $w = w^E$, where we have set $\beta = 0$. For $w = w^E$ the owners' reaction functions pass through the point α^E, and clearly joint returns are maximized by choosing $\overline{\alpha}(w^E) = \alpha^E$. Thus, w^E gives the value of entering the matching pool that induces owners to choose the efficient investment levels. Moreover, it is easy to see that $\overline{\alpha}(w) \neq \alpha^E$ for any other value of w, so it follows that $v(w) < v^E$ if $w \neq w^E$.

The function $v(w)$ in the overinvestment case is illustrated in Figure 4 under the assumptions $v(w^E) < w^E$ and $v(w^E) > w^E$. As shown, $v(w) = v^N$ for sufficiently small values of w, as a consequence of the assumption $s(\alpha^N) > 0$; further, $v(w)$ is uniquely maximized at $w = w^E$ in either case, as argued in the preceding paragraph. The value w^N indicates the level of w at which the effort constraint just binds when $\alpha = \alpha^N$. As shown in panels (a) and (b), an increase in w from the point w^N leads to a higher value of $v(w)$, until the maximizing point w^E is reached; $v(w)$ then falls as w rises further. Figure 5 depicts $v(w)$ for the underinvestment case. Note that $v(w)$ must fall as w rises above w^N, reflecting the fact that the corresponding shift in the effort constraint pulls α further from the efficient outcome.



(a) $v(w^E) < w^E$ (a) $v(w^E) > w^E$

Figure 4: Welfare in the overinvestment case.

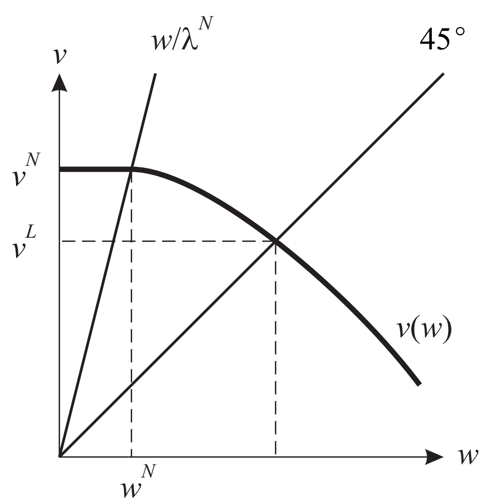


Figure 5: Welfare in the underinvestment case.

4.2 Effect of Market Friction

Market equilibrium w^*, as defined in (10), occurs at the point where the curve $v(w)$ intersects the ray $w/\overline{\lambda}$. In the two panels of Figure 4, the ray $w/\overline{\lambda}$ passing through the point (w^N, v^N) is labelled λ^N, and it follows at once that the market equilibrium has $w^* \leq w^N$ and $v(w^*) = v^N$ for $\overline{\lambda} \leq \lambda^N$. Thus, when market friction is high, in the sense that the probability of rematching is low, the effort constraint becomes relatively easy to satisfy, and in particular the unconstrained Nash equilibrium investment choices satisfy the constraint. Here the opportunistic behavior associated with the noncontractibility of investment is unaffected by the market interaction.

For $\overline{\lambda} > \lambda^N$, in contrast, the intersection occurs at $w^* > w^N$, and the market equilibrium must depart from the unconstrained Nash outcome. Panel (a) depicts the case $v(w^E) < w^E$: as $\overline{\lambda}$ approaches unity, welfare converges to a value v^L that gives the highest welfare level obtainable over the range of possible $\overline{\lambda}$. Observe however that $v^L < v^E$; in this case, taking $\overline{\lambda} = 1$, which eliminates all market friction, will maximize welfare, but the maximized level falls short of the efficient level.

The case $v(w^E) > w^E$ is considered in panel (b). Here welfare is uniquely maximized by $\overline{\lambda} = \lambda^E$, and moreover, the efficient level v^E is obtained. Thus, the market is able to mitigate against all adverse efficiency consequences of opportunism in forming relationships. Importantly, we have $\lambda^E < 1$, so that a positive degree of market friction is necessary in order to obtain the efficient welfare level. As $\overline{\lambda}$ rises from λ^E, equilibrium welfare declines, and in this example, setting $\overline{\lambda} = 1$ gives a limiting outcome v^L that minimizes welfare over the possible values of $\overline{\lambda}$. In this instance, eliminating market friction has very adverse welfare consequences.

