Econ 210C Homework 1

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Due: 4/12/2018 in class

1. Questions from textbook

1. Romer, Problem 5.8.
2. Romer, Problem 5.9.
3. Romer, Problem 5.11.

2. Permanent income hypothesis and the "excess smoothness" puzzle

Suppose consumption is governed by the permanent income hypothesis, i.e., the consumer maximizes a utility function given by

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s U(C_{t+s})$$

subject to the intertemporal budget constraint

$$\sum_{s=0}^{\infty} (1+r)^{-s}(C_{t+s} - Y_{t+s}) = A_t$$

where Y is labor income, A is non-human wealth, and r is the constant and known real interest rate. Also assume that the transversality condition holds.

1. What happens to saving (income less consumption) in response to a positive shock to income ($\epsilon_t > 0$) if income follows each of the processes below? In answering this question, you may assume that $\beta = (1+r)^{-1}$ and that the utility function is quadratic.

   (a) $Y_t = \mu \times t + \phi Y_{t-1} + \epsilon_t,\ \phi \in (0, 1)$;

   (b) $Y_t = Y_{t-1} + \epsilon_t$;

   (c) $\Delta Y_t = \phi \Delta Y_{t-1} + \epsilon_t,\ \phi \in (0, 1)$

   Hint: First trace out the impulse response functions for each of the three processes. For each, ask yourself what the intuition of the permanent income hypothesis predicts. Then solve algebraically, using both the first order conditions and the budget constraint. Note that since the budget constraint holds in actuality, it must also hold in expectation (the converse is not true).

2. In light of your results, discuss the following (pseudo) quotation: "The permanent income hypothesis implies that consumption want to spread out changes in income over the rest of their lives, so consumption should be smoother than income."

3. In the US data, consumption is significantly smoother than income, and income seems to follow approximately a random walk (a process like (b) above). Do these findings constitute a puzzle? How might you explain them?

3. Estimation of adjustment costs.

Consider the following setup:

$$\max E_0 \left\{ \sum_{t=0}^{\infty} R_t \left[(1-\tau)F(K_t, L_t) - w_t L_t - I_t \left(1 + \phi\left(\frac{I_t}{K_t}\right) \right) \right] \right\}$$

subject to

$$K_t = (1-\delta)K_{t-1} + I_{t-1}$$

where K_0 is given, $R_t = [\Pi_{s=0}^{t}(1+r_s)]^{-1}$ is the discount factor which allows for a time-varying interest rate, w is the real wage, δ is the rate of depreciation of capital, I is the gross investment. Variables w_t and r_t are stochastic. Suppose that F, ϕ are homogenous of degree one and the firm is a price taker. Suppose that F is Cobb-Douglas. Also assume that the function ϕ is linear. Specifically, $\phi(\cdot) = a(I_t/K_t - \delta)$ is a positive constant.

1. Let q be the Lagrange multiplier on the constraint (its also Tobins Q). Derive the first order conditions for optimization with respect to L, I and K. Given q, what information is needed to establish optimal level of investment? Log-linearize the FOC for investment. Suppose we observe I, K, and q. How would you estimate the parameter a using this equation? How would you interpret the error term in this regression? What are the properties of the error term given your interpretation (e.g., serial correlation, correlation with other variables, etc.)?

2. Rewrite the Euler equation in terms of observable quantities (use the FOC for I). Log- linearize this equation and substitute out for expected values in terms of realized ones and an expectational error (expectational error at time $t+1$ for variable x is given by $x_{t+1} - E_t x_{t+1}$).

3. Can you rearrange the equation in a way that makes it a valid regression i.e., one of the form $y_t = G(X_t, \beta) + \epsilon_t$ where y is the dependent variable, X is a group of independent variables, β is a vector of parameters, and ϵ_t is an error term with the property that $E(\epsilon_t|X_t) = 0$? How would you interpret the error term in this regression? Can this equation be used to identify the parameter a?

4. How would your results change in (3) if I adds to the capital stock immediately (not with a lag)? In other words, $K_t = (1-\delta)K_{t-1} + I_t$.

5. Discuss the pros and cons of estimating investment as a function of q versus estimating the Euler equation? What are the strengths and weaknesses of each approach?

4. Practice log-linearization.

Log-linearize the following equations

1. $Y = C + I + G + NX$ (resource constraint; assume $NX = 0$ in the steady state).
2. $Y = (\alpha K^\rho + (1-\alpha)(AL)^\rho)^{1/\rho}$ (CES production function; A is the level of technology; L is labor; K is capital; ρ, α are constants).
3. $K_t = (1-\delta)K_{t-1} + I_t - \psi\Big(\frac{I_t}{K_{t-1}} - \delta\Big)^2 I_t$ (capital adjustmentcost; I is investment; ψ, δ are constants).
4. $K_t = (1-\delta)K_{t-1} + I_t - \psi\Big(\frac{I_t}{K_{t-1}} - \delta\Big)^2 I_t - \phi\Big(\frac{I_t}{I_{t-1}} - 1\Big)^2 I_t$ (capital adjustmentcost; I is investment; ψ, δ, ϕ are constants).
5. $\exp(i_t) = \big(\frac{P_t}{P_{t-1}}\big)^{\phi_\pi}\big(\frac{Y_t}{Y_{t-1}}\big)^{\phi_y}\exp(\rho i_{t-1})$ (Policy reaction function for interest rate i as a function of prices P and output Y; ϕ_π, ϕ_y, ρ are constants).
6. $A \times F(L) = \Big(\frac{\partial U(C,1-L)}{\partial L}\Big)/\Big(\frac{\partial U(C,1-L)}{\partial C}\Big)$ (U is a utility function [which could be non-separable in consumption and leisure]; consumption C and leisure $(1-L)$ [where L is the labor supply]; F is a production function; A is the level of technology).
7. $Y_t = K_t^{\alpha_t} L_t^{1-\alpha_t}$ (Cobb-Douglass production function with **variable** capital share α_t).