Notes on the New Keynesian Model

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1 Model and equilibrium

a Marginal cost

The production technology of intermediate good firms is

$$Y_{it} = Z_t F(K_{i,t-1}, X_t L_{it}). \tag{1}$$

F satisfies standard assumptions, including constant returns to scale. Cost-minimizing choices of $K_{i,t-1}$ and L_{it} by firm i yield the following necessary conditions:

$$\phi_t Z_t F_K(K_{i,t-1}, X_t L_{it}) = R_t, \tag{2}$$

$$\phi_t Z_t F_L(K_{i,t-1}, X_t L_{it}) X_t = W_t, \tag{3}$$

where ϕ_t is the Lagrange multiplier on the constraint (1). Using (2), (3) and constant returns to scale, we can write

$$\frac{F_L(K_{i,t-1}, X_t L_{it}) X_t}{F_K(K_{i,t-1}, X_t L_{it})} = \frac{F_L(K_{i,t-1}/L_{it}, X_t) X_t}{F_K(K_{i,t-1}/L_{it}, X_t)}$$

$$= \frac{F_L(\kappa_t, X_t) X_t}{F_K(\kappa_t, X_t)} = \frac{W_t}{R_t},$$

where κ_t gives the capital-labor ratio chosen by each firm. Thus we can express ϕ_t as

$$\phi_t = \frac{R_t}{Z_t F_K(\kappa_t, X_t)} = \frac{W_t}{Z_t F_L(\kappa_t, X_t) X_t}, \tag{4}$$

and it follows that ϕ_t depends on neither i nor Y_{it}.

Minimized total cost can be written as

$$R_t K_{i,t-1} + W_t L_{it} = \phi_t Z_t F_K(\kappa_t, X_t) K_{i,t-1} + \phi_t Z_t F_L(\kappa_t, X_t) X_t L_{it}$$

$$= \phi_t Z_t \left(F_K(K_{i,t-1}, X_t L_{it}) K_{i,t-1} + F_L(K_{i,t-1}, X_t L_{it}) X_t L_{it} \right)$$

$$= \phi_t Z_t F(K_{i,t-1}, X_t L_{it}) = \phi_t Y_{it},$$

using Euler's theorem. Thus, ϕ_t gives marginal cost in period t for each firm i.

b Calvo price adjustment

Firm i faces the following demand function for its good:

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\sigma} Y_t.$$

Thus, its profit function in units of the final good is

$$\Pi_t(P_{it}) \;=\; \frac{P_{it}}{P_t} Y_{it} - \phi_t Y_{it} \;=\; \left(\left(\frac{P_{it}}{P_t} \right)^{1-\sigma} - \phi_t \left(\frac{P_{it}}{P_t} \right)^{-\sigma} \right) Y_t. \tag{5}$$

Assume that profits are paid out to the household as they are received.

In each period t, firm i seeks to maximize the market value of its current and future profit stream, given by

$$\mathbb{E}_t \sum\nolimits_{s=0}^{\infty} \frac{\beta^s U_{C,t+s}}{U_{Ct}} \Pi_{t+s},$$

where U_{Ct} indicates the household's marginal utility of consumption in period t. Note that $\beta^s U_{C,t+s}/U_{Ct}$ is the stochastic discount factor for period $t+s$ final goods priced in terms of period t final goods.

Price adjustment is subject to the restrictions introduced by Calvo (JME 1983). In any period, with probability $\omega > 0$, firm i is not allowed to adjust its price. In this case, $P_{it} = P_{i,t-1}$. With probability ω, firm i can choose any P_{it} that it wishes. These price adjustment draws are assumed to be independent of other random variables in the economy, including the firm's own past price adjustment draws. It follows that the value-maximizing choice of P_{it} satisfies the following necessary condition:

$$\mathbb{E}_t \sum\nolimits_{s=0}^{\infty} \frac{\beta^s U_{C,t+s}}{U_{Ct}} \omega^s \frac{\partial \Pi_{t+s}}{\partial P_{it}} = 0. \tag{6}$$

