Investment and the Real Business Cycle — Greenwood, Hercowitz and Huffmann (88 points)

2. Consider the canonical RBC model with changes to the marginal efficiency of investment: A consumer maximizes

$$\max \sum_{t}^{\infty} \beta^t \left(\ln C_t - \frac{(L_t^s)^{1+1/\eta}}{1+1/\eta} \right)$$
$$\text{s.t. } A_t + C_t = (1+R_t)A_{t-1} + W_t L_t^s + \pi_t$$
$$lim_{t\to+\infty} \left(\prod_{s=1}^{t} (1+R_s) \right)^{-1} A_t = 0$$

taking prices as given.

A representative firm maximizes profits,

$$\max \sum_{t}^{\infty} \left(\prod_{s=1}^{t} (1+R_s) \right)^{-1} (Y_t - W_t L_t^d - I_t)$$
$$\text{s.t. } Y_t = K_{t-1}^{\alpha} (L_t^d)^{1-\alpha}$$
$$K_t = (1-\delta)K_{t-1} + I_t(1+\epsilon_t).$$

taking prices as given. The variable ϵ_t denotes a shock to the marginal efficiency of investment. High values for ϵ_t convert any amount I_t you invest into more units of capital. Note that the price of output is normalized to 1, $P_t = 1$.

The market clearing conditions are:

$$L_t^s = L_t^d = L_t$$
$$K_t = A_t$$
$$Y_t = C_t + I_t$$

Standard parameter values apply:

$$0 < \alpha, \beta, \delta < 1$$
$$\eta > 0$$

Question continues on next page.

(a) (12 points) Set up the Lagrangian and derive the first order conditions for the household w.r.t. C_t, L_t^s, A_t. Use λ_t as the lagrange multiplier on the budget constraint. Briefly interpret each equation (one sentence each).

(b) (12 points) Set up the Lagrangian and derive the first order conditions for the firm w.r.t I_t, L_t^d, K_t. (Substitute for Y_t first.) Use q_t as the Lagrange multiplier on capital accumulation. Briefly interpret each equation (one sentence each).

3. The log-linearized equations of the RBC model in question 2 (with $\bar{\epsilon} = 0$) give rise to the following dynamic equations:

$$\Delta \check{K}_t = \left(\frac{\alpha(1-\alpha)}{\alpha + 1/\eta} \frac{\bar{Y}}{\bar{K}} + \alpha \frac{\bar{Y}}{\bar{K}} - \delta \right) \check{K}_{t-1} + \left(\frac{\bar{C}}{\bar{K}} + \frac{\bar{Y}}{\bar{K}} \frac{(1-\alpha)}{\alpha + 1/\eta} \right) \check{\lambda}_t + \delta \epsilon_t$$

$$\Delta \check{\lambda}_{t+1} = - \frac{\alpha \frac{\bar{Y}}{\bar{K}}}{\alpha \frac{\bar{Y}}{\bar{K}} + 1 - \delta} \left[(1-\alpha) \left(\frac{\alpha}{1/\eta + \alpha} - 1 \right) \check{K}_t + \frac{1-\alpha}{1/\eta + \alpha} \check{\lambda}_{t+1} \right]$$

(a) (10 points) Plot the dynamics of this system in a phase diagram as we did in class. I.e., plot the $\Delta K = 0$ and $\Delta \lambda = 0$ locus (K_t on x-axis, λ_t on y-axis). **Include all of the following**: axis labels, curve labels, steady-state (labeled), stable arm (labeled), and arrows showing the dynamics in each quadrant.

(b) (36 points) Suppose that there is a **temporary** increase in ϵ_t between t_0 and t_1. At t_1, ϵ_t returns to zero.

Using the phase diagram in the (λ, K) space as well as the labor market diagram, plot the response of the economy to the shock. **Label all curves, axis and points.** Summarize the impact, transition (before and after t_1), inflection, and steady-state responses for all variables in the table below (like we did in class).

Make the following assumptions:
1. At t_0, the economy jumps to a point **above** the new $\Delta K = 0$ locus.

Figure 1: RBC model response to a marginal efficiency of investment shock.

	Impact $t = t_0$	Transition I $t \in (t_0, t_1)$	Inflection $t = t_1$	Transition II $t \in (t_1, \infty)$	Steady State $t \to \infty$
λ					
K					
C					
L					
Y					
I					
W					
R					

(c) (6 points) **Briefly** interpret your results from part (b).

(d) (6 points) Are the impact responses ($t = t_0$) you derived consistent with the co-movement of output, labor, consumption and investment over the business cycle? **Explain your answer.** What does that tell you about the plausibility of attributing the business cycle to such shocks?

(e) (6 points) Given what you know from class and/or the Greenwood, Hercowitz and Huffmann (1988) paper, briefly describe how would you modify the model to make the responses consistent with business cycle co-movement.