Economics 210C - Macroeconomics Spring 2018-19

Prof. Garey Ramey

RBC Computational Assignment

Consider a version of the RBC model with indivisible labor supply and permanent technology shocks. In each period, the social planner solves the following problem:

$$\max_{C_t, L_t, K_t} \mathbb{E}_t \sum\nolimits_{s=0}^{\infty} \beta^s \left(\ln C_{t+s} + \chi \left(T - L_{t+s} \right) \right),$$

$$\text{s.t.} \quad K_{t+s-1}^{\alpha} \left(X_{t+s} L_{t+s} \right)^{1-\alpha} + (1-\delta) K_{t+s-1} = K_{t+s} + C_{t+s} + G_{t+s},$$

where $\chi, T > 0$ and $0 < \beta, \alpha, \delta < 1$. $\{X_t\}$ and $\{G_t\}$ are exogenous random processes given by

$$X_t = X_{t-1} e^{\mu + \zeta_t}, \quad \zeta_t = \rho \zeta_{t-1} + \varepsilon_t,$$

$$\frac{G_t}{X_t} = g^{1-\gamma} \left(\frac{G_{t-1}}{X_{t-1}} \right)^{\gamma} e^{\nu_t},$$

where $\mu, g > 0$, $0 < \rho, \gamma < 1$, and $\{\varepsilon_t\}$ and $\{\nu_t\}$ are exogenous white noise processes.

a. Derive the first-order necessary conditions for a solution to the planner's problem, letting Λ_t denote the Lagrange multiplier on the resource constraint.

b. Define detrended variables as follows:

$$c_t = \frac{C_t}{X_t}, \quad k_t = \frac{K_t}{X_t}, \quad g_t = \frac{G_t}{X_t},$$

$$\lambda_t = \Lambda_t X_t.$$

Restate your necessary conditions in terms of the detrended variables.

c. Output and gross investment are given by

$$Y_t = K_{t-1}^{\alpha} \left(X_t L_t \right)^{1-\alpha},$$

$$I_t = K_t - (1-\delta) K_{t-1}.$$

Restate these equations in terms of detrended variables:

$$y_t = \frac{Y_t}{X_t}, \quad i_t = \frac{I_t}{X_t}.$$

d. Derive the nonstochastic steady state solution of the detrended system in part b, along with the steady state values of detrended output and investment.

e. Log-linearize the equations from parts b and c around the steady state derived in part d. Express the log-linearized system in the form indicated in the "Rational Expectations Solution" notes.

f. Calculate the rational expectations solution matrices Φ_S and Φ_U for the log-linearized system under the following parameter values:

α	β	δ	χ	T	μ	ρ	g	γ
.344	$1.03^{-.25}$	.01	1	1,369	.004	.1	.8	.96 .

Calculate the associated laws of motion for detrended consumption, hours, output and investment.

g. Use your solution to calculate 40-period impulse responses of $\hat{y}_t$, $\hat{\lambda}_t$ and $\hat{c}_t$ to a one percent shock to ζ_t (i.e., $\varepsilon_{t_0} = .01$ and $\varepsilon_t = 0$ for $t \neq t_0$). Repeat your calculation for a one percent shock to $\hat{g}_t$.

h. The deviations of output, marginal utility of wealth, and consumption from their deterministic trends are given by

$$\hat{Y}_t = \hat{y}_t + S_t, \quad \hat{C}_t = \hat{c}_t + S_t, \quad \hat{\Lambda}_t = \hat{\lambda} - S_t,$$

where

$$S_t = \textstyle\sum_{j=1}^{t} \zeta_{t-j}.$$

Calculate 40-period impulse responses of $\hat{Y}_t$, $\hat{C}_t$ and $\hat{\Lambda}_t$ to a one percent shock to ζ_t, and repeat your calculation for a one percent shock to $\hat{g}_t$. Provide a brief intuitive discussion of the results.