Econ 210C Homework 3

Instructor: Johannes Wieland

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1. Variable labor supply in the RBC model

Modify your Dynare codes from the last homework to add elastic labor supply.

(a) Set the Frisch elasticity to $\eta = 1$ and simulate the model. What is the volatility of output, consumption, and hours worked? How does it compare to the data? (You can use moments from the lecture notes to compare.)

(b) Set the Frisch elasticity to $\eta = 2$ and simulate the model. Relative to your answer to (a), has the fit with the data improved?

(c) Set the Frisch elasticity to $\eta = 0.5$ and simulate the model. Relative to your answer to (a) and (b), has the fit with the data improved?

2. Variable capital utilization in an RBC model

The world is populated by many consumers who maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \frac{L_t^{1+\chi}}{1+\chi} \right\}$$

subject to the usual flow budget constraint:

$$C_t + A_t = W_t N_t + (1 + r_t) A_{t-1} + \Pi_t$$

C is consumption, A is the consumer's stock of assets (equal to the capital stock K in equilibrium), N is labor supply, r is the real rate of return on assets, W is the real wage, and Π is economic profit (if any).

There is no government consumption in this model, so aggregate output equals the sum of consumption and investment:

$$Y_t = C_t + I_t$$

Firms are competitive in output and factor markets, and have the production function:

$$Y_t = (U_t K_{t-1})^{\alpha} (Z_t N_t)^{1-\alpha}$$

where Z_t is the level of technology and U_t is the level of capital utilization. U_t is a choice variable for the firm.

The capital stock evolves according to $K_t = I_t + (1 - \delta(U_t))K_{t-1}$ where $\delta^{'} > 0$,$\delta^{''} > 0$.

(a) Solve for the first-order conditions for profit-maximization by firms. What is the rental rate of capital? Why does it depend on U?

(b) Log-linearize the firm's first-order condition for capital utilization. Let $\check{X}_t$ be the percent deviation of variable X from its steady state value $\bar{X}$. Express $\check{U}$ in terms of $\check{Y}$,$\check{K}$ and Δ, where

$$\Delta = \frac{\delta^{''}(\bar{U})\bar{U}}{\delta^{'}(\bar{U})}$$

(c) Log-linearize the production function, substitute the optimal choice of utilization (use you solution in (b)), and solve for $\check{Y}$ as a function of $\check{L}$, $\check{K}$, $\check{Z}$. How does $\check{Y}$ depend on Δ? Discuss the intuition, by focusing on the limiting cases of $\Delta = \infty$ and $\Delta = 0$. Interpret these two cases.

(d) A necessary condition for indeterminacy in one-sector business-cycle models is that labor supply and labor demand curves must "cross the wrong way". (You need not prove this assertion.) Assuming that $\chi \geqslant 0$, can we get indeterminacy in this model (for any value of Δ)? Explain why or why not. If not, what features can we add to the model that would make indeterminacy possible? Does variable capital utilization make indeterminacy more likely? Explain.

3. Homework in macroeconomics

Suppose a household has the following per-period utility function:

$$C - \left(\frac{1}{\eta} + 1\right)^{-1} (L_h + L_m)^{\frac{1}{\eta}+1}$$

where

C = consumption aggregate, $= (C_m^{\rho} + C_h^{\rho})^{\frac{1}{\rho}}$;

$1 - L_m - L_h$ = leisure;

C_m = consumption of goods bought in the market (e.g., purchase of childcare services);

C_h = consumption of goods produced at home (e.g., raise children at home);

L_m = supply of labor in formal markets (e.g., work for UCSD);

L_h = supply of labor to produce goods at home (e.g., spend time raising children);

The elasticity between market and home consumption is $1/(1-\rho)$.

Assume that the household maximizes this utility subject to these constraints:

$$C_m \leqslant W L_m$$
$$C_h \leqslant L_h$$

where W is market wages (taken as given the household). The first condition is the budget constraint for the "market" consumption goods. The second condition is the budget constraint for "home-produced" goods.

(a) Find first order conditions for C_m, C_h, L_h, L_m. (Denote multipliers on the first and second constraints with λ and ξ respectively.)

(b) What is the relationship between λ, ξ, W? (hint: use optimality conditions for L_h, L_m)

(c) What is the relationship between λ, ξ given the optimality conditions for C_m, C_h?

(d) Can you express C_h and L_h as a function of W and C_m? (hint: use the budget constraints)

(e) Express L_m as a function of wages W and λ.

(f) What is the elasticity of L_m with respect to wages (holding λ constant; that is, holding the marginal utility of market consumption C_m constant)?

(g) In the standard RBC model without home production, the elasticity of labor supply is given by η. Why is it potentially different in the model with home production? Explain.

