News of Savings Taxes in the RBC model (72 points)

2. Consider the canonical RBC model with shocks to the tax rate on saving: A consumer maximizes

$$\max \sum_{t}^{\infty} \beta^t \left(\ln C_t - \chi \frac{(L_t^s)^{1+1/\eta}}{1+1/\eta} \right)$$
$$\text{s.t. } A_t + C_t = (1+R_t)(1-\boldsymbol{\tau}_t)A_{t-1} + W_t L_t^s + \pi_t$$
$$lim_{t\to+\infty} \left(\prod_{s=1}^{t} (1+R_s) \right)^{-1} A_t = 0$$

taking prices as given. The variable τ_t denotes a tax on savings.

A representative firm maximizes profits,

$$\max \sum_{t}^{\infty} \left(\prod_{s=1}^{t} (1+R_s) \right)^{-1} (Y_t - W_t L_t^d - I_t)$$
$$\text{s.t. } Y_t = K_{t-1}^{\alpha} (L_t^d)^{1-\alpha}$$
$$K_t = (1-\delta)K_{t-1} + I_t.$$

taking prices as given. Note that the price of output is normalized to 1, $P_t = 1$.

The market clearing conditions are:

$$L_t^s = L_t^d = L_t$$
$$K_t = A_t$$
$$Y_t = C_t + I_t$$

Standard parameter values apply:

$$0 < \alpha, \beta, \delta < 1$$
$$\eta > 0$$

Question continues on next page.

(a) (36 points) Set up the Lagrangian and derive the first order conditions for the household w.r.t. C_t, L_t^s, A_t. Use λ_t as the Lagrange multiplier on the budget constraint. Briefly interpret each equation (one sentence each).

(b) (0 points) The following equations are the first-order conditions for firms. These are a useful reference for the phase diagram.

$$(1-\alpha)K_{t-1}^{\alpha}(L_t^d)^{-\alpha} = W_t$$
$$\alpha K_t^{\alpha-1}(L_{t+1}^d)^{1-\alpha} = R_{t+1} + \delta$$

(c) (22 points) Log-linearize the first of the equations above ($(1-\alpha)K_{t-1}^{\alpha}(L_t^d)^{-\alpha} = W_t$).

3. The log-linearized FOCs of the RBC model in question 2 give rise to the following dynamic equations:

$$\Delta \check{K}_t = \left(\frac{\alpha(1-\alpha)}{\alpha + 1/\eta} \frac{\bar{Y}}{\bar{K}} + \alpha \frac{\bar{Y}}{\bar{K}} - \delta \right) \check{K}_{t-1} + \left(\frac{\bar{C}}{\bar{K}} + \frac{\bar{Y}}{\bar{K}} \frac{(1-\alpha)}{\alpha + 1/\eta} \right) \check{\lambda}_t$$

$$\Delta \check{\lambda}_{t+1} = -\frac{\alpha \frac{\bar{Y}}{\bar{K}}}{\alpha \frac{\bar{Y}}{\bar{K}} + 1 - \delta} \left[(1-\alpha) \left(\frac{\alpha}{1/\eta + \alpha} - 1 \right) \check{K}_t + \frac{1-\alpha}{1/\eta + \alpha} \check{\lambda}_{t+1} \right] + \tau_{t+1}$$

   (a) (20 points) Plot the dynamics of this system in a phase diagram as we did in class. I.e., plot the $\Delta K = 0$ and $\Delta \lambda = 0$ locus (K_t on x-axis, λ_t on y-axis). **Include all of the following**: axis labels, curve labels, steady-state (labeled), stable arm (labeled), and arrows showing the dynamics in each quadrant.

(b) (90 points) Suppose that at t_0 agents learn of a future increase in the rate of savings taxes τ. The increase is expected to be enacted at $t_1 > t_0$ and is expected to be permanent.

Using the phase diagram in the (λ, K) space as well as the labor market diagram, plot the response of the economy to the shock. **Label all curves, axis and points.** Summarize the impact, transition (before and after t_1), inflection, and steady-state responses for all variables in the table below (like we did in class).

Figure 1: RBC model response to news of a savings tax shock.

	Impact $t = t_0$	Transition I $t \in (t_0, t_1)$	Inflection $t = t_1$	Transition II $t \in (t_1, \infty)$	Steady State $t \to \infty$
λ					
K					
C					
L					
Y					
I					
W					
R					

(c) (25 points) **Intuitively** (no equations), explain the broad behavior of the representative consumer and firm following the news shock. **Do not write more than two paragraphs.**

(d) (20 points) Are the impact responses ($t = t_0$) you derived consistent with the co-movement of output, labor, consumption, investment and the real wage over the business cycle? **Explain your answer.** What does that tell you about the plausibility of attributing the business cycle to such shocks?