The following proposition summarizes these observations.

Proposition 3. Suppose that the overinvestment case holds, and $s(\alpha^N) > 0$.
(a) There exists $\lambda^N > 0$ such that $v(w^*) = v^N$ if $\overline{\lambda} \leq \lambda^N$.
(b) If $v(w^E) < w^E$, then equilibrium welfare is maximized at $\overline{\lambda} = 1$, and the maximized value of welfare necessarily exceeds v^N if $v^N > w^N$.
(c) If $v(w^E) > w^E$, then equilibrium welfare is maximized at $\overline{\lambda} = \lambda^E \in (0, 1)$, and the efficient welfare level v^E is attained.[8]

[8] There may be multiple intersections between $v(w)$ and rays $w/\overline{\lambda}$; for the proposition we take w^* to be the intersection at which w is largest. If $v^N \leq w^N$, then $\lambda^N \geq 1$, and equilibria with $v(w^*) = v^N$ exist for all $\overline{\lambda}$. As long as $v(\dot{w}) > \dot{w}$ for *any* $\dot{w} > w^N$, however, then there will also exist equilibria in which $v(w^*) > v^N$, along the lines indicated in parts

The key mechanism driving these results is the effect of market friction on the effort constraint. As market friction declines, corresponding to a rise in λ, locating a new trading partner becomes easier should the existing relationship be severed, shrinking the range of investment levels that satisfy the effort constraint. Newly-matched owners are thereby forced to overinvest relative to their reaction functions in order to ensure that cooperation will obtain in the ensuing relationship. This market-induced overinvestment counteracts the underinvestment incentives arising from the owners' inability to write complete contracts, and a decline in market friction can thereby increase the efficiency of trading relationships.

As Proposition 3 indicates, the precise nature of market-friction effects is influenced by the relation between $v(w^E)$ and w^E, which in turn is determined by the relative attractiveness to the owners of low effort. This relationship is clarified in the following proposition.

Proposition 4. Suppose that the overinvestment case holds. Then we have

$$v(w^E) - w^E = \frac{x(\alpha^E) - z(\alpha^E)}{\delta} - \alpha_1^E - \alpha_2^E. \tag{12}$$

Proof. Given in the Appendix.

The right-hand side of (12) indicates the extent to which the effort constraint binds at the efficient investment point. If the function $x(\alpha)$ is relatively small, then the right-hand side of (12) is negative and $v(w^E) < w^E$ is implied. In this case, owners have little incentive to choose low effort, and a high level of w^E is required in order to make the effort constraint binding at α^E. If $x(\alpha)$ is relatively large, in contrast, then the right-hand side of (12) becomes positive and we have $v(w^E) > w^E$. With large $x(\alpha)$, there is a strong incentive to choose low effort, and the effort constraint binds at small values of w^E.

Thus, when incentives to choose low effort are weak, we have $v(w^E) < w^E$ and welfare is maximized when market frictions are eliminated. If, however, the owners face strong incentives to choose low effort, to the extent that $v(w^E) > w^E$, then it is possible to reduce market friction by too great an amount. In this case, there is a positive degree of friction sustaining efficient relationships, and further reductions serve to lower welfare by forcing the

(b) and (c) of the proposition.

owners to choose inefficiently high investment levels in order to ensure cooperation. In fact, as Figure 4b illustrates, reducing market friction to zero may yield the lowest attainable welfare level among equilibria with positive investment.

The latter conclusion will hold necessarily in the underinvestment case, to which we now turn. As shown in Figure 5, it continues to be true that $v(w) = v^N$ arises for a range of values $w \leq w^N$, but now $v(w)$ must fall as w rises above w^N, reflecting the fact that the reaction functions shift inward toward the origin, away from the point α^E. Thus, for $\overline{\lambda} \geq \lambda^N$, equilibrium welfare $v(w^*)$ falls as $\overline{\lambda}$ rises, and in the $\overline{\lambda} = 1$ limit, welfare is minimized at v^L. The following proposition states these results.