(h) Eliminate λ in your answer in (e) so that employment is just a function of wages W. Now you can calculate the elasticity of labor supply holding the marginal utility of total consumption C constant (here it's equal to one because utility is linear in consumption).

(i) What is the elasticity of L_m with respect to wages? Why is it potentially different from η?

(j) RBC models need a high labor supply elasticity to match data. What values of ρ can help to deliver it?

4. q-Theory with Variable Capital Utilization

Consider the following neoclassical business-cycle model. The world is populated by many consumers, who maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \frac{L_t^{1+1/\eta}}{1+1/\eta} \right\}$$

subject to the usual flow budget constraint:

$$A_t = W_t L_t + (1+r_t) A_{t-1} + \Pi_t - C_t$$

C is consumption, A is the consumer's stock of assets (equal to the capital stock K in equilibrium), L is labor supply, r is the real rate of return on assets, W is the real wage, and Π is economic profit (if any).

There is no government consumption in this model, so aggregate output equals the sum of consumption and investment:

$$Y_t = C_t + I_t.$$

Firms are competitive in output and factor markets, and have the production function:

$$Y_t = Z_t (U_t K_{t-1})^{\alpha} L_t^{1-\alpha}$$

where Z is the level of technology and U is the level of capital utilization. U is a choice variable for the firm.

In order to install investment goods and transform them into productive capital, firms must pay an adjustment cost. The total cost of adding I_t units of new capital is

$$I_t \left[1 + \phi \left(\frac{I_t}{K_{t-1}} \right) \right]$$

where $\phi^{'} > 0$ and $\phi^{''} > 0$.

The capital stock evolves according to $K_t = I_t + [1 - \delta(U_t)]K_{t-1}$ where $\delta^{'} > 0$, $\delta^{''} > 0$.

Let the shadow value of installed capital be termed q_t. for any variable X, define the percent deviation of X from its steady state level $\bar{X}$ as $\check{X}_t = (X_t - \bar{X})/\bar{X}$. Also define

$$\Delta = \frac{\delta^{''}(\bar{U})\bar{U}}{\delta'(\bar{U})}$$

where $\bar{U}$ is the steady state level of capital utilization.

(a) What is the profit-maximization problem that firms solve? Why is it truly dynamic, instead of being a sequence of one-period static problems?

(b) Solve for the first-order conditions for profit-maximization by firms (both the static first order conditions and the Euler equation). What is the rental rate of capital (equal to capitals marginal product)? Why does it depend on U and q?

(c) Log-linearize the firm's first-order condition for capital utilization. Express $\check{U}_t$ in terms of $\check{Y}_t$, $\check{K}_{t-1}$, $\check{q}_t$ and Δ. Why does capital utilization depend on q? Explain the economics.

Even *if* you could not derive all the first-order conditions, try to answer parts (d)-(e) anyway.

(d) Julio Rotemberg has argued that capital utilization cannot be procyclical because we observe that investment is procyclical (NBER Macro Annual, 1991, p. 140). Using the first-order conditions you have derived, explain why Rotemberg might make this statement. Is he unambiguously right? Why or why not?

(e) Suppose business cycles are driven by shocks to Z. Under what sort of time-series processes for Z would you expect capital utilization to go up in response to a positive innovation to technology? When, if ever, might it go down? Explain the economics.

5. Fiscal multiplier in the RBC model

In the class we analyzed the RBC model with elastic labor supply. We would like to know the size of the fiscal multiplier for government spending in this model. Log-linearize the model around the steady state. You can take parameter values from the lecture note. You can also assume that government spending is 20% of output, $\check{G}_t = \rho_g \check{G}_{t-1} + \epsilon_t^g$ and the persistence of government spending shocks is $\rho_g = 0.9$. Without loss of generality you can assume that technology is fixed at $Z = 1$ in all times so that $\check{G}_t$ is the only source of fluctuations in this economy.

(a) Use the method of underdetermined coefficients to find the reduced form solution to the log-linearized model. You do not need to derive the analytical solution. Numeric solutions are fine.

(b) Using your reduced form solution to the log-linearized model, compute and plot the impulse response of consumption, output, investment, wages, interest rate, capital stock, and government spending to a unit (i.e., 1 percent) shock in government spending. Discuss your results.

(c) Note that impulse responses are percent deviations from the steady state. Convert the impulse response of output from percent deviations to dollars by multiplying the impulse response by $(\bar{Y}/\bar{G})$. Discuss the size of the fiscal multiplier for the transitory government spending shocks.

(d) Repeat (a)-(c) with $\rho_g = 1$ which would mean that government spending shocks are permanent. Discuss your results and compare them to what you found in (a)-(c). (You can ignore that fact that you need to log-linearize the model around stochastic trend in government spending. In other words, you can use the same reduced form solution but now plug in $\rho_g = 1$.)