

# Econ 210C Homework 2

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Due: Thursday, April 19, 2018

## 1. Investment and the Housing Market

Consider the following model of the housing market (due to James Poterba):

$$\begin{aligned}I &= \psi(P), \quad \psi' > 0 \\r + \delta &= (R + \dot{P})/P \\R &= R(H), \quad R' < 0 \\\dot{H} &= I - \delta H\end{aligned}$$

where  $\dot{x} \equiv \partial x / \partial t$ ,  $H$  is the available quantity of housing stock,  $I$  is the gross investment in housing,  $P$  is the price of a house,  $R$  is the rental cost of a house, and  $r$  is the rental interest rate.  $\delta$  and  $r$  are fixed parameters, while  $I$ ,  $R$ , and  $H$  may vary over time.

- (a) Explain why each of the equations of the model is reasonable.
- (b) Rewrite the model in terms of two variables (a state variable  $H$  and a costate variable  $P$ ) and two equations of motion.
- (c) In a phase diagram, display the system dynamics.
- (d) What is the steady-state effect on  $H$ ,  $P$ ,  $I$ , and  $R$  of an increase in the real interest rate  $r$ ?
- (e) What is the effect on  $H$ ,  $P$ ,  $I$ , and  $R$  over time of an unanticipated, permanent increase in the real interest rate  $r$ ?
- (f) What is the effect of an unanticipated, temporary increase in the real interest rate?
- (g) What is the effect of an announced, future, permanent increase in the real interest rate?
- (h) Suppose that instead of having rational expectations (here, perfect foresight) about the price of a house, people have static expectations—they expect that the price of a house will never change from what it is now. Which equation will this change? Redo part (e) under this new assumption.
- (i) How could you use this model to analyze the housing crisis?

## 2. Discount Factor Shocks

Consider the model developed in class and allow the discount factor  $\beta$  to exogenously vary over time. This variation captures the idea that impatience to consume can fluctuate and it can be interpreted as a demand shock. Log-linearize the model to include variation in  $\beta$  (you can denote it with  $\tilde{\beta}_t$ ) as a part of the system. Using the phase diagram in the  $(\lambda, K)$  space as well as the labor market diagram, make qualitative predictions for the responses of consumption, output, investment, employment, wages, interest rate to a permanent and a transitory increase in the discount factor  $\beta$ . For both shocks (i.e., permanent and transitory), summarize the impact, transition, and steady-state responses for all variables in a table (like we did in class) and explain the intuition behind the response.

## 3. News Shocks

Recent research in macroeconomics suggests that business cycle fluctuations may be driven by news shocks, i.e. changes in fundamentals which will occur in the future rather than current periods. Suppose that economic agents expect a permanently higher level of productivity after  $t_1$  periods from now ( $t = 0$ ). However at time  $t_1$ , the expected increase in the level of productivity is not realized. Using the phase diagram in the  $(\lambda, K)$  space as well as the labor market diagram, make qualitative predictions for the responses of consumption, output, investment, employment, wages, interest rate to a news shock about future productivity. Summarize the impact, transition (before and after  $t_1$ ), inflection, and steady-state responses for all variables in a table (like we did in class) and explain the intuition behind the response. You can make the same assumptions regarding the movements of the  $\Delta K = 0$  and  $\Delta \lambda = 0$  locus as we did in class.

## 4. Labor Supply

Suppose an individual with a  $T$  period horizon wants to consume a constant level of consumption  $C$  per period. She earns  $w$  per hour, she must decide how many hours  $N$  to work per period. The path of the wage rate  $w_t$  is known at  $t = 0$ . The flow disutility of work is  $\log(1 + N_t)$ . The rate of time preference and of interest are zero. Accordingly, she minimizes

$$\sum_{t=0}^T \ln(1 + N_t)$$

subject to the constraints

$$\begin{aligned} \sum_{t=0}^T w_t N_t &= C \times T \\ N_t &\geq 0 \end{aligned}$$

- (a) What is the stochastic first-order condition (Euler equation of this problem)?
- (b) Give the Euler equation an economic interpretation.

- (c) What are the econometric implications of this theory? What is the timing of the labor supply? How would you test them?
- (d) What is the response of labor to an unexpected, permanent increase in the level of the wage?
- (e) What would the response of labor be if the increase in the level of the wage were expected rather than unexpected?
- (f) What does this example teach us about labor supply elasticity?

## 5. Impulse Responses

Consider the model analyzed in the class (with inelastic labor supply).

- (a) Given reduced-form solution to the log-linearized model and calibrated parameter values, simulate the model and compare moments from the generated series with moments in the data (you may use data from Stock and Watson paper in the syllabus). Specifically, compare volatility of output, consumption, investment, and wages. Discuss your results.
- (b) Using reduced form solution to the log-linearized model, compute and plot the impulse response of consumption, output, investment, wages, and technology to a unit shock in technology. Discuss your results.
- (c) Suppose that the persistence of technology is now given by  $\rho = 1$ . Compute and plot the impulse responses to a unit shock in technology. (You can ignore that fact that you need to log-linearize the model around stochastic trend in technology. In other words, you can use the same reduced form solution but now plug in  $\rho = 1$ .)
- (d) Repeat (a)-(c) with capital share  $\alpha = 2/3$ . Contrast and discuss the differences with results in (a)-(b).
- (e) How would you estimate the vector of structural parameters  $(\alpha, \beta, \delta, \rho, \sigma_\epsilon)$  that matches the volatility of series in the data?

## 6. Impulse Responses (2)

Analyze the model with inelastic labor supply using Dynare. You should read Eric Sims' excellent notes on using Dynare on TED. For our purposes the information up to and including section 6 suffices.

- (a) Solve the non-linear model in Dynare. Compute and plot the impulse response functions to a unit technology shock.
- (b) Compare these impulse response functions with those from 5(b). Comment on the difference.
- (c) OPTIONAL Solve the planner's problem using value function iteration. Construct the impulse response functions starting in the steady-state. Compare these results to 6(c) and comment on the difference.