Notes on Guerrieri and Lorenzoni (2015)

Johannes Wieland

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1 Introduction

- Agents fact idiosyncratic and aggregate risk.
- Markets are incomplete markets.
  - Only asset is a one-period bond.
  - Agents face a borrowing constraint.
- Credit crunch implemented by tightening borrowing limit.
  - Heterogeneity matters a lot since this shock would be irrelevant in a complete market economy.
- First paper (?) that combines nominal rigidities with precautionary savings.
- Computational challenge solved by shooting.

2 Model

2.1 Households

- Continuum of consumers $j \in [0, 1]$
- Preferences

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{jt}^{1-\sigma} - 1}{1-\sigma} + \chi \frac{(1-l_{jt})^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}} \right]$$

- Households produce consumption goods $y_{jt} = \theta_{jt} l_{jt}$.

- Budget constraint:

$$q_t b_{jt} + c_{jt} - \tau_{jt} \leq b_{j,t-1} + y_{jt}$$

- Taxes:

$$\tau_{jt} = \tau_t \text{ if } \theta_{jt} > 0$$
$$\tau_{jt} = \tau_t - v_t \text{ if } \theta_{jt} = 0$$

  Interpret as unemployment benefit.

- Borrowing constraint:

$$b_{jt} \geq -\phi$$

- FOC for consumers

$$c_{jt}^{-\sigma} \geq \beta q_t^{-1} \mathbb{E}_t[c_{j,t+1}^{-\sigma}]$$
$$\chi(1 - l_{jt})^{-\frac{1}{\eta}} \geq \theta_{jt} c_{jt}^{-\sigma}$$

  with equality if $b_{jt} > -\phi$ and $l_{jt} > 0$ respectively.

2.2 Government

- Evolution of bonds:

$$B_{t-1} + v_t u = q_t B_t + \tau_t$$

  where $u_t = u$ is the (time-invariant) fraction of unemployed agents.

- Initially fix B and v and let τ adjust to balance budget.

$$\tau_t = B(1 - q_t) + vu$$

2.3 Shocks

- Idiosyncratic income shocks follow an AR(1) when $\theta > 0$,

$$\theta_{jt} = \rho \theta_{j,t-1} + \varepsilon_{jt}, \qquad \varepsilon_{jt} \sim N(0, \sigma_\varepsilon^2$$

- In practice they will approximate this with a Markov process using the approximation in Tauchen (1986).

2.4 Equilibrium An equilibrium is a sequence of bond price $\{q_t\}$, a sequence of consumption and labor supply policies $\{C_t(b, \theta), N_t(b, \theta)\}$, a sequence of taxes $\{\tau_t\}$, and a sequence of distributions for bond holdings and productivity levels $\{\Psi_t\}$ such that, given the initial distribution $\{\Psi_0\}$:

1. $C_t(b, \theta)$ and $N_t(b, \theta)$ are optimal given $\{q_t\}$ and $\{\tau_t\}$.

2. Ψ_t is consistent with the consumption and labor supply policies,

3. the tax satisfies the government budget constraint,

$$\tau_t = B(1 - q_t) + vu$$

4. the bonds market clears,

$$\int b d\Psi_t(b, \theta) = B$$

Note:

- Here we define a sequence equilibrium.
- Dependence of distribution in policy function suppressed in the time subscript.

3 Computation

Computational difficulty:

- As in Krusell and Smith (1998) the distribution is now a time-varying state-variable.
- The trick used here is that there is idiosyncratic uncertainty, but everything happening at the macro level is deterministic.
- There are no further macro shocks after $t = 0$, so agents can perfectly forecast the path of interest rates and distributions.
- Thus, we only have to solve for a path of the real interest rate, rather than a function of the state variables.
- There is a cost in that we can no longer invoke stationarity and we have to resolve the model whenever we run a different experiment. We also study a 0-probability event from the perspective of the agents.

4 Calibration and and steady state algorithm

- If you want to see how a calibration section should be written, this paper is a nice example.
- $\beta = 0.9713$
- $\sigma = 4$
- $\eta = 1/1.88$
- $\chi = 12.48$
- $\rho = 0.967$
- $\sigma_\varepsilon = 0.017$
- Transition to unemployment $\pi_{e,u} = 0.057$
- Transition to employment $\pi_{u,e} = 0.882$
- $v = 0.10$
- $\phi = 1.04$
- $B = 1.60$

Some of these parameters are derived by targeting steady-state statistics:

1. Start with initial parameter values.
2. Find the optimal policy function:
   (a) Start with initial guess $c(b, \theta)$ on grids for b and the set of values for θ.
   (b) Calculate $E_\theta[c(b', \theta')^{-\sigma}]$. This is expected utility if b' is the optimal choice.
   (c) Determine c from $c^{-\sigma}(b', \theta) = \beta(1 + r)E_\theta[c(b', \theta')^{-\sigma}]$. This is the optimal choice of c if b' is optimal.
   (d) Calculate b^* from the budget constraint. So c is optimal choice given initial bond holdings b^*.
   (e) Interpolate $c(b^*, \theta)$ onto the grid for b. This is the new policy function. Repeat until convergence.
3. Find the steady-state distribution:

(a) From the policy function iteration we get the inverse bond function, $b^* = f^{-1}(b')$, given θ.

(b) Start with an initial cdf $\Psi_0(b', \theta)$ whose grid points are next periods bond choice b'. So think of this as the distribution of b next period.