Note that ω^s is the probability that P_{it} will not be adjusted through period $t+s$, so that Π_{t+s} will depend on P_{it}. Differentiating (5) gives

$$\frac{\partial \Pi_{t+s}}{\partial P_{it}} = \frac{1}{P_{it}} \left(\frac{P_{it}}{P_t}\right)^{-\sigma} \left((1-\sigma) \frac{P_{it}}{P_t} \left(\frac{P_t}{P_{t+s}}\right)^{1-\sigma} + \sigma \phi_{t+s} \left(\frac{P_t}{P_{t+s}}\right)^{-\sigma} \right) Y_{t+s}. \tag{7}$$

Substitute (7) into (6), cancel terms and rearrrange to obtain

$$\mathbb{E}_t \sum\nolimits_{s=0}^{\infty} (\beta\omega)^s U_{C,t+s} \left((1-\sigma) \frac{P_{it}}{P_t} \left(\frac{P_t}{P_{t+s}}\right)^{1-\sigma} + \sigma \phi_{t+s} \left(\frac{P_t}{P_{t+s}}\right)^{-\sigma} \right) Y_{t+s} = 0,$$

which can be expressed as

$$\frac{P_{it}}{P_t} = \frac{\sigma}{\sigma-1} \frac{\sum\nolimits_{s=0}^{\infty} (\beta\omega)^s \mathbb{E}_t U_{C,t+s} \phi_{t+s} \left(\frac{P_t}{P_{t+s}}\right)^{-\sigma} Y_{t+s}}{\sum\nolimits_{s=0}^{\infty} (\beta\omega)^s \mathbb{E}_t U_{C,t+s} \left(\frac{P_t}{P_{t+s}}\right)^{1-\sigma} Y_{t+s}}. \tag{8}$$

Observe that taking the limit of (8) as $\omega \to 0$ gives

$$\frac{P_{it}}{P_t} = \frac{\sigma}{\sigma-1} \phi_t, \tag{9}$$

or

$$\frac{P_{it}/P_t}{\phi_t} = \frac{\sigma}{\sigma-1}.$$

This is the profit-maximizing markup derived earlier under unrestricted price setting. Thus, the current model may be viewed as a generalization of the earlier imperfect competition model. For $\omega > 0$, (8) shows that firm i chooses its price as a markup on an <u>index</u> of current and future marginal costs, weighted to reflect the prospects for future price adjustment.

Let $P_{it} = P_t^*$ denote the solution to (8). The resulting price setting equation can be written as

$$\frac{P_t^*}{P_t}\,\mathbb{E}_t \sum\nolimits_{s=0}^{\infty} (\beta\omega)^s\, U_{C,t+s} \left(\frac{P_t}{P_{t+s}}\right)^{1-\sigma} Y_{t+s} \tag{10}$$

$$= \frac{\sigma}{\sigma-1}\mathbb{E}_t \sum\nolimits_{s=0}^{\infty} (\beta\omega)^s\, U_{C,t+s}\phi_{t+s} \left(\frac{P_t}{P_{t+s}}\right)^{-\sigma} Y_{t+s}.$$

c Final good price

Cost minimization by the final good firm, together with perfect competition in the final good market, imply that P_t satisfies

$$P_t^{1-\sigma} = \int_0^1 P_{it}^{1-\sigma} di. \tag{11}$$

Proportion ω of the intermediate good firms must have $P_{it} = P_{i,t-1}$, while proportion $1-\omega$ choose $P_{it} = P_t^*$. Moreover, since nonadjustment events are determined independently of period $t-1$ prices, it follows that the period $t-1$ price distribution is preserved among the nonadjusting firms. Thus, the final good price may be expressed as

$$P_t = \left(\omega \int_0^1 P_{i,t-1}^{1-\sigma} di + (1-\omega) \int_0^1 (P_t^*)^{1-\sigma} di\right)^{\frac{1}{1-\alpha}}$$

$$= \left(\omega P_{t-1}^{1-\sigma} + (1-\omega)\,(P_t^*)^{1-\sigma}\right)^{\frac{1}{1-\alpha}}.$$

Observe that the state variable P_{t-1} summarizes the distribution of prices at nonadjusting firms. In this way, the Dixit-Stiglitz composite commodity specification makes for very convenient price aggregation.