Proposition 5. Suppose that the underinvestment case holds, and $s(\dot{\alpha^F}) > 0$.
(a) There exists $\lambda^N > 0$ such that $v(w^*) = v^N$ if $\overline{\lambda} \leq \lambda^N$.
(b) If $v^N > w^N$, then $\lambda^N < 1$ and equilibrium welfare is decreasing in $\overline{\lambda}$ on the range $\overline{\lambda} \geq \lambda^N$.

In the underinvestment case, the market induces owners to reduce investment in order to ensure cooperation, so that the distortions associated with opportunism are compounded. Reducing market friction to zero assures the lowest welfare obtainable from among the positive-investment equilibria.

5 Integrated Organization

To this point we have assumed that the two assets needed in a production relationship are separately owned. We now introduce the possibility that, when a match is first made, owners can arrange a transaction whereby one owner sells his asset to the other, creating an integrated organizational form. This section considers how the incentives of the owners, and the profitability of productive relationships, depend on the organizational form. We first determine the value of an integrated relationship, and then we model the choice of organizational form by newly-matched owners as part of a market equilibrium.

Suppose there is a single agent who owns both assets; this agent had originally owned one of the assets and was matched with an owner of the other asset, from whom the other asset was purchased. The owner of the two assets is able to invest in one of the assets, say asset i, but he must hire a manager to invest in the other. The structure of trading between the owner and the manager follows the nonintegrated case of Section 2: the agents first make investments; a long-term contract is negotiated; and production commences. As in the nonintegrated case, we assume that at the end of each period, the asset match becomes obsolete with probability β. In this event, the owner divests himself of one of the assets and reenters the matching pool.

Consider first the contract-negotiation stage, where the owner and manager have chosen investment levels α_i and α_j, respectively. Assume that managers who own no assets are abundant in the marketplace, so that the owner is able to costlessly locate and hire a manager in any period. Further, let the prevailing wage for managers be normalized to zero. Recall that costs and benefits of effort accrue to asset owners, not managers; thus, managers have a reservation wage of zero for exerting either high or low effort in managing assets. After the asset j manager has invested α_j in the asset, the owner can replace the manager without forfeiting the investment in asset j, since the investment is specific to the asset; this gives the owner all of the bargaining power during contract negotiation. The owner will use this power to appropriate the full returns from both assets, and high effort on the part of the manager will be induced, while the owner himself will prefer to exert high effort. Thus, the owner's payoff from the contract negotiation is $g(\alpha)$, reflecting the stream of returns $z(\alpha)$ obtained as long as the match continues, together with the price w_j obtained from selling asset j to a nonowning manager in the period following obsolescence.

Although the owner can costlessly induce high effort from the asset j manager under our assumptions, he cannot induce this manager to make a positive investment in asset j at the beginning of their relationship, since the contract negotiation leaves the manager with zero returns from the investment. Thus, recalling that $g(\alpha) = z(\alpha)/\gamma + \beta\delta w/\gamma$, the owner chooses an investment level $\widehat{\alpha}_i$ that solves the following problem:

$$\max_{\alpha_i \geq 0} z(\alpha)/\gamma - \alpha_i \text{ given } \alpha_j = 0.$$

Further, the owner makes a profit-maximizing selection of which asset to invest in personally; thus, the value of an integrated relationship to the owner is given by:

$$v_3(w) = \max\{z(\widehat{\alpha}_1, 0)/\gamma - \widehat{\alpha}_1, z(0, \widehat{\alpha}_2)/\gamma - \widehat{\alpha}_2\} + \beta\delta w/\gamma.$$

We now reconsider the market equilibrium when newly-matched owners have the option of forming an integrated organization. Immediately following the making of a match, the owners choose whether or not to exchange the assets. If they opt for exchange, then they bargain over division of the value $v_3(w)$, and the production relationship assumes the integrated form, while otherwise the owners proceed with the nonintegrated relationship described in Section 2. We assume that in the latter case, the owners cannot carry out an asset exchange at any future time (e.g., the nonintegrated contract may contain provisions that excessively complicate future asset transfer); thus, an integrated organization can be formed only when a match is first made. As long as $v(w) > v_3(w)$, the owners will choose nonintegrated organization, while integration arises when $v(w) < v_3(w)$. A market equilibrium is therefore given by a pair θ^* and w^* satisfying (8) and:

$$w^* = \overline{\lambda} \max\{v(w^*), v_3(w^*)\}. \tag{13}$$

Conditions for existence of positive-investment equilibria are easily derived, along the lined of Propositions 1 and 2.