(c) Interpolate to get the corresponding cdf for current bond holdings $\Psi_1(b, \theta)$. (The distribution has to be stationary.)

(d) Given initial grid point (b, θ), next period move to (b', θ').

(e) Use the transition matrix P for θ to capture the change from θ to θ'. Further, everyone at (b, θ) picks b' next period, so $\Psi_0(b', \theta) = P\Psi_1(b, \theta)$ is next periods distribution.

(f) Iterate until convergence.

4. Aggregate and calculate relevant statistics.

5. Update parameters and iterate until convergence.

5 Steady-State

- Analyzing the steady-state is very helpful for understanding how the transitions ultimately play out.

- Two key features of this economy: For a given θ the consumption function is concave and the labor supply function is convex.

- For low levels of b income effect dominates substitution effect.

- Implicit in this set-up are marginal propensities to work (MPLs) which may or may not be reasonable. Unlike MPCs we have much less evidence on that statistic and thus much less data to discipline the model. But these do matter for model outcomes, so you should try multiple versions and understand to what extent it matters.

- Significant mass of agents are at or near the borrowing constraint. Why the difference to KS? In KS consumers can engage in self-insurance by accumulating capital. This leads the economy itself to have more capital and thus more means for insurance. Here the supply of assets is fixed so this margin is turned off.

  Further, the consumers are very impatient, $\beta = 0.977$, so the equivalent complete-markets annualized interest rate is almost 10%. But the incomplete-markets interest

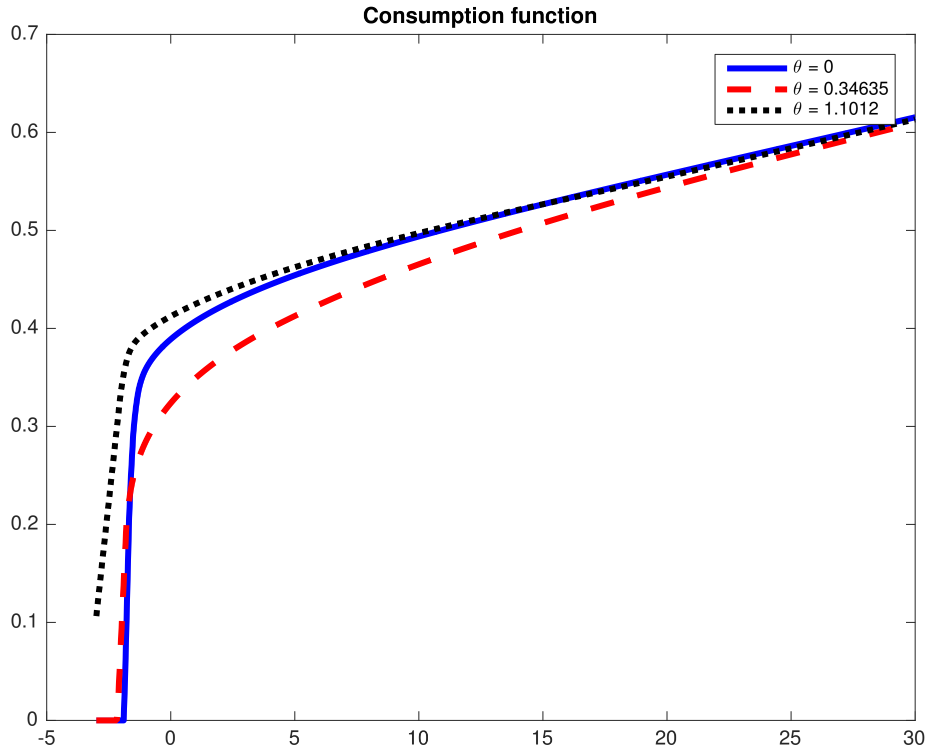

Figure 1 – Consumption function in GL model

rate is only 2.5% reflecting the large demand for precautionary assets. So many more consumers here look like hand-to-mouth given how unattractive the savings asset is.

- Note: we could have a higher β and a smaller drop the interest rate if consumers were more willing to substitute intertemporally. But, as we will see, this economy is already "too sensitive" to the interest rate.

- As in KS, high MPCs only for those individuals close to the borrowing constraint.

- Very large changes in labor income in the tail. Change in labor income equal to minus 20-25% of the change wealth.