Rearrange the preceding equation to obtain

$$1 = \omega\pi_t^{\sigma-1} + (1-\omega)\left(\frac{P_t^*}{P_t}\right)^{1-\sigma}. \tag{12}$$

d Factor market clearing

Firm i's output can be expressed as

$$Y_{it} = Z_t F(\kappa_t, X_t) L_{it}.$$

Thus, aggregate demand for labor in period t satisfies

$$L_t = \int_0^1 L_{it} di = \int_0^1 \frac{Y_{it}}{Z_t F(\kappa_t, 1)} di = \frac{Y_t}{Z_t F(\kappa_t, X_t)} \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\sigma} di. \tag{13}$$

The labor market clears if L_t equals the household's desired labor supply. Note that the price distribution does not aggregate in this case.

For the capital market, we have

$$\int_0^1 K_{i,t-1} di = \kappa_t \int_0^1 L_{it} di = \kappa_t L_t.$$

Hence the capital market clears if the factor ratio chosen by individual firms equals the aggregate factor ratio:

$$\kappa_t = \frac{K_{t-1}}{L_t}. \tag{14}$$

Using (14), (13) may be rewritten as

$$Z_t F(K_{t-1}, X_t L_t) = Y_t \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\sigma} di. \tag{15}$$

e Nonstochastic steady state

In the nonstochastic steady state equilibrium, we have $\pi_t = \pi$ for all t. Thus,

$$\frac{P_t}{P_{t+s}} = \frac{1}{\pi_{t+1}\pi_{t+2}\cdots\pi_{t+s}} = \frac{1}{\pi^s},$$

and (10) becomes

$$\Delta^* \sum_{s=0}^{\infty} \left(\beta\omega\pi^{\sigma-1}\right)^s = \frac{\sigma\phi}{\sigma-1} \sum_{s=0}^{\infty} \left(\beta\omega\pi^{\sigma}\right)^s, \tag{16}$$

where $\Delta^* = P_t^*/P_t$ for all t. Moreover, using (12) we have

$$\Delta^* = \left(\frac{1-\omega\pi^{\sigma-1}}{1-\omega}\right)^{\frac{1}{1-\sigma}}. \tag{17}$$

If $\pi = 1$, then $P_t^* = P_t = P_{t-1}$ for all t, and hence a single price is chosen by all firms in all periods. If $\pi \neq 1$, then $P_t^* \neq P_t$, and a nontrivial price distribution obtains in the steady state.

Using (15), steady state aggregate labor demand is

$$L = \frac{Y}{ZF(\kappa, X)} \int_0^1 d\Delta(i), \tag{18}$$

where $\Delta(i)$ is the steady state distribution of $(P_{it}/P_t)^{-\sigma}$. If $\pi = 1$, then $\Delta(i)$ becomes degenerate at unity. In this case, combining (18) and (14) gives

$$ZF(\kappa, X)L = ZF(K, XL) = Y. \tag{19}$$

If $\pi \neq 1$, then $ZF(K, XL) \neq Y$ will hold in general; i.e., the production technology does not aggregate in the steady state.

2 Log-linearized equilibrium conditions

The equilibrium conditions will now be log-linearized around the nonstochastic steady state, using the following functional forms for U and F:

$$U = \frac{C_t^{1-\xi}-1}{1-\xi} + \omega\frac{m_t^{1-\gamma}-1}{1-\gamma} - \frac{L_t^{1+1/\eta}}{1+1/\eta}, \tag{20}$$

$$F(K_{t-1}, X_t L_t) = K_{t-1}^{\alpha}\left(X_t L_t\right)^{1-\alpha}, \tag{21}$$

where $\xi, \omega, \gamma, \chi, \eta > 0$, and $0 < \alpha < 1$.

