The "Great Vacation" in the Real Business Cycle model (72 points)

2. Consider the canonical RBC model with shocks to the disutility of labor: A consumer maximizes

$$\max \sum_{t}^{\infty} \beta^t \left(\ln C_t - \boldsymbol{\chi}_t \frac{(L_t^s)^{1+1/\eta}}{1+1/\eta} \right)$$
$$\text{s.t. } A_t + C_t = (1+R_t)A_{t-1} + W_t L_t^s + \pi_t$$
$$lim_{t \to +\infty} \left(\prod_{s=1}^{t} (1+R_s) \right)^{-1} A_t = 0$$

taking prices as given. The variable χ_t denotes a shock to the disutility of labor. High values for χ_t imply higher disutility from the same labor supply L_t^s — everyone really wants to go on vacation rather than work.

A representative firm maximizes profits,

$$\max \sum_{t}^{\infty} \left(\prod_{s=1}^{t} (1+R_s) \right)^{-1} (Y_t - W_t L_t^d - I_t)$$
$$\text{s.t. } Y_t = K_{t-1}^{\alpha} (L_t^d)^{1-\alpha}$$
$$K_t = (1-\delta)K_{t-1} + I_t.$$

taking prices as given. Note that the price of output is normalized to 1, $P_t = 1$.

The market clearing conditions are:

$$L_t^s = L_t^d = L_t$$
$$K_t = A_t$$
$$Y_t = C_t + I_t$$

Standard parameter values apply:

$$0 < \alpha, \beta, \delta < 1$$
$$\eta > 0$$

Question continues on next page.

(a) (12 points) Set up the Lagrangian and derive the first order conditions for the household w.r.t. C_t, L_t^s, A_t. Use λ_t as the Lagrange multiplier on the budget constraint. Briefly interpret each equation (one sentence each).

(b) (0 points) The following equations are the first-order conditions for firms. These are a useful reference for the phase diagram.

$$
\begin{aligned}
(1-\alpha)K_{t-1}^{\alpha}(L_t^d)^{-\alpha} &= W_t \\
\alpha K_t^{\alpha-1}(L_{t+1}^d)^{1-\alpha} &= R_{t+1} + \delta
\end{aligned}
$$

3. The log-linearized FOCs of the RBC model in question 2 (with $\bar{\chi} = 1$) give rise to the following dynamic equations:

$$\Delta \check{K}_t = \left(\frac{\alpha(1-\alpha)}{\alpha + 1/\eta} \frac{\bar{Y}}{\bar{K}} + \alpha \frac{\bar{Y}}{\bar{K}} - \delta \right) \check{K}_{t-1} + \left(\frac{\bar{C}}{\bar{K}} + \frac{\bar{Y}}{\bar{K}} \frac{(1-\alpha)}{\alpha + 1/\eta} \right) \check{\lambda}_t - \left(\frac{\bar{Y}}{\bar{K}} \frac{(1-\alpha)}{\alpha + 1/\eta} \right) \check{\chi}_t$$

$$\Delta \check{\lambda}_{t+1} = -\frac{\alpha \frac{\bar{Y}}{\bar{K}}}{\alpha \frac{\bar{Y}}{\bar{K}} + 1 - \delta} \left[(1-\alpha) \left(\frac{\alpha}{1/\eta + \alpha} - 1 \right) \check{K}_t + \frac{1-\alpha}{1/\eta + \alpha} \check{\lambda}_{t+1} - \frac{1-\alpha}{1/\eta + \alpha} \check{\chi}_{t+1} \right]$$

(a) (10 points) Plot the dynamics of this system in a phase diagram as we did in class. I.e., plot the $\Delta K = 0$ and $\Delta \lambda = 0$ locus (K_t on x-axis, λ_t on y-axis). **Include all of the following**: axis labels, curve labels, steady-state (labeled), stable arm (labeled), and arrows showing the dynamics in each quadrant.

(b) (36 points) Suppose that there is a **temporary** increase in χ_t between t_0 and t_1 (a "laziness" shock). At t_1, χ_t returns to zero.

Using the phase diagram in the (λ, K) space as well as the labor market diagram, plot the response of the economy to the shock. **Label all curves, axis and points.** Summarize the impact, transition (before and after t_1), inflection, and steady-state responses for all variables in the table below (like we did in class).

Make the following assumptions:
1. The new steady-state (between t_0 and t_1) is above and to the left of the old steady-state.
2. The new stable arm (between t_0 and t_1) lies above the old stable arm.

Figure 1: RBC model response to a "laziness" shock.

	Impact $t = t_0$	Transition I $t \in (t_0, t_1)$	Inflection $t = t_1$	Transition II $t \in (t_1, \infty)$	Steady State $t \to \infty$
λ					
K					
C					
L					
Y					
I					
W					
R					

(c) (8 points) **Briefly** interpret your results from part (b).

(d) (6 points) Are the impact responses ($t = t_0$) you derived consistent with the co-movement of output, labor, consumption, investment and the real wage over the business cycle? **Explain your answer.** What does that tell you about the plausibility of attributing the business cycle to such shocks?