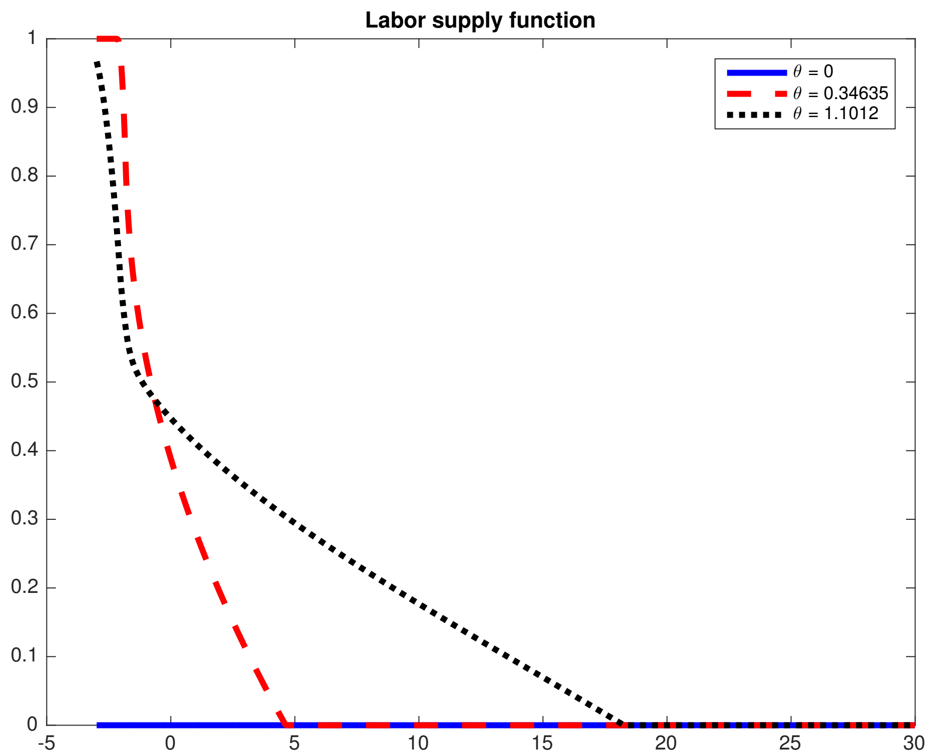

Figure 2 – Labor supply function in GL model

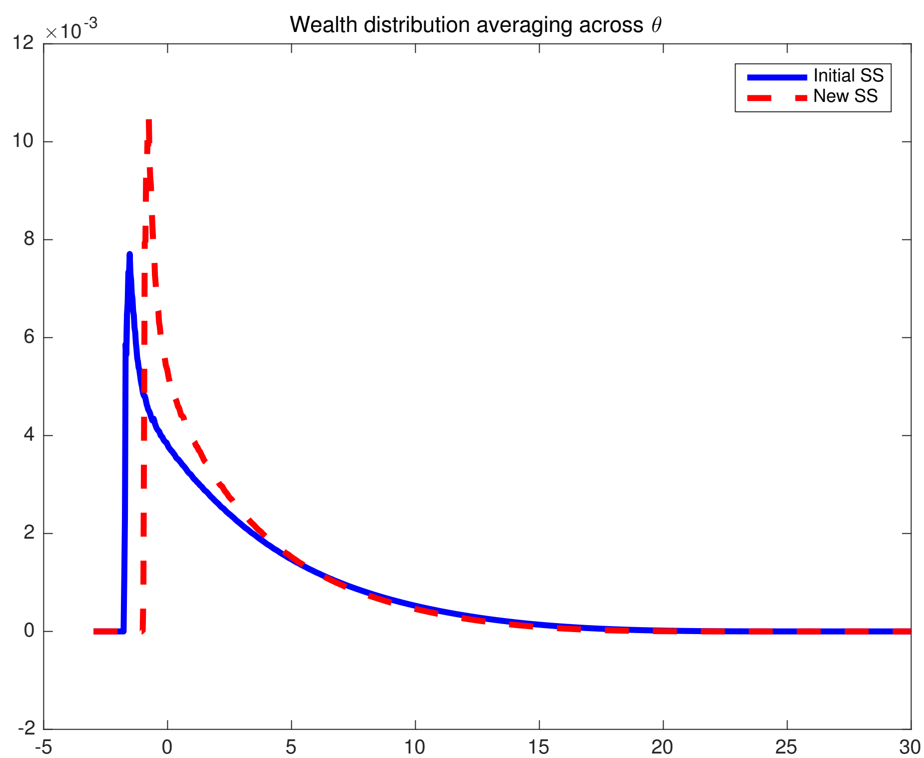

Figure 3 – Wealth distribution in GL model

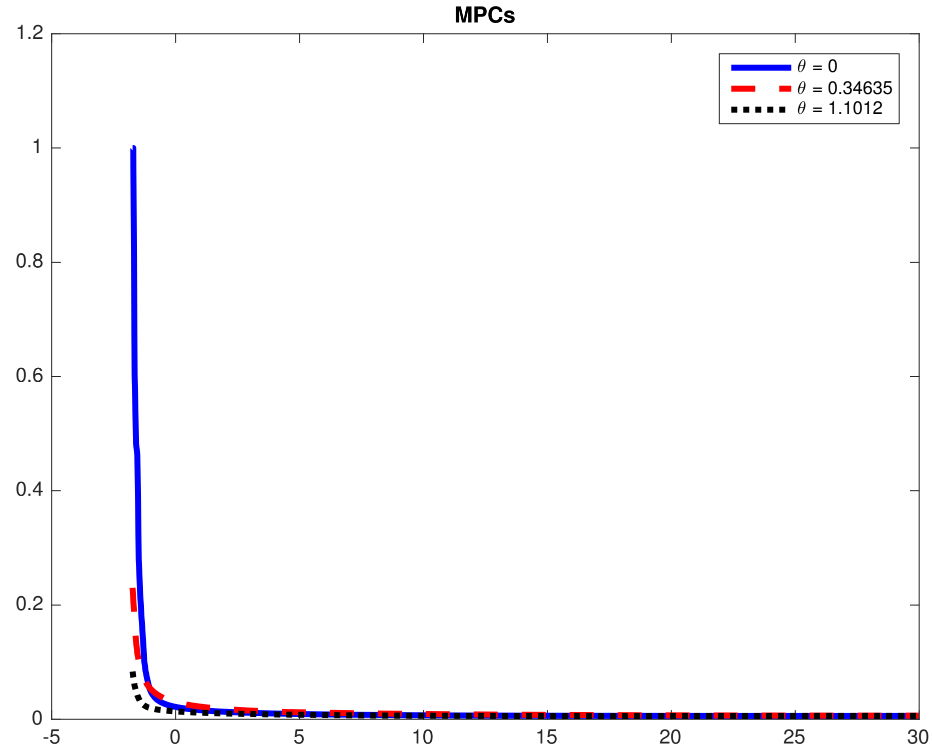

Figure 4 – MPCs in GL model

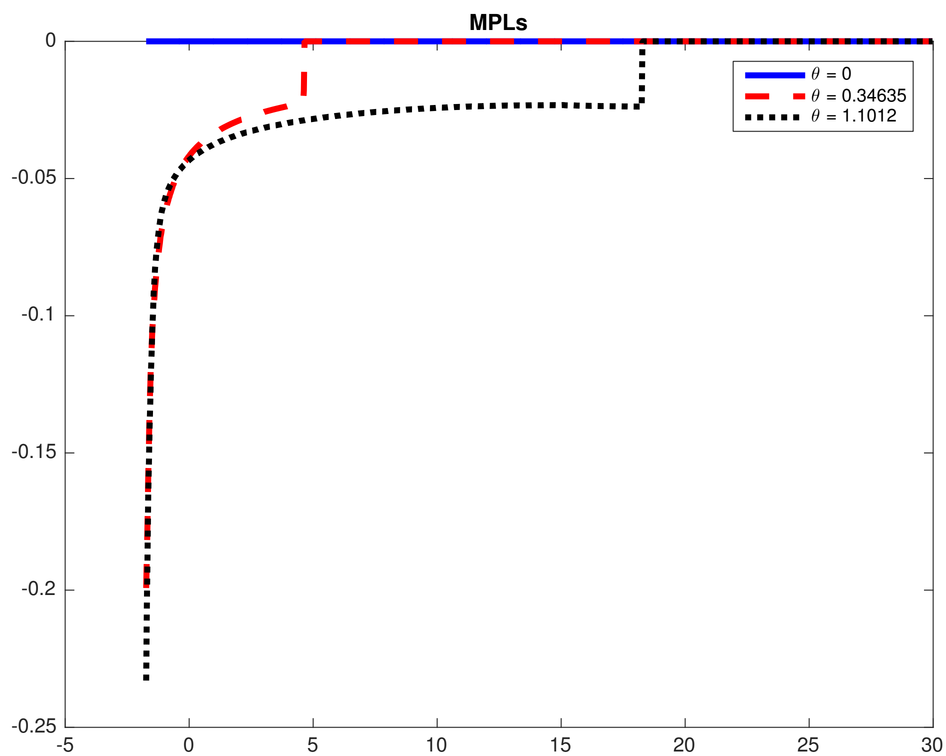

Figure 5 – MPLs in GL model

6 Dynamics

6.1 Experiment

- Path for borrowing limit:

$$\phi_t = \max\{\phi', \phi - \Delta t\}$$

  where $\phi' = 0.58 < \phi$.

- Designed to create a 10% fall in the debt-to-GDP ratio (why this target?)

- Chose Δ so the adjustment lasts 6 quarters to slow down repayment and prevent default from being an equilibrium outcome.

- Pretty big drop in the steady-state real interest rate from 2.5% to 1.5% (see figure 6).

- Interest rates drop because bond demand by all consumers with holdings less than ϕ' has to rise *and* because of greater precautionary savings by those with bond holdings more than ϕ'. Intuitively, everyone wants to borrow less so the price of borrowing (the real rate) declines.

6.2 Algorithm

1. Solve for the two steady-state of the economy given ϕ and ϕ'.

2. Pick a large T such that the economy is essentially in the new steady state after T periods, $C_T(b, \theta) = C(b, \theta)$, $N_T(b, \theta) = N(b, \theta)$

3. Guess a sequence of bond prices $\{r_t\}$ (which implies a sequence of tax rates).

4. Calculate optimal policies $C_t(b, \theta)$, $N_t(b, \theta)$ using policy function iteration going backwards (T-1 to t=0).

5. Solve for the distribution of bond holdings going forward given Ψ_0 and your policy functions.

6. Compute aggregate bond demand each period and update the bond price. If $B_t > B$ raise the bond price, else lower the bond price.