a Price equations

Log-linearizing the left-hand side of (10) around the steady state gives

$$\Delta^* C^{-\xi} Y \sum\nolimits_{s=0}^{\infty} \left(\beta\omega\pi^{\sigma-1}\right)^s (\hat{p}_t^* - \hat{p}_t)$$

$$+\Delta^* C^{-\xi} Y \sum\nolimits_{s=0}^{\infty} \left(\beta\omega\pi^{\sigma-1}\right)^s (-\xi\mathbb{E}_t\hat{c}_{t+s} + (1-\sigma)\hat{p}_t - (1-\sigma)\mathbb{E}_t\hat{p}_{t+s} + \mathbb{E}_t\hat{y}_{t+s}),$$

and log-linearizing the right-hand side gives

$$\frac{\sigma\phi}{\sigma-1} C^{-\xi} Y \sum\nolimits_{s=0}^{\infty} (\beta\omega\pi^{\sigma})^s \left(-\xi\mathbb{E}_t\hat{c}_{t+s} + \mathbb{E}_t\hat{\phi}_{t+s} - \sigma\hat{p}_t + \sigma\mathbb{E}_t\hat{p}_{t+s} + \mathbb{E}_t\hat{y}_{t+s}\right).$$

After equating the formulas, canceling terms and rearranging, we have

$$\Delta^* \frac{1}{1-\beta\omega\pi^{\sigma-1}} (\hat{p}_t^* - \hat{p}_t) \tag{22}$$

$$= \frac{\sigma\phi}{\sigma-1} \sum\nolimits_{s=0}^{\infty} (\beta\omega\pi^{\sigma})^s \mathbb{E}_t\hat{\phi}_{t+s} + \Delta^* \sum\nolimits_{s=0}^{\infty} \left(\beta\omega\pi^{\sigma-1}\right)^s (\mathbb{E}_t\hat{p}_{t+s} - \hat{p}_t)$$

$$+\sum\nolimits_{s=0}^{\infty} (\beta\omega)^s \left(\frac{\sigma\phi}{\sigma-1}\pi^{\sigma s} - \Delta^*\pi^{(\sigma-1)s}\right) (-\xi\mathbb{E}_t\hat{c}_{t+s} - \sigma\hat{p}_t + \sigma\mathbb{E}_t\hat{p}_{t+s} + \mathbb{E}_t\hat{y}_{t+s}).$$

For (12), we have

$$\hat{p}_t^* - \hat{p}_t = \frac{\omega\pi^{\sigma-1}}{1-\omega\pi^{\sigma-1}}\hat{\pi}_t. \tag{23}$$

b Aggregate labor demand equation

The price distribution term in (15) can be dealt with by means of a log-linear approximation of the integrand around $P_t^{-\sigma}$. We can write

$$\int_0^1 \ln P_{it}^{-\sigma} di \;=\; \int_0^1 \left(\ln P_t^{-\sigma} + \frac{1}{P_t^{-\sigma}}\left(P_{it}^{-\sigma} - P_t^{-\sigma}\right)\right) di$$

$$= \ln P_t^{-\sigma} + \frac{1}{P_t^{-\sigma}} \int_0^1 \left(P_{it}^{-\sigma} - P_t^{-\sigma} \right) di$$

$$= \ln P_t^{-\sigma} + \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\sigma} di - 1,$$

and hence (15) may be expressed as

$$Z_t K_{t-1}^{\alpha} \left(X_t L_t \right)^{1-\alpha} = Y_t \left(\sigma \left(\ln P_t - \int_0^1 \ln P_{it} di \right) + 1 \right). \tag{24}$$

Applying these same steps to the final good price aggregator (11) gives

$$\int_0^1 \ln P_{it} di = \ln P_t,$$

so that we can write

$$Z_t K_{t-1}^{\alpha} \left(X_t L_t \right)^{1-\alpha} = Y_t. \tag{25}$$

Log-linearizing around the steady state and rearranging gives

$$\hat{l}_t = \frac{1}{(1-\alpha)} \left(\hat{y}_t - \hat{z}_t - \alpha \hat{k}_{t-1} - (1-\alpha) \hat{x}_t \right). \tag{26}$$

c Labor market clearing equations

The Euler equations for the household's optimal choice of labor supply, given the functional form (20), is

$$C_t^{\xi} L_t^{1/\eta} = W_t.$$

The corresponding labor demand equation is obtained by substituting the functional form (21) into (4), and using (14):

$$\phi_t Z_t (1-\alpha) K_{t-1}^{\alpha} X_t^{1-\alpha} L_t^{-\alpha} = W_t.$$

Equating these expressions and log-linearizing around the steady state gives

$$\xi \hat{c}_t + \left(\frac{1}{\eta} + \alpha \right) \hat{l}_t = \hat{\phi}_t + \hat{z}_t + \alpha \hat{k}_{t-1} + (1-\alpha) \hat{x}_t. \tag{27}$$

