Rotemberg and Woodford (1995): markup shocks in the RBC model

2. Consider the canonical RBC model with imperfect competition and no depreciation $\delta = 0$. Notation is as in our class: A consumer maximizes

$$\max \sum_{t}^{\infty} \beta^t \left(\ln C_t - \frac{L_t^{1+1/\eta}}{1+1/\eta} \right)$$
$$\text{s.t. } C_t + I_t = \frac{R_t}{P_t} K_{t-1} + \frac{W_t}{P_t} L_t + \Pi_t$$
$$K_t = K_{t-1} + I_t.$$
$$lim_{t \to +\infty} \left(\prod_{s=1}^{t} (1 + R_s) \right)^{-1} K_t = 0$$

taking prices R_t, W_t, P_t as given.

A representative firm maximizes period-by-period profits by hiring labor L_t and renting capital K_{t-1} in a spot market,

$$\max\ \Pi_t = P_t(Y_t)Y_t - W_t L_t - R_t K_{t-1}$$
$$\text{s.t. } P_t(Y_t) = Y_t^{\frac{1}{\mu_t}-1}$$
$$Y_t = K_{t-1}^{\alpha} L_t^{1-\alpha}$$

where $\mu_t > 0$ is the equilibrium markup.

The market clearing conditions are:

$$Y_t = C_t + I_t$$

Standard parameter values apply:

$$0 < \alpha < 1$$
$$0 < \beta < 1$$
$$\eta > 0$$
$$\mu_t \geq 1$$

Question continues on next page.

(a) (25 points) Set up the Lagrangian and derive the first order conditions for the household w.r.t. C_t, L_t, K_t. Use λ_t as the Lagrange multiplier on the budget constraint. *You should substitute for I_t using the accumulation equation for capital.* Briefly interpret each equation (one sentence each).

(b) (15 points) Derive the first order conditions for the firm w.r.t L_t, K_{t-1}. (Substitute for Y_t first.) Then show that they can be written as

$$\frac{(1-\alpha)}{\mu_t}\frac{Y_t}{L_t} = \frac{(1-\alpha)}{\mu_t}K_{t-1}^{\alpha}L_t^{-\alpha} = \frac{W_t}{P_t}$$
$$\frac{\alpha}{\mu_t}\frac{Y_t}{K_{t-1}} = \frac{\alpha}{\mu_t}K_{t-1}^{\alpha-1}L_t^{1-\alpha} = \frac{R_t}{P_t}$$

Briefly interpret each of the two equations above (one sentence each).

(c) (15 points) Log-linearize the labor demand equation, $\frac{(1-\alpha)}{\mu_t} K_{t-1}^{\alpha} L_t^{-\alpha} = \frac{W_t}{P_t}$.

3. The log-linearized equations of the RBC model in question 2 give rise to the following dynamic equations:

$$\Delta \check{K}_t = \left(\frac{\alpha(1-\alpha)}{\alpha + 1/\eta} \frac{\bar{Y}}{\bar{K}} + \beta^{-1} - 1 \right) \check{K}_{t-1} + \left(\frac{\bar{C}}{\bar{K}} + \frac{\bar{Y}}{\bar{K}} \frac{(1-\alpha)}{\alpha + 1/\eta} \right) \check{\lambda}_t - \frac{\bar{Y}}{\bar{K}} \frac{(1-\alpha)}{\alpha + 1/\eta} \check{\mu}_t$$

$$\Delta \check{\lambda}_{t+1} = -\frac{\alpha \frac{\bar{Y}}{\bar{K}}}{\alpha \frac{\bar{Y}}{\bar{K}} + 1} \left[(1-\alpha) \left(\frac{\alpha}{1/\eta + \alpha} - 1 \right) \check{K}_t + \frac{1-\alpha}{1/\eta + \alpha} \check{\lambda}_{t+1} - \frac{1-\alpha}{1/\eta + \alpha} \check{\mu}_{t+1} \right]$$

(a) (20 points) Plot the dynamics of this system in a phase diagram as we did in class. I.e., plot the $\Delta K = 0$ and $\Delta \lambda = 0$ locus (K_t on x-axis, λ_t on y-axis). **Include all of the following**: axis labels, curve labels, steady-state (labeled), stable arm (labeled), and arrows showing the dynamics in each quadrant.

(b) (120 points) At t_0 agents learn that markups will be temporarily higher between $[t_0, t_1)$.

Using the phase diagram in the (λ, K) space as well as the labor market diagram in the $(W/P, L)$ space, plot the response of the economy to the shock. **Label all curves, axis and points.** Summarize the impact, transition, and steady-state responses for all variables in the table below (like we did in class).

Make the following assumptions:
1. At t_0 the economy jumps to a point **below** the new $\Delta K = 0$ locus and **below** the new $\Delta\lambda = 0$ locus.
2. The economy does not cross either the new $\Delta\lambda = 0$ or the $\Delta K = 0$ locus between t_0 and t_1.

Table 1: RBC model response to temporary markup shock.

	Impact $t = t_0$	Transition I $t \in (t_0, t_1)$	Inflection $t = t_1$	Transition II $t \in (t_1, \infty)$	Steady State $t \to \infty$
λ					
K					
C					
L					
Y					
I					
W/P					
R/P					

(c) (15 points) Explain *intuitively* how the economy adjusts to a temporary higher markup (max 2 paragraphs).

(d) (10 points) Are the impact responses ($t = t_0$) you derived consistent with the co-movement of output, labor, consumption and investment over the business cycle? **Explain your answer.** What does that tell you about the plausibility of attributing the business cycle to markup shocks?

(e) (10 points) Other than your answer to part d, for what other reasons are markup shocks a plausible or implausible source of the business cycle? (max 2 paragraphs.)

4. Suppose you added production externalities to the model. For an individual firm, the production function is $Y_{i,t} = E_t K_{i,t-1}^{\alpha} L_{i,t}^{1-\alpha}$ where $E_t = Y_t^{1-\frac{1}{\gamma}} \equiv \left[\int_0^1 Y_{i,t} di\right]^{1-\frac{1}{\gamma}}$.

   (a) (25 points) Show how this modification would affect the labor market diagram on impact. *Briefly*, explain your answer. (You do not need to show any derivations here.)

   (b) (15 points) Would this modification worsen or improve the ability of the model to match the business cycle using markup shocks? *Briefly*, explain your answer. (max 2 paragraph)