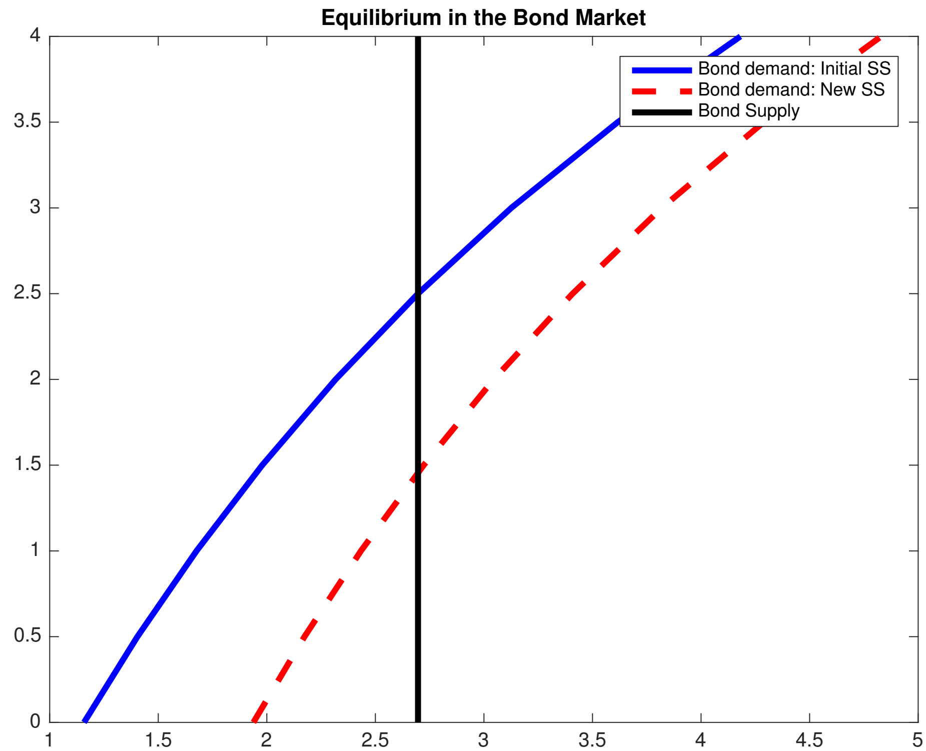

Figure 6 – Equilibrium in the bond market

6.3 Results

- Key result: real interest rate overshoots. It has to drop by more than the new steady-state interest rate.
- Output, consumption declines, labor supply expands.
- To understand why interest rates have to overshoot, note that the initial steady-state wealth distribution is a mean-preserving spread of the initial steady-state wealth distribution. (They both have the same mean B.) Figure 3 shows this.
- The bond-accumulation function $b' - b$ at the new steady-state is convex (figure 8). So integrating using the initial wealth-distribution gives a bond demand larger than B, since $b' - b$ is convex and we know integrating using the new wealth-distribution yields B. In English, at the new interest rate the bond demand by the deleveraging consumers exceeds the bond supply by the wealthy agents willing to decumulate. Thus, the interest rate has to fall further to clear the market.
- The same logic also explains why labor supply overshoots. The labor supply policy is convex, so integrating using the initial bond distribution results in larger average employment. This is where MPLs are important. If these were not as steep then the labor supply policy becomes concave and labor supply would fall following the shock.

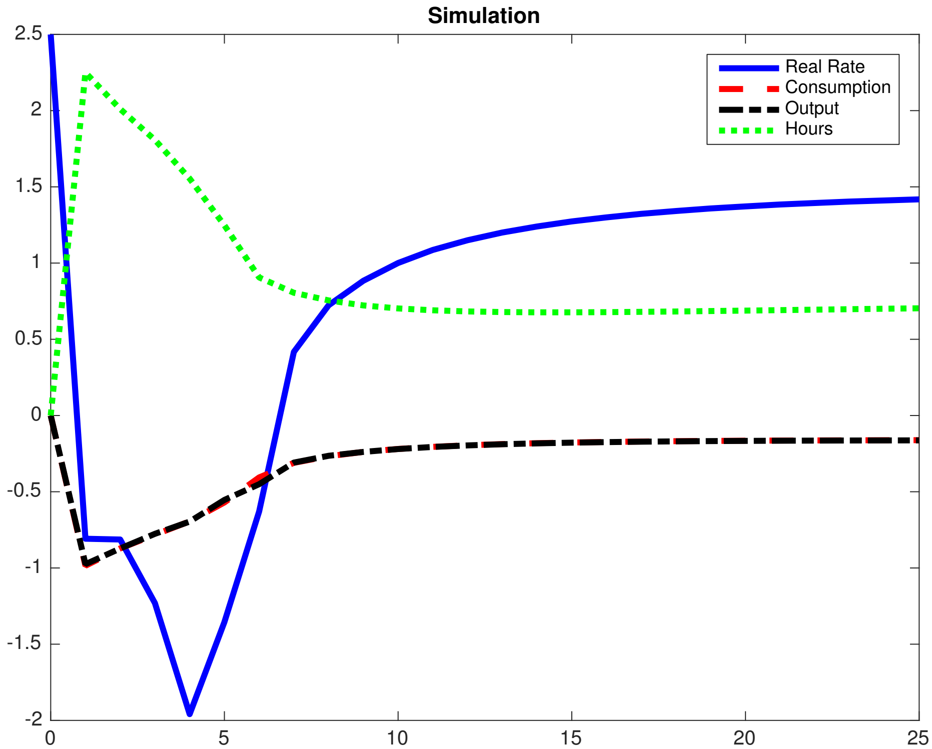

Figure 7 – Transition path following decline in debt limit

- Note: output declines despite the labor supply increase because it is mainly low-productivity agents who expand labor supply. So, average labor productivity declines in the recession.

- Overall the economy is very sensitive to changes in the interest rate. Figure 9 compares the outcome with a partial equilibrium simulation where the real interest rate is held fixed (so the bond market does not clear). Here output falls by more than 4%. The interest-rate elasticity of consumption is near 1, which is very high.

