Economics 210C - Macroeconomics Spring 2018-19

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In-Class Final Exam - Solutions

1. Deficit finance

a. The Lagrangian for the household's problem is

$$\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\ln C_t + \omega \ln \left(\frac{M_t^d}{P_t} \right) + \lambda_t \left(Y + \frac{M_{t-1}^d}{P_t} + R_{t-1}^n \frac{B_{t-1}^d}{P_t} \right.\right.$$
$$\left.\left. - C_t - \frac{M_t^d}{P_t} - \frac{B_t^d}{P_t} \right) \right).$$

The first-order necessary conditions for C_t, M_t^d, B_t^d and λ_t are, after canceling terms and rearranging:

$$\frac{1}{C_t} = \lambda_t,$$

$$\frac{\omega}{\left(M_t^d/P_t\right)} + \beta \mathbb{E}_t \lambda_{t+1} \frac{P_t}{P_{t+1}} = \lambda_t,$$

$$\beta \mathbb{E}_t \lambda_{t+1} R_t^n \frac{P_t}{P_{t+1}} = \lambda_t,$$

$$Y + \frac{M_{t-1}^d}{P_t} + R_{t-1}^n \frac{B_{t-1}^d}{P_t} = C_t + \frac{M_t^d}{P_t} + \frac{B_t^d}{P_t}.$$

b. The government budget constraint implies:

$$\frac{M_{t-1}}{P_t} + R_{t-1}^n \frac{B_{t-1}}{P_t} = \frac{B_t}{P_t} + \frac{M_t}{P_t} - G.$$

Substitute into the household wealth constraint:

$$Y - G = C_t.$$

c. Define

$$m_t = \frac{M_t}{P_t}, \quad b_t = \frac{B_t}{P_t}, \quad \pi_t = \frac{P_t}{P_{t-1}}.$$

The household's intertemporal Euler equations may be written as

$$\frac{\omega}{m_t} + \beta \mathbb{E}_t \lambda_{t+1} \frac{1}{\pi_{t+1}} = \lambda_t,$$

$$\beta \mathbb{E}_t \lambda_{t+1} R_t^n \frac{1}{\pi_{t+1}} = \lambda_t.$$

In a steady state equilibrium, we have

$$\frac{\omega C}{m} + \beta \frac{1}{\pi} = 1, \tag{1}$$

$$\beta R^n \frac{1}{\pi} = 1. \tag{2}$$

If the government choses $M_t = M_{t-1}$ for all t, then its budget constraint becomes

$$G + R_{t-1}^n \frac{B_{t-1}}{P_t} = \frac{B_t}{P_t},$$

or

$$G + R_{t-1}^n b_{t-1} \frac{1}{\pi_t} = b_t.$$

In a steady state equilibruim, we have

$$G + R^n b \frac{1}{\pi} = b. \tag{3}$$

Combining (2) and (3) gives

$$G = b - R^n b \frac{1}{\pi} = \frac{\beta - 1}{\beta} b \leq 0,$$

and hence $G \leq 0$ for all $b \geq 0$. Moreover, $b < 0$ is infeasible, since government has no resources to lend in the initial period.

Intuitively, since $1/\beta > 0$, the government must pay a positive net real interest rate on its debt. Thus, payments on maturing debt necessarily exceed proceeds from new debt in each period of the steady state. With no other sources of financing, the government cannot sustain either positive debt or positive puchases of goods.

d. If the government chooses $B_t = 0$ for all t, then its budget constraint becomes

$$P_t G = M_t - M_{t-1},$$

or

$$G = m_t - m_{t-1}\frac{1}{\pi_t}.$$

In a steady state equilibrium, we have

$$G = \frac{\pi - 1}{\pi} m. \tag{4}$$

which implies $\pi \geq 1$. Moreover, rearranging equation (1) and substituting $C = Y - G$ gives

$$m = \frac{\pi\omega}{\pi - \beta} C = \frac{\pi\omega}{\pi - \beta} (Y - G). \tag{5}$$

Solve (4) and (5) for G:

$$G = \frac{(\pi - 1)\,\omega}{\pi - \beta + (\pi - 1)\,\omega} Y = \varphi(\pi) Y.$$

Since $0 \leq \varphi(\pi) < 1$ whenever $\pi \geq 1$, it follows that $0 \leq G < Y$ must hold.