Finally, substitute (26) into (27) to obtain

$$\xi\hat{c}_t + \frac{1/\eta+\alpha}{1-\alpha}\hat{y}_t \;=\; \hat{\phi}_t + \frac{1/\eta+1}{1-\alpha}\left(\hat{z}_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{x}_t\right). \tag{28}$$

d Asset pricing equations and resource constraint

The Euler equations for the household's optimal choices of capital, real balances and nominal debt are

$$\beta\mathbb{E}_t C_{t+1}^{-\xi}\left(Z_{t+1}\alpha K_t^{\alpha-1}\left(X_{t+1}L_{t+1}\right)^{1-\alpha} + 1 - \delta\right) \;=\; C_t^{-\xi},$$

$$\omega C_t^{\xi} m_t^{-\gamma} \;=\; \frac{R_t^n - 1}{R_t^n},$$

$$\beta\mathbb{E}_t C_{t+1}^{-\xi}\frac{R_t^n}{\pi_{t+1}} \;=\; C_t^{-\xi}.$$

Log-linearizing these equations around the steady state and rearranging gives

$$E_t\hat{c}_{t+1} - \hat{c}_t \;=\; \frac{1-\beta\left(1-\delta\right)}{\xi}\left(\mathbb{E}_t\hat{z}_{t+1} - (1-\alpha)\hat{k}_t + (1-\alpha)(\mathbb{E}_t\hat{x}_{t+1} + \mathbb{E}_t\hat{l}_{t+1})\right), \tag{29}$$

$$\hat{m}_t \;=\; \frac{1}{\gamma}\left(\xi\hat{c}_t - \frac{\beta}{\pi-\beta}\hat{r}_t^n\right), \tag{30}$$

$$\mathbb{E}_t\hat{c}_{t+1} - \hat{c}_t \;=\; \frac{1}{\xi}\left(\hat{r}_r^n - \mathbb{E}_t\hat{\pi}_{t+1}\right). \tag{31}$$

Finally, the resource constraint is

$$Y_t + (1-\delta)K_{t-1} \;=\; C_t + K_t + G_t,$$

and log-linearization gives

$$Y\hat{y}_t + (1-\delta)K\hat{k}_{t-1} \;=\; C\hat{c}_t + K\hat{k}_t + G\hat{g}_t. \tag{32}$$

e Summary

The log-linearized equilibrium conditions consist of equations (22), (23), (26) and (28)-(32), along with

$$\hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1}. \tag{33}$$

This system of nine equations determines the ten endogenous variables $\hat{p}_t^*$, $\hat{p}_t$, $\hat{\phi}_t$, $\hat{c}_t$, $\hat{y}_t$, $\hat{\pi}_t$, $\hat{l}_t$, $\hat{k}_t$, $\hat{m}_t$ and $\hat{r}_r^n$, given the exogenous variables $\hat{z}_t$, $\hat{x}_t$ and $\hat{g}_t$. To close the model, a monetary policy equation must be specified, typically involving the variables $\hat{m}_t$, $\hat{\pi}_t$, $\hat{r}_r^n$ and $\hat{y}_t$, along with additional exogenous variables.