Let us now focus on the case of very small β, where equilibrium welfare approaches $\max\{v(w^*), v_3(w^*)\}$; further, $\beta = 0$ implies that the value of an integrated relationship is independent of w. We denote the latter value by v^I, where:

$$v^I = \max\{z(\widehat{\alpha}_1, 0)/\gamma - \widehat{\alpha}_1, z(0, \widehat{\alpha}_2)/\gamma - \widehat{\alpha}_2\}.$$

Figure 6 considers market equilibria in the over- and underinvestment cases, under the assumption $v^I < v^N$; here nonintegrated organization is preferable as long as the effort constraint does not bind. Observe that integrated organization emerges in equilibrium if and only if $\overline{\lambda} \geq \lambda^I$: a low level of market friction makes nonintegrated relationships very unprofitable, and owners are able to respond by shifting to the integrated form. With low levels of market friction, satisfying the effort constraint may involve large investment distortions under nonintegrated organization, making integrated forms relatively more profitable. Thus, integrated forms may tend to emerge when market frictions are low, but welfare would be greater if market frictions were high enough to support more efficient nonintegrated forms.[9]

[9] The results are altered under the alternative assumption $v^I > v^N$. In the overinvestment case, integration emerges in market equilibrium for very low levels of $\overline{\lambda}$; nonintegration obtains for an intermediate range of $\overline{\lambda}$ if $v^L < v^I$, or for the entire upper range of $\overline{\lambda}$ if $v^L > v^I$, where v^L is indicated in Figure 4. In the underinvestment case, integration occurs in every market equilibrium when $v^I > v^N$.

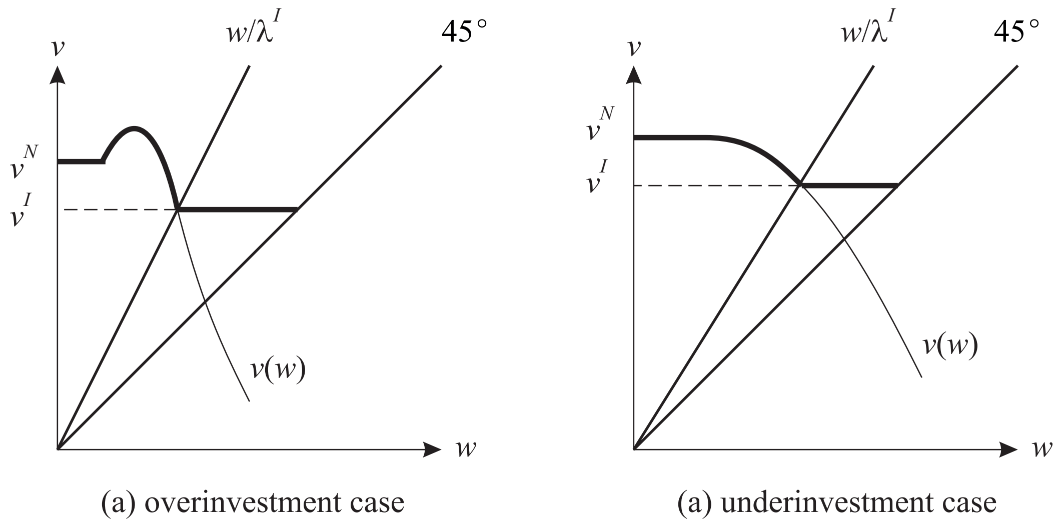


Figure 6: Equilibrium organizational form.

6 Structural Changes

6.1 Contractual Fragility

The immutability of investments made by owners at the start of a trading relationship introduces a fragility into relationships that can generate abrupt transition effects when parameters of the model are altered. This can be vividly seen in the case of a rise in λ to a higher level λ', which may be interpreted as an institutional liberalization or an introduction of improved communication technology. Such a reduction in market friction may or may not improve welfare in the long-run, as discussed in Section 4, but in the short-run a welfare-reducing "crash" may ensue, as a consequence of destabilization of the preexisting trading relationships.