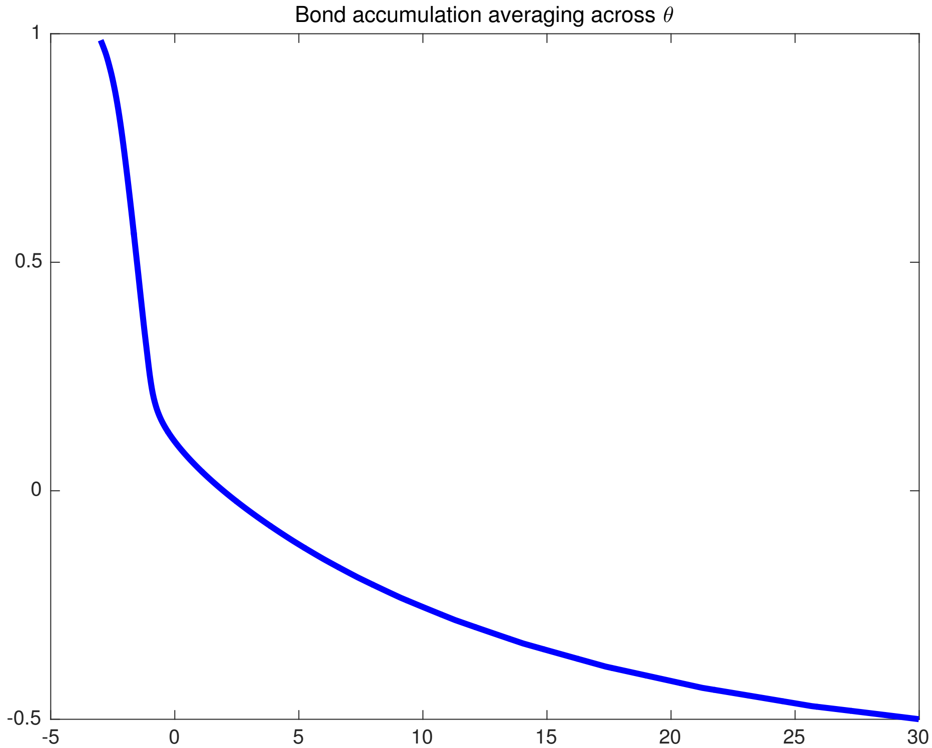

Figure 8 – Bond accumulation at new steady-state

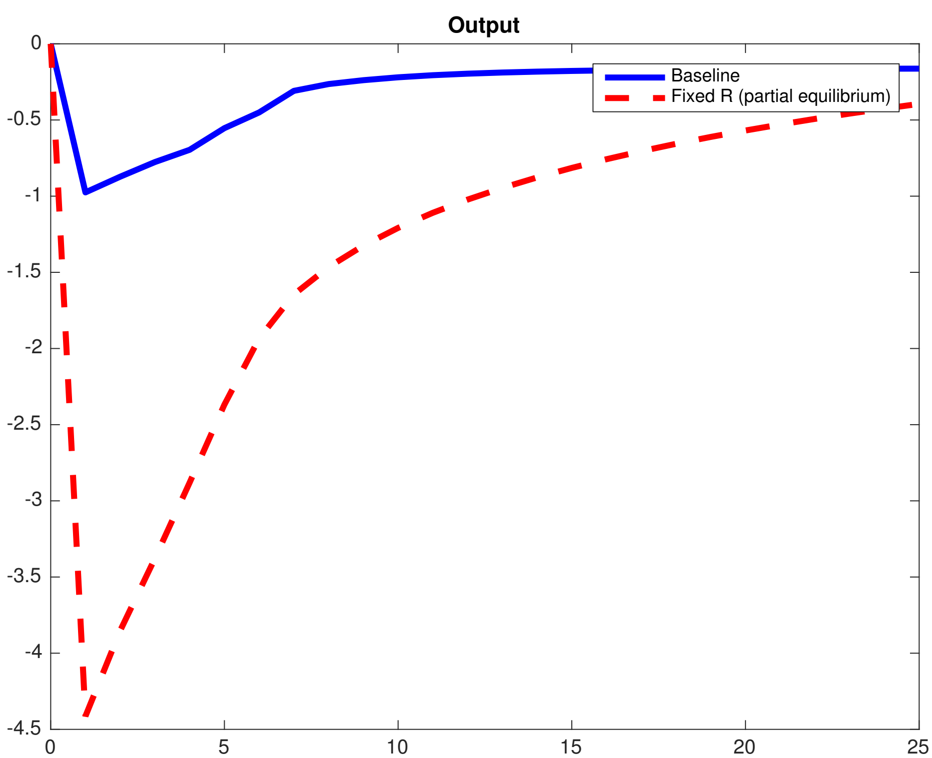

Figure 9 – Comparison to partial equilibrium fixed real interest rate economy.

7 Sticky-price economy

- We next study the same economy with sticky prices. In particular, we take the extreme case where all nominal prices are fixed.

- Even with fixed prices, monetary policy could replicate the flexible price allocation by setting the nominal interest rate equal to the natural rate of interest.

- Here the zero lower bound will prevent that policy.

7.1 Households

- Households earn a real wage $\frac{w_t}{p_t}$ for each efficiency unit of labor supplied.

- Households earn a fixed share of profits π_t.

