"Demand" Shocks in the RBC model (270 points)

2. Consider the canonical RBC model with shocks to marginal utility ξ_t. In steady-state $\bar{\xi} = 1$. Notation is as in our class: A consumer maximizes

$$\max \sum_t^{\infty} \beta^t \left(\xi_t \ln C_t - \frac{(L_t^s)^{1+1/\eta}}{1+1/\eta} \right)$$
$$\text{s.t. } A_t + C_t = (1+R_t)A_{t-1} + W_t L_t^s + \Pi_t$$
$$lim_{t\to+\infty} \left(\prod_{s=1}^{t} (1+R_s) \right)^{-1} A_t = 0$$

taking prices as given.

A representative firm maximizes profits,

$$\max \sum_t^{\infty} \left(\prod_{s=1}^{t} (1+R_s) \right)^{-1} [Y_t - W_t L_t^d - I_t]$$
$$\text{s.t. } Y_t = K_{t-1}^{\alpha} (L_t^d)^{1-\alpha}$$
$$K_t = (1-\delta)K_{t-1} + I_t.$$

taking prices as given.

The market clearing conditions are:

$$L_t^s = L_t^d = L_t$$
$$K_t = A_t$$
$$Y_t = C_t + I_t$$

So tax revenues are rebated lump-sum to households.

Standard parameter values apply:

$$0 < \alpha < 1$$
$$0 < \beta < 1$$
$$0 < \delta < 1$$
$$\eta > 0$$

Question continues on next page.

(a) (30 points) Set up the Lagrangian and derive the first order conditions for the household w.r.t. C_t, L_t^s, A_t. Use λ_t as the Lagrange multiplier on the budget constraint. Briefly interpret each equation (one sentence each).

(b) (30 points) Set up the Lagrangian and derive the first order conditions for the firm w.r.t I_t, L_t^d, K_t. (Substitute for Y_t first.) Use q_t as the Lagrange multiplier on capital accumulation. Briefly interpret each equation (one sentence each).

(c) (15 points) Log-linearize the household first order condition for consumption.

3. The log-linearized equations of the RBC model in question 2 give rise to the following dynamic equations:

$$\Delta \check{K}_t = -\frac{\bar{C}}{\bar{K}}\check{\xi} + \left(\frac{\alpha(1-\alpha)}{\alpha+1/\eta}\frac{\bar{Y}}{\bar{K}} + \beta^{-1} - 1\right)\check{K}_{t-1} + \left(\frac{\bar{C}}{\bar{K}} + \frac{\bar{Y}}{\bar{K}}\frac{(1-\alpha)}{\alpha+1/\eta}\right)\check{\lambda}_t$$

$$\Delta \check{\lambda}_{t+1} = -\frac{\alpha\frac{\bar{Y}}{\bar{K}}}{\alpha\frac{\bar{Y}}{\bar{K}} + 1 - \delta}\left[(1-\alpha)\left(\frac{\alpha}{1/\eta+\alpha} - 1\right)\check{K}_t + \frac{1-\alpha}{1/\eta+\alpha}\check{\lambda}_{t+1}\right]$$

(a) (20 points) Plot the dynamics of this system in a phase diagram as we did in class. I.e., plot the $\Delta K = 0$ and $\Delta \lambda = 0$ locus (K_t on x-axis, λ_t on y-axis). **Include all of the following**: axis labels, curve labels, steady-state (labeled), stable arm (labeled), and arrows showing the dynamics in each quadrant.

(b) (80 points) Suppose that ξ_t falls permanently to a lower level, so marginal utility is lower for any level of consumption.

Using the phase diagram in the (λ, K) space as well as the labor market diagram, plot the response of the economy to the shock. **Label all curves, axis and points.** Summarize the impact, transition, and steady-state responses for all variables in the table below (like we did in class) **and explain the intuition behind the response.**

Table 1: RBC model response to a permanent labor tax shock.

	Impact $t = t_0$	Transition $t \in (t_0, \infty)$	Steady State $t \to \infty$
λ			
K			
C			
L			
Y			
I			
W			
R			

(c) (20 points) Explain **intuitively** how the economy adjusts to a negative marginal utility shock.

(d) (10 points) Are the impact responses ($t = t_0$) you derived consistent with the co-movement of output, labor, consumption and investment over the business cycle? **Explain your answer.** What does that tell you about the plausibility of attributing the business cycle to demand shocks?

4. We now add **capital utilization** to our model. This leaves the household first order conditions unchanged. We therefore focus on the firm's problem.

   A representative firm maximizes profits,

$$\max \sum_{t}^{\infty} \left(\prod_{s=1}^{t} (1+R_s) \right)^{-1} \left[(u_t K_{t-1})^{\alpha} (L_t^d)^{1-\alpha} - W_t L_t^d - I_t \right]$$
$$\text{s.t. } K_t = (1-\delta_t)K_{t-1} + I_t.$$
$$\delta_t = \frac{1}{\theta} u_t^{\theta}$$

   taking prices as given. Additional parameter restrictions are $\theta > \alpha$

   (a) (17 points) Set up the Lagrangian and derive the first order conditions for the firm w.r.t u_t, I_t, L_t^d, K_t. Use q_t as the Lagrange multiplier on capital accumulation.

(b) (8 points) Using the firm first order conditions, derive the labor demand curve for the firm, substituting out for utilization u_t.

5. In this economy, a permanent reduction in the marginal utility shock ξ_t generates qualitatively the same phase diagram as our previous model. We therefore focus on the labor market.

   (a) (15 points) Draw the labor market diagram for t_0 (i.e., when the marginal utility shock arrives). How is it different from the labor market in the economy without utilization?

(b) (5 points) For what parameter values is the real wage acyclical or procylical?

(c) (20 points) Given that parameter range, are shocks to marginal utility of consumption a plausible source of business cycles? Explain why or why not.