To illustrate contractual fragility, consider the overinvestment case, and suppose agents are restricted to nonintegrated relationships. The economy begins in market equilibrium with $w^* = w$ and $\alpha^* = \alpha^N$; this situation is shown in Figure 3, where in the present instance the effort constraint requires that α lie above the curve w. When λ rises to λ', the new market equilibrium corresponds to $w^* = w' > w$, reflecting the higher investment levels that are chosen by newly-matched owners. In Figure 3, α must now lie above the curve w' in order to satisfy the effort constraint, meaning that *all* of the preexisting relationships, in which unconstrained Nash investment levels have been chosen, become unsustainable. Here the structural change undercuts preexisting relationships, initiating a phase of costly restructuring or rematching during which relationships must be reformed in conformity with the altered conditions.

6.2 Transition Effects with Nonrestructurability

The nature of transition effects following structural changes is easily studied under the assumption that relationships are nonrestructurable, meaning that agents are precluded from reinvesting in their relationship once the initial investment have been made. This assumption makes sense if reforming a preexisting relationship is subject to significantly greater costs than forming a new relationship. Such costs may include opportunism in exploiting existing institutional arrangements to affect the terms of the new contract (e.g., an owner might exploit provisions of the old contract to tie up his partner's outside lines of business). Added constraints due to opportunism may be compounded by difficulties in preserving the trading partners' specific "reputation capital" in the context of a contentious restructuring.

Thus, when an increase in λ causes preexisting relationships to be unsustainable, nonrestructurability implies that every pair of partners will immediately sever their relationship and reenter the matching pool. Correspondingly, welfare drops abruptly following the reduction in market friction, and the economy must pass through a low-welfare rematching phase unless the new value λ' is very close to unity. Figure 7 depicts the transition effect on welfare in the small β case, where $v(w^*)$ gives welfare in the market equilibria. We consider a rise in λ to the level $\lambda' < 1$ occurring at time t'. In this example, the reduction in market friction raises long-run welfare to $v(w')$. In the short-run, however, welfare declines steeply, and the benefits of reduced market friction are realized only gradually, as new matches are formed.

Adverse transition effects may be mitigated to some extent if trading partners can restructure their relationship without reentering the matching pool, allowing relationship-specific "matching capital" to be preserved. Restructuring may, however, be limited by scarcity of specialized factors, such as legal personnel, that are needed for implementing new contracts; this possibility is briefly considered in the next subsection. More broadly, structural change may be accompanied by a high demand for capital to reinvest in relationships, and if agents are assumed not to have sufficient outside income to finance their own investments, then capital rationing may become necessary. Either sort of restriction might slow the transition to the new steady-state and generate the kinds of transition dynamics shown in Figure 7, even when trading partners can preserve their matches. In any case, the structural change renders the stock of existing investments obsolete, and makes necessary some sort of costly reinvestment.

6.3 Limited Contracting Capacity

We have assumed thus far that asset owners face no added costs of investment and contract negotiation, with respect to either newly-formed or restructured relationships. This assumption may be unrealistic, however, to the extent that investment and negotiation require the input of specialized personnel such as lawyers, consultants and brokers. Scarcity of such personnel may create a bottleneck that limits the restructuring of preexisting relationships following a structural change, and thereby forcing a gradual low-output transition to the new steady-state.

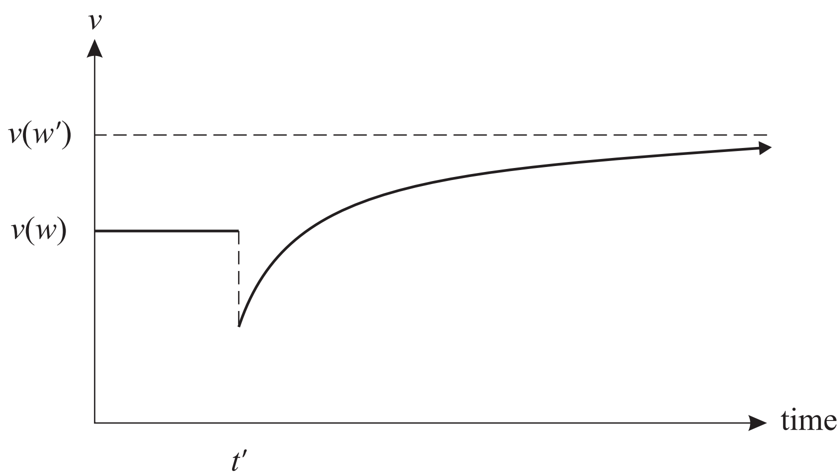


Figure 7: Effects of a reduction in market friction.

