Economics 210C - Macroeconomics Spring 2018-19

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Exercises on the New Keynesian Model - Solutions

1. Taylor principle and indeterminacy

a. Solve for $\mathbb{E}_t \hat{\pi}_{t+1}$:

$$\mathbb{E}_t \hat{\pi}_{t+1} = -\frac{\hat{\theta}}{\beta}\chi_t + \frac{1}{\beta}\hat{\pi}_t.$$

Substitute this and the policy rule into the bond pricing equation:

$$\mathbb{E}_t \chi_{t+1} = \left(1 + \frac{\hat{\theta}}{\xi\beta}\right)\chi_t + \left(\frac{\rho_\pi}{\xi} - \frac{1}{\xi\beta}\right)\hat{\pi}_t + \frac{1}{\xi}v_t + \hat{\gamma}(1-\rho)\hat{z}_t.$$

Thus we have

$$\mathcal{M} = \begin{bmatrix} 1 + \frac{\hat{\theta}}{\xi\beta} & \frac{\rho_\pi}{\xi} - \frac{1}{\xi\beta} \\ -\frac{\hat{\theta}}{\beta} & \frac{1}{\beta} \end{bmatrix}.$$

b. The eigenvalues λ of $\mathcal{M}$ are the solutions to

$$\det\left(\begin{bmatrix} 1 + \frac{\hat{\theta}}{\xi\beta} - \lambda & \frac{\rho_\pi}{\xi} - \frac{1}{\xi\beta} \\ -\frac{\hat{\theta}}{\beta} & \frac{1}{\beta} - \lambda \end{bmatrix}\right) = 0.$$

Calculating the determinant gives

$$\left(\left(1 + \frac{\hat{\theta}}{\xi\beta}\right) - \lambda\right)\left(\frac{1}{\beta} - \lambda\right) + \frac{\hat{\theta}}{\beta}\left(\frac{\rho_\pi}{\xi} - \frac{1}{\xi\beta}\right)$$

$$= c + b\lambda + \lambda^2,$$

where

$$b = -\left(1 + \frac{1}{\beta} + \frac{\hat{\theta}}{\xi\beta}\right), \quad c = \frac{1}{\beta} + \frac{\hat{\theta}\rho_\pi}{\xi\beta}.$$

Note that $b < -2$ and $c > 1$. The eigenvalues are given by

$$\lambda^{+} = \frac{1}{2}\left(-b + \left(b^2 - 4c\right)^{1/2}\right),$$

$$\lambda^{-} = \frac{1}{2}\left(-b - \left(b^2 - 4c\right)^{1/2}\right).$$

If $b^2 \geq 4c$, then the eigenvalues are real, and $\lambda^{+} \geq \lambda^{-}$. Moreover, $\lambda^{+} > 1$ follows from $-b > 2$. If $b^2 < 4c$, then the eigenvalues are complex, and $\left|\lambda^{+}\right| = \left|\lambda^{-}\right|$. Moreover,

$$\left|\lambda^{+}\right|\left|\lambda^{-}\right| = \frac{1}{4}\left(b^2 + \left(4c - b^2\right)\right) = c > 1.$$

c. If $b^2 \geq 4c$, then λ^{-} is the smallest eigenvalue. We have $\lambda^{-} > 1$ if and only if

$$-b - 2 > \left(b^2 - 4c\right)^{1/2},$$

which is equivalent to $\rho_\pi > 1$.

If $b^2 < 4c$, then part b shows that the eigenvalues must exceed unity in modulus. Moreover, we can write

$$b^2 < 4c < 4c + (1-c)^2 = (1+c)^2,$$

or

$$-b < 1 + c,$$

which implies $\rho_\pi > 1$.

