Economics 210C - Macroeconomics — Spring 2018-19

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Take-Home Final Exam

1. Technology growth regimes Consider the following version of the RBC model with transitory and permanent technology shocks. The social planner's problem is

$$\max_{\{C_t, L_t, K_t\}} \mathbb{E}_0 \sum\nolimits_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}-1}{1-\sigma},$$

$$\text{s.t.} \quad Z_t K_{t-1}^{\alpha} \left(X_t L_t\right)^{1-\alpha} + (1-\delta) K_{t-1} = C_t + K_t,$$

$$X_t = X_{t-1}\mu_t, \quad \mu_t = \mu^{1-\gamma}\mu_{t-1}^{\gamma} e^{\zeta_t},$$

$$Z_t = Z_{t-1}^{\rho} e^{\varepsilon_t},$$

where $0 < \beta, \delta, \alpha, \gamma, \rho < 1$, $\sigma > 0$, $\mu \geq 1$, and $\{\zeta_t\}$ and $\{\varepsilon_t\}$ are exogenous white noise processes. Assume that the household supplies one unit of labor inelastically in each period. Assume also that $\mu^{\sigma-1} > \beta$.

a. Derive the first-order necessary conditions for a solution to the planner's problem, letting Λ_t denote the Lagrange multiplier on the resource constraint.

b. Define detrended variables as follows:

$$c_t = \frac{C_t}{X_t}, \quad k_t = \frac{K_t}{X_t}, \quad \lambda_t = \Lambda_t X_t^{\sigma}, \quad \mu_t = \frac{X_t}{X_{t-1}}.$$

Restate the necessary conditions in terms of detrended variables. Use the restated conditions to characterize the nonstochastic steady state solution.

c. Use your conditions from part b to solve for $\Delta\lambda_t$ and Δk_t as functions of the variables λ_t, k_{t-1}, μ_t and Z_t, under the assumption that μ_t and Z_t are perfectly-anticipated deterministic paths. Derive continuous time approximations of these equations; i.e., use the equations to obtain expressions in continuous time for $\dot{\lambda}_t$ and $\dot{K}_t$ as functions of λ_t, k_t, μ_t and Z_t.

d. Assume there is a unique $\bar{k}_t > 0$ such that $\dot{\lambda}_t = 0$ for $k_t = \bar{k}_t$. Assume also that $\dot{\lambda}_t < 0$ for $k_t < \bar{k}_t$, and $\dot{\lambda}_t > 0$ for $k_t > \bar{k}_t$. Calculate either the partial derivative or partial log-derivative of $\bar{k}_t$ with respect to μ_t. Show that for μ_t in the neighborhood of the steady state, $\bar{k}_t$ is increasing in μ_t if σ is sufficiently small, and decreasing in μ_t if σ is sufficiently large.

e. Assume there is a uniquly defined function $\bar{\lambda}_t(k_t) > 0$ such that $\dot{k}_t = 0$ for $\lambda_t = \bar{\lambda}_t(k_t)$. Assume also that $\dot{k}_t < 0$ for $\lambda_t < \bar{\lambda}_t(k_t)$, and $\dot{k}_t > 0$ for $\lambda_t > \bar{\lambda}_t(k_t)$. Calculate either the partial derivatives or partial log-derivatives of $\bar{\lambda}_t$ with respect to k_t and μ_t. Show that $\bar{\lambda}_t$ is strictly decreasing in k_t for values of k_t, μ_t and Z_t in a neighborhood of the steady state (the assumption $\mu^{\sigma-1} > \beta$ is used here). Show also that $\bar{\lambda}_t$ is strictly increasing in μ_t.

f. Sketch the $\dot{\lambda}_t = 0$ and $\dot{k} = 0$ curves in a graph having k_t on horizontal axis and λ_t on the vertical axis. Show the directions of motion of the endogenous variables at the various points in the graph, along with the saddlepoint path.

g. Suppose the economy begins in a steady state, and μ is increased permanently to μ'. Use your graph to locate the new steady state values of k_t and λ_t, considering in turn the cases of low σ and high σ. Describe also how the steady state value of c_t is affected in each case.

h. Suppose the economy begins in a steady state. At period t_0, μ_t increases from μ to μ', and it returns permanently to μ at period $t_1 > t_0$. Assume that Z_t remains constant. Trace out the continuous time perfect foresight dynamics generated by this technology path for the case of large σ. How does c_t adjust along the transition path? How does the transition path change if t_1 is increased? Provide economic intuition for your answer. (Remember that the variables are measured relative to trend.)

i. (Extra credit) Verify the assumptions made in parts d and e, by analyzing the $\dot{\lambda}_t$ and $\dot{k}_t$ equations, together with the steady state characterization.

2. Optimal policy in the New Keynesian model In the equilibrium of a benchmark New Keynesian model, the variables χ_t, $\hat{\pi}_t$ and $\hat{r}_t^n$ satisfy

$$\chi_t = \mathbb{E}_t\chi_{t+1} - \frac{1}{\xi}(\hat{r}_t^n - \mathbb{E}_t\hat{\pi}_{t+1}) + u_t,$$

$$\hat{\pi}_t = \theta\chi_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} + \varepsilon_t,$$

where ξ, $\theta > 0$, $0 < \beta < 1$, $\{u_t\}$ is an exogenous process, and $\{\varepsilon_t\}$ is a white noise process with variance $\sigma_\varepsilon^2 > 0$. The household's equilibrium expected utility is approximated by

$$\mathcal{U} = -\mathbb{E}_0 \sum\nolimits_{t=0}^{\infty} \beta^t \left(\varphi\left(\chi_t - \chi^*\right)^2 + \hat{\pi}_t^2\right),$$

where $\chi^* \geq 0$.

a. Suppose that the policymaker controls $\hat{\pi}_t$ directly, according to a rule of the form

$$\hat{\pi}_t = a_0 + a_1\varepsilon_t,$$

where a_0 and a_1 are policy parameters. Further, the two parameters can be permanently and publicly fixed at the start of period 0 to maximize $\mathcal{U}$. Set up the policymaker's problem, and derive the optimal values of the parameters. Discuss the effects on the optimal rule of (i) an increase in θ; (ii) an increase in φ; and (iii) an increase in χ^*. Provide intuition.

b. Suppose instead that for each t, the policymaker observes ε_t and then chooses $\hat{\pi}_t$ to maximize

$$-\left(\varphi(\chi_t - \chi^*)^2 + \hat{\pi}_t^2\right).$$

The household is assumed to ignore past inflation when forming expectations in period t (so there are no "trigger strategies"). Characterize the equilibrium in terms of $\hat{\pi}_t$ and $E_t\hat{\pi}_{t+1}$, and then use the method of undetermined coefficients to solve for $\hat{\pi}_t$ as a policy rule of the form

$$\hat{\pi}_t = b_0 + b_1\varepsilon_t.$$

Compare the solutions b_0 and b_1 to the optimal values obtained in part a. Provide intuition.

c. Derive an interest rate rule of the form

$$\hat{r}_t^n = c_0 + c_1\varepsilon_t + c_2u_t$$

that implements the policy rule in part b. Discuss the economic rationale for the policymaker's interest rate adjustments in response to ε_t and u_t.