To investigate this possibility, we modify our model slightly to incorporate a new class of agents, called negotiators. In order to negotiate a new contract or restructure an existing one, a matched pair of owners must locate a negotiator, which is done on a separate matching market operating in parallel to the asset-matching market described in Section 2. For simplicity, we assume that negotiators provide their service at no charge. The need to match with a negotiator can be interpreted simply as reflecting limited capacity to consummate contracts in the economy.

Suppose that an asset match must occur first before a negotiator match can be sought, and let λ give the probability of making a match in the negotiator matching pool. The value of entering the asset matching pool becomes:

$$w_i = \lambda^2 v_i(w) + \lambda(1-\lambda)\delta\widetilde{w}_i + (1-\lambda)^2\delta w_i, \tag{14}$$

where $\widetilde{w}_i$ gives the value of beginning a period in the negotiator matching pool. Here we specify that a newly-matched pair of asset owners may seek a negotiator within the same period, so that a productive relationship begins in the current period with probability λ^2. The value of entering the negotiator matching pool is given by:

$$\widetilde{w}_i = \lambda v_i(w) + (1-\lambda)\delta\widetilde{w}_i. \tag{15}$$

Combining (14) and (15), it follows that market equilibrium is again given by (10), where in this case we have:

$$\overline{\lambda} = \frac{\lambda^2}{1-(1-\lambda)^2\delta}\left[1+\frac{(1-\lambda)\delta}{1-(1-\lambda)\delta}\right].$$

With this modification, the results of Sections 3 and 4 continue to hold as stated, and the framework can be straightforwardly extended to allow for integrated organization. The key new effect is that, following a structural change in which preexisting relationships become unsustainable, productivity can be restored only gradually even when existing asset matches can be preserved, due to the limited contract-writing capacity implicit in the need to match with a negotiator.

7 Conclusion

We have developed a model in which agents form pairwise repeated trading relationships and make relation-specific investments as part of a dynamic market equilibrium in which agents must search for new trading partners when a relationship terminates. Market frictions play a central role in resolving incentive problems associated with contractual incompleteness: if incentives to choose low effort in the ongoing relationship are sufficiently strong, then there is an optimal positive level of market friction. Moreover, welfare may be minimized should market friction be lowered to zero. Integrated organizational forms may emerge as a response to low market friction, and a reduction in market friction may further lead to adverse transition effects by disrupting preexisting relationships.

The interaction between bilateral incentives and market friction that drives our results would appear to have broad relevance to a range of economic problems. Principal-agent relationships, for example, might be usefully viewed as growing out of a market process whereby pools of potential principals and agents seek to make matches. Further, explicit consideration of contracting issues in a market context could be important in clarifying the impact of institutional constraints on aggregate phenomena such as nominal rigidity and business cycles. These issues and others suggest that our approach may prove to be important in a variety of future applications.

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Appendix

Proof of Proposition 1. Let w^F be defined by:

$$s(\alpha^F) = (1-\beta)\delta(1-\delta)w^F.$$

It is not difficult to show that $\overline{\alpha}(w^F) = \alpha^F$. Since $w^F \geq 0$, and by the definition of α^F, we have:

$$v(w^F) = \frac{\beta\delta w^F}{\gamma} \leq w^F. \tag{16}$$

Further, in the overinvestment case, the effort constraint is weaker for $w = 0$ than for $w = w^F$, so at $w = 0$ we have $AB \cap P \neq \emptyset$, assuring $\overline{\alpha}(0) \in P$. Thus:

$$v(0) = \frac{z(\overline{\alpha}(0))}{\gamma} - \overline{\alpha}_1(0) - \overline{\alpha}_2(0) \geq 0. \tag{17}$$

As long as $s(\alpha^F) > 0$, we have $w^F > 0$ and the inequalities in (16) and (17) are strict. Further, since $v(w)$ is a continuous function of w, there exists $\overline{w} \in (0, w^F)$ such that $v(w) = w$. We have $w/v(w)$ continuous on $[0, \overline{w}]$, with $w/v(w) = 0$ at $w = 0$ and $w/v(w) = 1$ at $w = \overline{w}$, so there exists w^* such that $w^*/v(w^*) = \overline{\lambda}$. If $s(\alpha^F) = 0$, then $w^F = 0$ and we have an equilibrium in which $w^* = 0$, $\alpha^* = \alpha^F$ and $v_i(\alpha^*) = 0$ for $i = 1, 2$.