- New budget constraint:

$$q_t b_{jt} + c_{jt} - \tau_{jt} \leq b_{jt} + \frac{w_t}{p_t}\theta_{jt} l_{jt} + \Pi_t$$

  (q_t is known since prices are fixed)

- FOC for labor supply:

$$\chi(1 - l_{jt})^{-\frac{1}{\eta}} \geq \frac{w_t}{p_t}\theta_{jt} c_{jt}^{-\sigma}$$

- Every household holds a fixed share. This is imposed and not the outcome of optimization. But it is an important assumption because shocks will revalue assets differently.

7.2 Firms

- Monopolistic competition means ideal mark-up is,

$$\frac{p_{it}^*}{p_t} = \frac{\varepsilon}{\varepsilon - 1}\frac{w_t}{p_t}$$

  Think of the equilibrium we study as one where menu costs are sufficiently high that no firm changes prices.

- In steady-state the real wage is

$$\frac{w}{p} = \frac{\varepsilon - 1}{\varepsilon}$$

and real profits are

$$\Pi = \frac{1}{\varepsilon} Y$$

- But when the zero lower bound constraint is binding, then we get an equilibrium with

$$\frac{w_t}{p_t} < \frac{w}{p}$$

and profits

$$\Pi_t = \left(1 - \frac{w_t}{p_t}\right) Y_t$$

(So even when $\varepsilon \to \infty$ these firms make profits on the equilibrium path.)

- Intuitively, at that interest rate bond demand is too high. So something else has to give to clear the bond market. By lowering the wage we make everyone poorer and reduce savings.

7.3 Algorithm

- Solve the same as before, but now when $q_t = 1$ update the wage as opposed to the interest rate.

7.4 Results

- Larger contraction in output and consumption at ZLB.
- Real interest rates too high to clear the bond market. Instead wages and employment fall. This reduces income and desired saving thereby clearing the bond market.
- There is no involuntary unemployment. Everyone is on their labor supply curve.

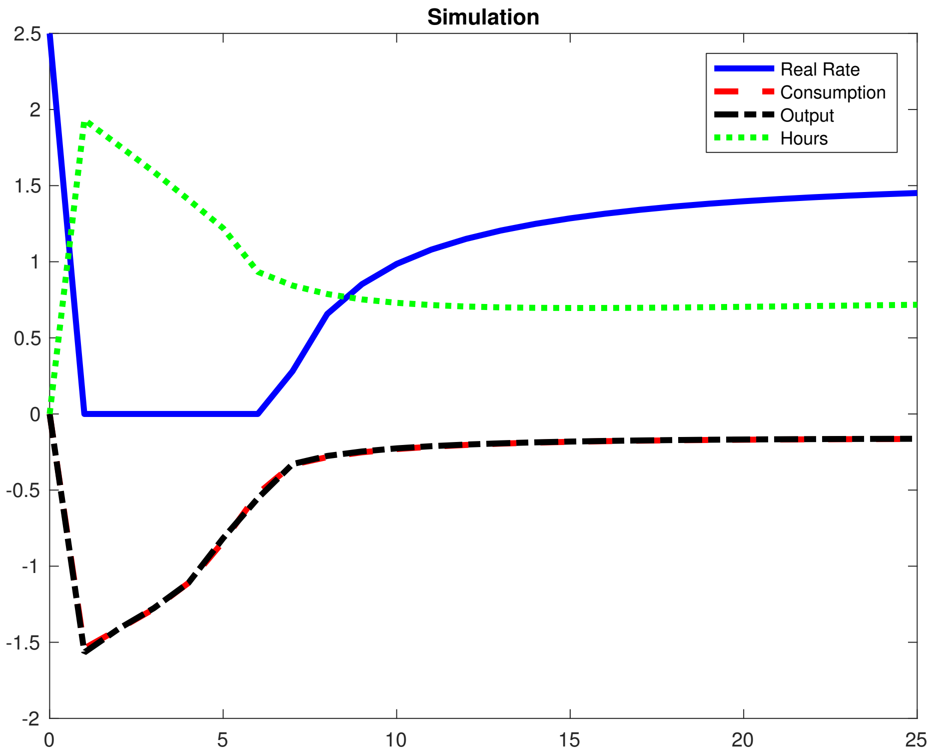

Figure 10 – Transition path following decline in debt limit in sticky-price ZLB economy

8 Sticky-price and sticky-wage economy

- Even with sticky prices, workers are on their labor supply curve, so there is no notion of unemployment.

- We can extend the model on that dimension by positing that the nominal wage cannot fall. Since prices are then constant this is equivalent to the real wage cannot fall.

- The difficulty is determining how this will affect individual labor supply. Absent a deeper theory, here we will simply assume that labor supply falls proportionally for all workers. (That said in our previous model we had to rule out trades in shares, so it's not clear that either model dominates the other.)

8.1 Households

- Budget constraint:

$$q_t b_{jt} + c_{jt} - \tau_{jt} \leq b_{jt} + \frac{w}{p}\theta_{jt} l_{jt} + \Pi_t$$

- Ideal labor supply:

$$\chi(1-l_{jt}^*)^{-\frac{1}{\eta}} \geq \frac{w_t}{p_t}\theta_{jt}c_{jt}^{-\sigma}$$

- Actual labor supply:

$$l_{jt} = \gamma_t l_{jt}^*, \quad \gamma_t \leq 1$$

- This still allows self-insurance in the sense that workers with more ideal labor supply also work more in equilibrium.

