

# Econ 210C Homework 4

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Due: Thursday 5/10/2017

## 1. Labor supply problem

Consider the problem of an individual maximizing lifetime utility:

$$V \equiv \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

subject to the usual budget constraint:

$$\sum_{t=0}^{\infty} (1+r)^{-t} (C_t - w_t L_t) = 0$$

where  $C$  is consumption,  $L$  is labor supply,  $r$  is the real rate of return on assets,  $w$  is the real wage. Assume that  $\beta(1+r) = 1$  and that wages follow

$$w_t = \begin{cases} w^H, & t = 1, 3, 5, \dots \\ w^L, & t = 0, 2, 4, \dots \end{cases}$$

where  $w^H > w^L$ . Compare the pattern of labor supply of a person with time-separable utility

$$U(C_t, L_t) = \log C_t + \log(1 - L_t)$$

with non-time-separable preferences of the following form:

$$U(C_t, L_t) = \log C_t + \log(1 - 0.5[L_t + L_{t-1}])$$

The individual is sophisticated in the sense that he knows that his choice of labor supply in period  $t$  will affect his utility in period  $t - 1$ .

- (a) Do non-separable preferences allow us to get large changes in labor input in response to small shocks?
- (b) Do non-separable preferences allow us to solve the problem that estimated individual labor supply elasticities are small?

## 2. Demand shocks

Suppose that economic fluctuations take place in an economy where many identical consumers maximize an intertemporal function of consumption  $C$  and labor supply  $L$ :

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - v_t \frac{L_t^{1+\chi}}{1+\chi} \right\}$$

where  $v_t$  is a random variable that is always positive and  $\chi > 0$ . Consumers are subject to the usual series of budget constraints:

$$P_t K_{t+1} = W_t L_t + (P_t + d_t) K_t + \Pi_t - C_t$$

$K$  is physical capital;  $P$  is the relative price of capital (in units of output);  $d$  is a dividend (equal to the marginal product of capital) that is paid to holders of capital;  $W$  is the real wage; and  $\Pi$  is economic profit (if any).

The capital stock of the economy is fixed at  $K_t = \bar{K}$  for all  $t$ . Capital does not depreciate and investment is impossible.

- (a) What are the consumer first-order conditions for optimal consumption and labor supply at a point in time?
- (b) Consider a perturbation where consumers reduce consumption by one unit, invest in capital and consume the next period. Use this perturbation to derive the intertemporal first-order condition for consumption.
- (c) What is the equilibrium condition in the capital market? What is the interpretation of the Euler equation in part (b) given the capital market equilibrium condition?
- (d) Solve for  $L_t$  as a function of  $v_t$ , the real wage and the marginal utility of wealth. What is the interpretation of  $v_t$ ? How does  $v_t$  affect the position and the slope of the labor supply curve?
- (e) Discuss the following statement: “In any model where consumers choose quantities freely, the real wage must be procyclical in order for consumption and labor supply to be procyclical.” Is the statement true in the model presented in this question? What are the implications for business cycle modeling?

## 3. Business cycle and external returns to scale

Continue the previous problem. There is a continuum of identical, competitive firms in the economy indexed by  $i \in [0, 1]$ . All firms have the production technology:

$$Y_{it} = E_t K_{it}^{\alpha} (Z_t L_{it})^{1-\alpha}$$

where  $Z$  is exogenous technical change (common across firms), and  $E$  is an external effect that each firm takes as exogenous.  $E$  depends on aggregate output:

$$E_t = Y_t^{1-1/\gamma}, \quad \gamma \geq 1$$

There is no government consumption in this model, so aggregate output just equals consumption (recall that there is no investment in this model):  $Y_t = C_t$ .

- (a) State the condition for firms profit maximizing use of labor. Use it to derive the firms labor demand curve in this economy. (You may assume that the standard properties of the Cobb-Douglas production function are common knowledge).
- (b) Solve for the aggregate production function. Are there economic profits in this model? What is the social labor demand curve?
- (c) Suppose you wish to explain the following stylized facts about business cycles:
  - a. Consumption is procyclical
  - b. Leisure is countercyclical
  - c. Labor productivity is slightly procyclical
  - d. Real wages are slightly procyclical

Suppose there are shocks only to  $v_t$ . Is the model presented above consistent with all these facts? Why or why not? If any answer is ambiguous, explain why.

- (d) Now suppose a standard RBC economy:  $\gamma = 1$  and there are only technology shocks. (Assume that  $v_t$  is constant at 1.) Is the model above consistent with all the facts? Why or why not?
- (e) Discuss the following statement: “As an RBC modeler, I am puzzled. For reasonable labor supply elasticities, my models predict that labor productivity should be strongly procyclical. But in the data, labor productivity is only mildly procyclical. I dont know how to fix this problem.” Is this statement correct? In the model presented here, would a combination of  $v$  and  $Z$  shocks help solve the problem? How does your answer depend on the value of  $\gamma$ ?

## 4. Problems from Romer

- 1. Romer, Problem 6.10.
- 2. Romer, Problem 6.11.
- 3. Romer, Problem 6.12.