Economics 210C - Macroeconomics Spring 2018-19

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Phase Diagram Exercise v.2

Consider a version of the RBC model in which the household owns the capital stock and rents capital to the firm. In each period, the household solves the following problem:

$$\max_{C_t, L_t, K_t} \mathbb{E}_t \sum\nolimits_{s=0}^{\infty} \beta^s \left(\ln C_{t+s} - \frac{L_{t+s}^{1+1/\eta}}{1+1/\eta} \right)$$

$$\text{s.t.} \quad W_{t+s}L_{t+s} + (1-\tau_{t+s})R_{t+s}K_{t+s-1} + \Pi_{t+s} + (1-\delta)K_{t+s-1}$$

$$= K_{t+s} + C_{t+s} + T_{t+s},$$

and the firm chooses K_t and L_t to maximize

$$\Pi_t = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} - W_t L_t - R_t K_{t-1},$$

where R_t is the capital rental rate and Z_t is exogenous. Moreover, in each period, the government chooses the lump-sum tax T_t to balance its budget:

$$\tau_t R_t K_{t-1} + T_t = G_t,$$

where τ_t and G_t are exogenous. The parameters satisfy $\eta > 0$ and $0 < \beta, \delta, \alpha < 1$, while $Z_t, G_t > 0$ and $0 \leq \tau_t < 1$ for each t.

a. Derive first-order necessary conditions for solutions to the household's and firm's problems, assuming that the labor and capital markets clear in each period.

b. Eliminate the variables W_t, R_t and T_t from your equations in order to express the necessary conditions in terms of the variables C_t, L_t, λ_t, K_{t-1}, Z_t, τ_t and G_t.

c. Solve for $\Delta\lambda_t$ and ΔK_t as functions of the variables C_t, L_t, λ_t, K_{t-1}, Z_t, τ_t and G_t, under the assumption that Z_t, τ_t and G_t are perfectly-anticipated deterministic paths.

d. Convert the necessary conditions from parts b and c to continuous time. Express your conditions as four equations that determine C_t, L_t, λ_t, K_t, $\dot{\lambda}_t$ and $\dot{K}_t$ given Z_t, τ_t and G_t.

e. Let $\dot{\lambda}_t = \dot{K}_t = 0$. Calculate the total log-derivatives of the four equations with respect to the remaining variables at given positive values of the variables. Use the total log-derivatives to calculate the partial log-derivatives of the $\dot{\lambda}_t = 0$ and $\dot{K}_t = 0$ curves in the K_t-λ_t plane (i.e., where the curves determine values of λ_t as functions of K_t and the exogenous variables). Assume that the two curves are well-defined.

f. Suppose the economy begins in a steady state. At period t_0, τ is <u>reduced</u> to a lower level, and then it is restored to its previous level at period $t_1 > t_0$. Assume that Z_t and G_t remain constant. Show that the $\dot{\lambda}_t = 0$ curve shifts upward in the K_{t-1}-λ_t plane when τ_t increases, while the $\dot{K}_t = 0$ curve is unaffected by changes in τ_t.

g. Trace out the continuous time perfect foresight dynamics in the K_t-λ_t plane generated by the tax rate path from part f, assuming that the economy converges to the original steady state. Describe the corresponding paths of C_t, L_t, Y_t and W_t.