8.2 Firms

- With sticky wages we can study the competitive equilibrium where price is equal to marginal cost, $P = MC = W$ so $W/P = 1$.

8.3 Algorithm

- Solve the same as before, but now when $q_t = 1$ update γ_t as opposed to the interest rate.

8.4 Results

- Larger contraction in output and consumption at ZLB.

- Real interest rates too high to clear the bond market. Now employment falls directly through the rationing coefficient γ_t. This reduces income and desired saving thereby clearing the bond market.

- There is involuntary unemployment (at least in the form of hours). Agents want to supply l_{it}^* but can only supply $\gamma_t l_{it}^*$.

- Quantitatively, this looks similar to the sticky-price case. The contraction is a bit larger in the sticky-wage case where hours do not rise as much. For something more different we need to turn off self-insurance through labor supply.

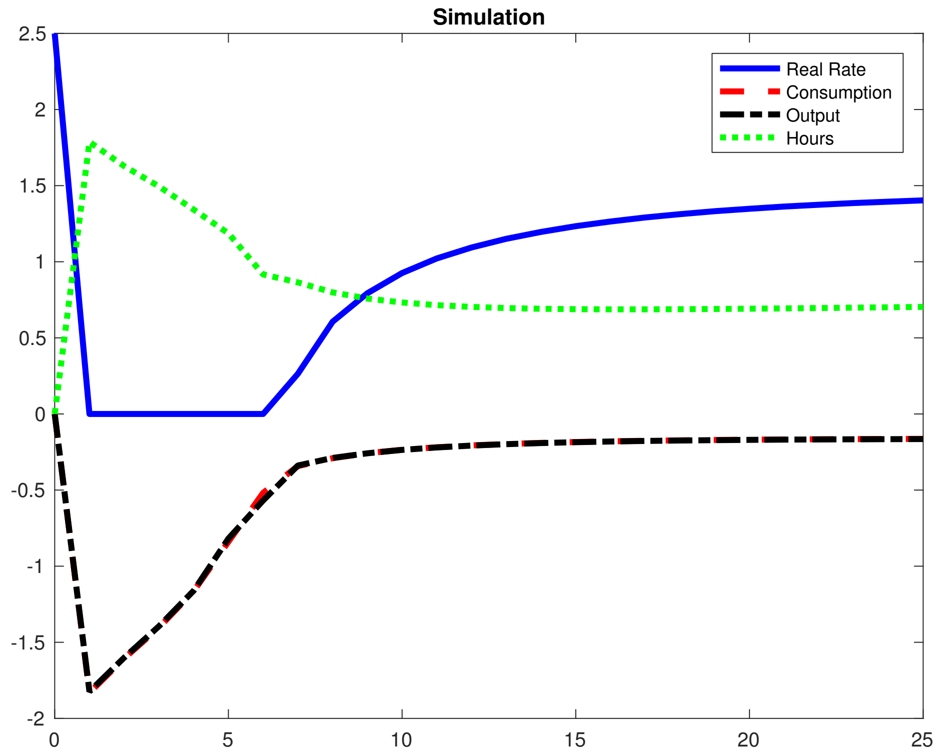

Figure 11 – Transition path following decline in debt limit in sticky-wage ZLB economy

9 Policy

- Paper has a section on fiscal policy.
- Policy is to some extent simply in this class of models. For the tightness of the borrowing constraint $\phi + B$ is the relevant quantity. So any decline in ϕ could be offset by an increase in B.
- Absent that it helps to funnel resources to unemployed agents, who have high mpcs. Note there are no distortionary effect on labor supply since unemployed agents always supply 0 and once θ (exogenously) changes they no longer receive benefits.

10 Durables

- Can we get a "flight to liquidity" whereby agents will want to reduce their holdings of durable assets and increase their holdings of bonds?
- Getting this is more complicated as it sounds. Both bonds and durables are savings vehicles, so we need consumers to increase one form of saving but reduce the other.
- The first step taken is make adjustment in durables costly. So bonds have a liquidity advantage and the two savings vehicles are imperfect substitutes.

- Whether we get a “flight to liquidity” depends on parameters and shocks. A decline in the debt limit causes an increase in durables purchases as consumers switch towards that form of saving.

- They then run a different experiment that increases the borrowing spread, either temporary or permanent. This essentially increases the cost of acquiring durables through leverage and allows the economy to produce a decline in durables. With a temporary shock the decline is even larger as agents will want to wait out the increase in costs.

- Note this economy can self-insure somewhat through the production of durables. This is similar to the KS economy with capital.

11 Conclusion

- Heterogeneity matters. We do not have this type of shock in a complete markets economy.

- Nice analysis combining incomplete markets with Keynesian elements.

- Quantitative analysis only a first step:

  - Is the shock size reasonable?
  - Is the wealth distribution reasonable?
  - Is the mpc distribution reasonable?
  - Is the income risk reasonable?
  - Is the interest-rate sensitivity reasonable?

- Unlike KS-algorithm, there are no numerical errors in this solution. Everything is rational, no ex ante mistakes are made.

- Many important elements can be captured with shooting but some are missing:

  - Shock has ex ante probability zero, so agents would never be prepared for such an event. (No self-insurance.)
  - No aggregate risk, so has some difficulty speaking to relationship of asset markets with macroeconomy.

12 Further Reading

- Auclert (2017) shows how adding long-term bonds reduces the interest rate sensitivity of the economy to more reasonable levels.

References

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