3 Zero inflation steady state

The price setting equation (10) represents a complex relationship between current price choices and future marginal costs, marginal utilities, prices and outputs. This relationship can be greatly simplified by restricting attention to the $\pi = 1$ steady state; i.e., the steady state in which the net inflation rate is zero. In this case, steady state prices satisfy $P_t^* = P_t = P_{t-1}$ for all t, and the log-linearized price setting equation (22) simplifies to

$$\frac{1}{1-\beta\omega}\left(\hat{p}_t^* - \hat{p}_t\right) = \sum\nolimits_{s=0}^{\infty} (\beta\omega)^s \left(\mathbb{E}_t\hat{\phi}_{t+s} + \mathbb{E}_t\hat{p}_{t+s} - \hat{p}_t\right). \tag{34}$$

Furthermore, from the final good price equation (23) we obtain

$$\hat{p}_t^* - \hat{p}_t = \frac{\omega}{1-\omega}\hat{\pi}_t. \tag{35}$$

a Inflation and marginal costs

A very simple formula relating inflation to marginal costs can now be derived by manipulating (34) and (35). Using the equations to eliminate the variable $\hat{p}_t^* - \hat{p}_t$ yields

$$\hat{\pi}_t \;=\; \theta \sum\nolimits_{s=0}^{\infty} (\beta\omega)^s \left(\mathbb{E}_t\hat{\phi}_{t+s} + \mathbb{E}_t\hat{p}_{t+s} - \hat{p}_t\right), \tag{36}$$

where

$$\theta = \frac{(1-\omega)(1-\beta\omega)}{\omega}.$$

Using (36), we can write

$$\hat{\pi}_{t+1} \;=\; \theta \sum\nolimits_{s=0}^{\infty} (\beta\omega)^s \left(\mathbb{E}_{t+1}\hat{\phi}_{t+s+1} + \mathbb{E}_{t+1}\hat{p}_{t+s+1} - \hat{p}_{t+1}\right)$$

$$=\; \theta \sum\nolimits_{u=1}^{\infty} (\beta\omega)^{u-1} \left(\mathbb{E}_{t+1}\hat{\phi}_{t+u} + \mathbb{E}_{t+1}\hat{p}_{t+u} - \hat{p}_t - \hat{\pi}_{t+1}\right).$$

Manipulating the latter formula gives

$$\beta\omega\left(1+\frac{\theta}{1-\beta\omega}\right)\hat{\pi}_{t+1} \;=\; \theta \sum\nolimits_{u=1}^{\infty} (\beta\omega)^{u} \left(\mathbb{E}_{t+1}\hat{\phi}_{t+u} + \mathbb{E}_{t+1}\hat{p}_{t+u} - \hat{p}_t\right).$$

Thus, (36) can be written as

$$\hat{\pi}_t \;=\; \theta\hat{\phi}_t + \theta \sum\nolimits_{s=1}^{\infty} (\beta\omega)^s \left(\mathbb{E}_t\hat{\phi}_{t+s} + \mathbb{E}_t\hat{p}_{t+s} - \hat{p}_t\right) \tag{37}$$

$$=\; \theta\hat{\phi}_t + \beta\omega\left(1+\frac{\theta}{1-\beta\omega}\right)\mathbb{E}_t\hat{\pi}_{t+1}.$$

Finally, substituting for θ gives

$$\beta\omega\left(1+\frac{\theta}{1-\beta\omega}\right) = \beta,$$

and hence (37) simplifies to

$$\hat{\pi}_t = \theta\hat{\phi}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1}. \tag{38}$$

Equation (38) links fluctuations in inflation to fluctuations in marginal cost. This connects the nominal and real sectors of the economy in a manner analogous to the original Phillips curve, which related inflation and unemployment. The equation also incorporates a forward-looking component. Solving forward gives

$$\hat{\pi}_t \;=\; \theta \sum\nolimits_{s=0}^{\infty} \beta^s \mathbb{E}_t\hat{\phi}_{t+s}.$$

It follows that current inflation is driven by the expected future path of marginal costs.