2. Policy objectives and divine coincidence

a. Solving the money demand equation for $\hat{r}_t^n$, substituting into the bond pricing equation, and rearranging gives

$$\mathbb{E}_t \chi_{t+1} + \frac{1}{\xi}\mathbb{E}_t \hat{\pi}_{t+1} + \frac{(1-\beta)\gamma}{\xi}\hat{m}_t \tag{1}$$

$$= (2-\beta)\chi_t + (2-\beta-\rho)\hat{\mu}\hat{z}_t + \frac{1-\beta}{\xi}\zeta_t.$$

The other two equations are

$$\mathbb{E}_t\hat{\pi}_{t+1} = -\frac{\hat{\theta}}{\beta}\chi_t + \frac{1}{\beta}\hat{\pi}_t, \tag{2}$$

$$\hat{m}_t = -\hat{\pi}_t + \hat{m}_{t-1} + \hat{\psi}_t. \tag{3}$$

b. The equilibrium satisfies $\chi_t = 0$ for all t if and only if

$$\frac{1}{\xi}\mathbb{E}_t\hat{\pi}_{t+1} + \frac{(1-\beta)\,\gamma}{\xi}\hat{m}_t = (2-\beta-\rho)\hat{\mu}\hat{z}_t + \frac{1-\beta}{\xi}\zeta_t,$$

$$\mathbb{E}_t\hat{\pi}_{t+1} = \frac{1}{\beta}\hat{\pi}_t,$$

$$\hat{m}_t = -\hat{\pi}_t + \hat{m}_{t-1} + \hat{\psi}_t.$$

Eliminating $\mathbb{E}_t\hat{\pi}$, rearranging, and using (3) gives

$$\begin{aligned}\hat{m}_t &= \frac{1}{(1-\beta)\,\gamma}\left(-\frac{1}{\beta}\hat{\pi}_t + \xi(2-\beta-\rho)\hat{\mu}\hat{z}_t + (1-\beta)\,\zeta_t\right) \\ &= -\hat{\pi}_t + \hat{m}_{t-1} + \hat{\psi}_t.\end{aligned} \tag{4}$$

Thus, for <u>any</u> path of $\hat{\pi}_t$, choosing $\hat{\psi}_t$ to satisfy (4) generates an equilibrium having $\chi_t = 0$ for all t.

c. If $\hat{\pi}_t = 0$ for all t, then (2) implies $\chi_t = 0$ for all t. Substituting into (1), rearranging, and using (3) gives

$$\begin{aligned}\hat{m}_t &= \frac{\xi}{(1-\beta)\,\gamma}(2-\beta-\rho)\hat{\mu}\hat{z}_t + \frac{1}{\gamma}\zeta_t \\ &= \hat{m}_{t-1} + \hat{\psi}_t.\end{aligned} \tag{5}$$

Thus, choosing $\hat{\psi}_t$ to satisfy (5) generates an equilibrium having $\hat{\pi}_t = \chi_t = 0$ for all t. Moreover, (5) implies that $\hat{\pi}_t = 0$ holds necessarily, in view of (3).

d. Suppose the economy begins at the steady state. According to (5), if $\hat{z}_t = \varepsilon_t > 0$, then

$$\hat{\psi}_t = \frac{\xi}{(1-\beta)\,\gamma}(2-\beta-\rho)\hat{\mu}\varepsilon_t > 0.$$

Thus the policymaker raises money growth in response to the shock. In equilibrium, the bond pricing equation implies

$$\hat{r}_t^n = -\xi(1-\rho)\hat{\mu}^{\hat{z}_t}\varepsilon_t,$$

and hence the rise in $\hat{\psi}_t$ induces a reduction in $\hat{r}_t^n$. Intuitively, lower $\hat{r}_t^n$ is needed in order to offset higher $\hat{z}_t$ in the bond pricing equation, making it possible for the bond market to clear with $\chi_t = \mathbb{E}_t\chi_{t+1} = 0$. This preserves $\pi_t = \mathbb{E}_t\pi_{t+1} = 0$ in the NKPC.

Broadly speaking, zero inflation eliminates the outut gap via the NKPC. This causes the stochastic discount factor to be constant in this simple model. Since a positive productivity shock would ordinarily raise the real interest rate, the policymaker raises money growth to offset this effect and maintain a constant real interest rate.

e. Beginning in the steady state, if $\zeta_t > 0$, then (5) gives

$$\hat{\psi}_t = \frac{1}{\gamma}\zeta_t > 0.$$

Thus the policymaker expands money growth to accommodate fully the money demand shock. This implies $\hat{r}_t^n = 0$, which must hold in equilibrium in view of the bond pricing equation.