Economics 210C - Macroeconomics Spring 2018-19

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MIU Exercises

1. Money growth and real activity Consider the following specification of the MIU model. The household problem is given by

$$\max_{\{C_t,L_t,K_t,M_t^d,B_t^d\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t + \omega \ln \frac{M_t^d}{P_t} - \chi \frac{L_t^{1+1/\eta}}{1+1/\eta} \right),$$

$$\text{s.t.} \quad P_t \left(W_t L_t + R_t K_{t-1} + (1-\delta) K_{t-1} \right) + M_{t-1}^d + R_{t-1}^n B_{t-1}^d$$

$$= P_t \left(C_t + K_t + T_t \right) + M_t^d + B_t^d,$$

and the production techology is

$$Y_t = K_{t-1}^{\alpha} \left(X_t L_t \right)^{1-\alpha},$$

where $0 < \beta, \delta, \alpha < 1$ and $\omega, \chi, \eta > 0$. The government budget constraint is

$$P_t G_t = P_t T_t + \left(\psi_t - 1 \right) M_{t-1}.$$

$\{X_t\}$, $\{G_t\}$ and $\{\psi_t\}$ are exogenous random processes specified by

$$X_t = X_{t-1} e^{\mu + \zeta_t},$$

$$\frac{G_t}{X_t} = g^{1-\gamma} \left(\frac{G_{t-1}}{X_{t-1}} \right) e^{\nu_t},$$

$$\psi_t = \psi^{1-\rho} \psi_{t-1}^{\rho} e^{\varepsilon_t},$$

where $\mu, g, \psi > 0$, $0 < \gamma, \rho < 1$, and $\{\zeta_t\}$, $\{\nu_t\}$ and $\{\varepsilon_t\}$ are exogenous white noise processes.

a. Derive the necessary conditions for an equilibrium of this model, letting λ_t denote the Lagrange multiplier on the wealth constraint.

b. Define detrended variables as follows:

$$C_t^c = \frac{C_t}{X_t}, \quad K_t^c = \frac{K_t}{X_t}, \quad m_t^c = \frac{M_t}{X_t P_t} = \frac{m_t}{X_t},$$

$$G_t^c = \frac{G_t}{X_t}, \quad \lambda_t^c = \lambda_t X_t.$$

Restate your necessary conditions in terms of the detrended variables.

c. Derive the nonstochastic steady state solution of the detrended system in part b.

d. Log-linearize the necessary conditions in part b around the steady state in part c.

e. Use your log-linearized necessary conditions to show that consumption, output and labor supply are unaffected by money growth if $\rho = 0$. Provide economic intuition for your answer.

f. Now suppose the production technology is

$$Y_t = X_t L_t.$$

(Thus $K_t = 0$ obtains for all t.) Show that consumption, output and labor supply are unaffected by money growth for <u>any</u> value of ρ.

2. Friedman rule and superneutrality Consider the following specification of the MIU model. The household problem is given by

$$\max_{\{C_t, L_t, K_t, M_t^d, B_t^d\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t^d}{P_t}, L_t\right),$$

$$\text{s.t.} \quad P_t\left(W_t L_t + R_t K_{t-1} + (1-\delta)K_{t-1}\right) + M_{t-1}^d + R_{t-1}^n B_{t-1}^d$$

$$= P_t\left(C_t + K_t + T_t\right) + M_t^d + B_t^d,$$

where $0 < \beta, \delta < 1$, and the production techology is

$$Y_t = F(K_{t-1}, L_t).$$

The functions U and F satisfy standard regularity conditions, and F exhibits constant returns to scale. The government budget constraint is

$$0 = P_t T_t + (\psi - 1)M_{t-1},$$

where $\psi > 0$.

a. Write down the necessary conditions for an equilibrium of this model, and derive the nonstochastic steady state equilibrium.

b. Write down the value of the household's lifetime utility in the steady state equilibrium. Derive the necessary condition for maximizing lifetime utility with respect to m. Also derive the value of ψ that would implement the solution in the steady state equilibrium. Explain intuitively why this policy is optimal.

c. What is the equilibrium value of the nominal interest rate under the optimal policy? Could a social planner implement the optimal policy by choosing ψ to achieve a target interest nominal interest rate?

d. Now suppose that the momentary utility function takes the form

$$U(C_t, L_t) V(m_t).$$

Show that a change in ψ can affect π, R^n and m, but not C, K or Y. Provide economic intuition for your answer.