b Effects of price rigidity

Deviations of marginal cost from the steady state can be expressed in terms of the distortions created by price rigidity relative to a flexible-price economy. If prices are fully flexible, then equilibrium price choices are given by (9), which may be written as

$$P_{it} = \frac{\sigma}{\sigma - 1}\phi_t P_t.$$

Using (11), we have

$$P_t^{1-\sigma} = \left(\frac{\sigma}{\sigma - 1}\phi_t P_t\right)^{1-\sigma},$$

or

$$1 = \left(\frac{\sigma}{\sigma - 1}\phi_t\right)^{1-\sigma}.$$

It follows that $\hat{\phi}_t = 0$ in any equilibrium with fully flexible prices. In this case, the labor market equilibrium condition (28) becomes

$$\xi \hat{c}_t^f + \frac{1/\eta + \alpha}{1-\alpha}\hat{y}_t^f \;=\; \frac{1/\eta + 1}{1-\alpha}\left(\hat{z}_t + \alpha \hat{k}_{t-1}^f + (1-\alpha)\hat{x}_t\right), \tag{39}$$

where $\hat{c}_t^f$, $\hat{y}_t^f$ and $\hat{k}_{t-1}^f$ are the values assumed by the variables in the flexible price equilibrium.

Now combine (28) and (39) to obtain

$$\hat{\phi}_t \;=\; \xi\left(\hat{c}_t - \hat{c}_t^f\right) + \frac{1/\eta + \alpha}{1-\alpha}\left(\hat{y}_t - \hat{y}_t^f\right) - \frac{1/\eta + 1}{1-\alpha}\alpha\left(\hat{k}_{t-1} - \hat{k}_{t-1}^f\right). \tag{40}$$

This shows how fluctuations in marginal cost are driven by gaps created by price rigidity: positive consumption and output gaps push marginal cost upward, while positive capital gaps work in the opposite direction. Substituting into (38) relates the gaps to inflation:

$$\hat{\pi}_t \;=\; \theta\xi\left(\hat{c}_t - \hat{c}_t^f\right) + \theta\frac{1/\eta + \alpha}{1-\alpha}\left(\hat{y}_t - \hat{y}_t^f\right) - \theta\frac{1/\eta + 1}{1-\alpha}\alpha\left(\hat{k}_{t-1} - \hat{k}_{t-1}^f\right) + \beta\mathbb{E}_t\hat{\pi}_{t+1}. \tag{41}$$

Thus, gaps that drive up marginal cost also increase the aggregate price level.

c Model without capital and government purchases

Equation (41) can be simplified by eliminating the variables K_t and G_t from the model, and specifying the intermediate good production function as

$$Y_{it} = Z_t L_{it}.$$

In this case, all final good output goes to consumption, i.e., $Y_t = C_t$, and (39) may be written as

$$\left(\xi + \frac{1}{\eta}\right) \hat{y}_t^f = \left(\frac{1}{\eta} + 1\right) \hat{z}_t.$$

This gives

$$\hat{y}_t^f = \frac{1/\eta + 1}{\xi + 1/\eta} \hat{z}_t, \tag{42}$$

and (41) becomes

$$\hat{\pi}_t = \theta \left(\xi + \frac{1}{\eta}\right) \chi_t + \beta \mathbb{E}_t \hat{\pi}_{t+1}. \tag{43}$$

where $\chi_t = \hat{y}_t - \hat{y}_t^f$ is called the output gap. (43) is called the New Keynesian Phillips curve (NKPC), and it establishes a positive relationship between inflation and the gap that separates equilibrium output from what it would be in a flexible-price economy.

The Euler equation for nominal debt can also be expressed in terms of the output gap. In the present setting, (31) can be written as

$$\mathbb{E}_t \chi_{t+1} - \chi_t = \frac{1}{\xi} \left(\hat{r}_r^n - E_t \hat{\pi}_{t+1}\right) - \left(\mathbb{E}_t \hat{y}_{t+1}^f - \hat{y}_t^f\right).$$

Substituting from (42) and rearranging gives

$$\chi_t = \mathbb{E}_t \chi_{t+1} - \frac{1}{\xi} \left(\hat{r}_r^n - E_t \hat{\pi}_{t+1}\right) + u_t, \tag{44}$$

where

$$u_t = \frac{1/\eta + 1}{\xi + 1/\eta} \left(\mathbb{E}_t \, \hat{z}_{t+1} - \hat{z}_t\right).$$

Equations (43) and (44), together with a policy rule that determines $\hat{r}_r^n$, form a three-equation model in the endogenous variables $\hat{\pi}_t$, χ_t and $\hat{r}_r^n$.