Economics 210C - Macroeconomics | Spring 2018-19

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Exercises on the New Keynesian Model

These exercises consider the following version of the New Keynesian model:

$$\chi_t = \mathbb{E}_t \chi_{t+1} - \frac{1}{\xi}\left(\hat{r}_t^n - \mathbb{E}_t \hat{\pi}_{t+1}\right) - \hat{\mu}(1-\rho)\hat{z}_t, \tag{1}$$

$$\hat{\pi}_t = \hat{\theta}\chi_t + \beta \mathbb{E}_t \hat{\pi}_{t+1}, \tag{2}$$

where $\chi_t = \hat{y}_t - \hat{y}_t^f = \hat{y}_t - \hat{\mu}\hat{z}_t$ is the output gap,

$$\hat{\theta} = \theta\left(\xi + \frac{1}{\eta}\right), \quad \hat{\gamma} = \frac{1/\eta + 1}{\xi + 1/\eta},$$

$\xi, \theta, \eta > 0,\ 0 < \rho, \beta < 1$, and $\{\hat{z}_t\}$ is an exogenous process of log labor productivity.

1. Taylor principle and indeterminacy Let the policy rule be given by

$$\hat{r}_t^n = \rho_\pi \hat{\pi}_t + v_t, \tag{3}$$

where $\rho_\pi \geq 0$, and $\{v_t\}$ is an exogenous process of policy control errors

a. Substitute (3) into (1) to express the model as a two-equation system in the endogenous variables χ_t and $\hat{\pi}_t$, and exogenous variables $\hat{z}_t$ and v_t. Express the system in the form

$$\begin{bmatrix} \mathbb{E}_t \chi_{t+1} \\ \mathbb{E}_t \hat{\pi}_{t+1} \end{bmatrix} = \mathcal{M} \begin{bmatrix} \chi_t \\ \hat{\pi}_t \end{bmatrix} + \begin{bmatrix} \frac{1}{\xi} v_t + \hat{\mu}(1-\rho)\hat{z}_t \\ 0 \end{bmatrix},$$

where $\mathcal{M}$ is a 2×2 coefficient matrix.

b. Show that the largest eigenvalue of $\mathcal{M}$ exceeds unity in modulus.

c. Show that the smallest eigenvalue of $\mathcal{M}$ exceeds unity in modulus if and only if $\rho_\pi > 1$. (Thus the system (1)-(3) has a unique nonexplosive REE if and only if the policy rule satisfies the <u>Taylor principle</u>; i.e., nominal interest rates should adjust more than one-to-one with inflation. Otherwise, there exist infinitely-many nonfundamental "sunspot" equilibria.)

2. Policy objectives and divine coincidence Let the model (1)-(2) be augmented with a monetary sector, given by the following money demand and market clearing equations:

$$\xi(\chi_t + \hat{\mu}\hat{z}_t) - \gamma\hat{m}_t + \zeta_t = \frac{1}{1-\beta}\hat{r}_t^n, \tag{4}$$

$$\hat{\pi}_t = \hat{m}_{t-1} - \hat{m}_t + \hat{\psi}_t,$$

where $\gamma > 0$, $\{\zeta_t\}$ is an exogenous white noise proces of money demand shocks, and $\hat{\psi}_t$ is the log of the growth rate of nominal balances, which is chosen by the policymaker.

a. Substitute (4) into (1) to express the model as a three-equation system in the endogenous variables χ_t, $\hat{\pi}_t$ and $\hat{m}_t$, exogenous variables $\hat{z}_t$ and ζ_t, and policy choice variable $\hat{\psi}_t$.

b. Suppose the policymaker wishes to neutralize the output gap, by chooosing $\hat{\psi}_t$ such that $\chi_t = 0$ for each t. Show that this policy is consistent with equilibria having any arbitrary paths of $\hat{\pi}_t$. (Thus the policy leads to indeterminacy.)

c. Now suppose the policymaker wishes to neutralize inflation, by chooosing $\hat{\psi}_t$ such that $\hat{\pi}_t = 0$ for each t. Show that the policy leads to a unique equilibrium, and this equilibrium has $\chi_t = 0$ for all t. (Thus the zero inflation policy gives uniqueness and no output gap as byproducts. This is called the divine coincidence.)

d. In the equilibrium of part c, how does the policymaker adjust $\hat{\psi}_t$ in response to a positive shock to $\hat{z}_t$? What is the corresponding adjustment in $\hat{r}_t^n$? By what channels does policy affect the economy? Provide intuition.

e. Repeat part d for the case of a positive shock to ζ_t.