We now verify that $w_i^* \geq 0$ for $i = 1, 2$. We also show how this implies that, at the solution w^* and $\alpha^* = \overline{\alpha}(w^*)$, the outside option constraints bind in neither the investment stage nor in the contract negotiation stage. The value of entering the matching pool for an asset i owner is given by:

$$w_i^* = \overline{\lambda} v_i(w^*) = \overline{\lambda}\left[\frac{z(\alpha^*)}{2\gamma} - \alpha_i^*\right] + \frac{\overline{\lambda}\beta\delta w^*}{2\gamma} \geq 0,$$

where nonnegativity of the term in braces follows from $\alpha^* \in P$. Thus:

$$\frac{g(\alpha^*)}{2} > \frac{g(\alpha^*)}{2} - \alpha_i^* = v_i(w^*) \geq \overline{\lambda} v_i(w^*) = w_i^* \geq \delta w_i^*.$$

Finally, each owner prefers the choice $\alpha_i^* > 0$ that satisfies the effort constraint, which gives a payoff of $v_i(w^*)$, to the profit-maximizing investment level that would violate the effort constraint, which is $\alpha_i = 0$, since in the latter case the relationship would be immediately severed, giving a payoff of $\delta w_i^* \leq v_i(w^*)$. *Q.E.D.*

Proof of Proposition 2. Let $\overline{w}$ be defined by $s(0,0) = (1-\beta)\delta(1-\delta)\overline{w}$. We have $\overline{w} > 0$ since $s(0,0) = z(0,0) - \gamma x(0,0) > 0$, using $z(0,0) > x(0,0) \geq 0$ and $\gamma < 1$. Further, $\overline{\alpha}(\overline{w}) = (0,0)$ since only this point satisfies (2). Thus:

$$v(\overline{w}) - \overline{w} = \frac{z(0,0)}{\gamma} - \frac{(1-\delta)\overline{w}}{\gamma} = \frac{-z(0,0) + x(0,0)}{(1-\beta)\delta} < 0,$$

using the definition of $\overline{w}$ and the assumption $z(0,0) \geq x(0,0)$. Next, if $s(\alpha^F) \leq 0$, then we have $w^F \leq 0$, where w^F is defined in the proof of Proposition 1. In this case $\overline{\alpha}(0) \in P$ and we have $v(0) > 0$ as in (17). If $s(\alpha^F) > 0$, then $w^F > 0$, and it follows that for all $w \leq w^F$, (2) is satisfied at every $\alpha \in P$. In this case, we have $\overline{\alpha}(0) = \alpha^N$, and (17) is satisfied since $\alpha^N \in P$. As in the proof of Proposition 1, $w/v(w)$ moves continuously from zero to a quantity greater than unity as w shifts from zero to $\overline{w}$, ensuring that $w^*/v(w^*) = \overline{\lambda}$ for some $w^* \in (0, \overline{w})$. *Q.E.D.*

Proof of Proposition 4. We establish the result for general values of β, in which case the right-hand side of (12) is given by:

$$r(\alpha) = \frac{x(\alpha) - z(\alpha)}{(1-\beta)\delta} - \alpha_1 - \alpha_2.$$

Since (2) binds at w^E, we have:

$$s(\alpha^E) = z(\alpha^E) - \gamma x(\alpha^E) = (1-\beta)\delta(1-\delta)w^E. \tag{18}$$

Further:

$$v(w^E) = \frac{z(\alpha^E)}{\gamma} + \frac{\beta\delta w^E}{\gamma} - \alpha_1^E - \alpha_2^E. \tag{19}$$

Using (18) and (19), it is straightforward to establish that:

$$v(w^E) - w^E = \frac{x(\alpha^E) - z(\alpha^E)}{(1-\beta)\delta} - \alpha_1^E - \alpha_2^E.$$

Q.E.D.

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