Differentiating φ gives

$$\frac{\partial \varphi}{\partial \pi} = \frac{\omega\,(1 - \beta)}{\left((1 + \omega)\,\pi - (\beta + \omega)\right)^2} > 0,$$

and hence increases in G require larger π. In the limit, we have

$$\lim_{\pi \to \infty} \varphi = \frac{\omega}{1 + \omega} < 1.$$

Thus, the upper bound of feasible G is

$$\overline{G} = \frac{\omega}{1 + \omega} Y < Y.$$

Intuitively, higher levels of government spending can be financed by raising money growth, which leads to higher inflation. In essence, this is a tax on real balances. Demand for real balances is reduced, both by reducing consumption, which lowers the marginal transaction benefit of money, and by reducing the real return on holding money. As the money growth rate approaches infinity, willingness to hold real balances reaches a limit, and this places an upper bound $\overline{G} < Y$ on feasible G.

2. Monopolistic competition with markup shocks

a. Firm i's profit maximization problem is

$$\max_{P_{it}} \Pi_{it} = (P_{it} - \phi_t)\, D_t(P_{it}).$$

The first order necessary condition for P_{it} is

$$\frac{\partial \Pi_t}{\partial P_{it}} = D_t(P_{it}) + (P_{it} - \phi_t)\,\frac{\partial D_t(P_{it})}{\partial P_{it}} = 0.$$

Rearranging and substituting for the constant price elasticity of D_t gives

$$P_{it}\left(1 + \frac{D_t(P_{it})}{\frac{\partial D_t(P_{it})}{\partial P_{it}} P_{it}}\right) = P_{it}\left(\frac{\sigma_t - 1}{\sigma_t}\right) = \phi_t.$$

Given $\sigma_t > 1$, the latter equality implies

$$\mu_{it} = \frac{P_{it}}{\phi_t} = \frac{\sigma_t}{\sigma_t - 1} > 1. \tag{6}$$

b. The Lagrangian for the household's problem is

$$\mathcal{L} = \mathbb{E}_t \sum\nolimits_{t=0}^{\infty} \beta^t (\ln C_t + \lambda_t (R_t K_{t-1} + W_t + \Pi_t^*$$

$$+ (1-\delta) K_{t-1} - C_t - K_t)).$$

The first-order necessary conditions for C_t, K_t and λ_t are, after canceling terms and rearranging:

$$\frac{1}{C_t} = \lambda_t,$$

$$\beta \mathbb{E}_t \lambda_{t+1} \left(R_{t+1} + 1 - \delta\right) = \lambda_t,$$

$$R_t K_{t-1} + W_t + \Pi_t^* + (1-\delta)K_{t-1} = C_t + K_t.$$

c. The profit function of firm i is

$$\Pi_{it} = (P_{it} - \phi_t)\, Y_{it} = P_{it}Y_{it} - R_t K_{i,t-1} - W_t L_{it}.$$

In a symmetric pricing equilibrium such that factor markets clear, we have

$$\Pi_t^* = P_t^* Y_t^* - R_t K_{t-1}^* - W_t L_t^*$$

$$= P_t^* Y_t^* - R_t K_{t-1} - W_t.$$

Thus, household income satisfies

$$R_t K_{t-1} + W_t + \Pi_t^* = P_t^* Y_t^*.$$

Next, cost minimization by firm i implies

$$\phi_t Z_t \alpha K_{i,t-1}^{\alpha-1} L_{it}^{1-\alpha} = \phi_t Z_t \alpha \left(\frac{K_{i,t-1}}{L_{it}}\right)^{\alpha-1} = R_t, \tag{7}$$

and hence we can express the capital-labor ratio as

$$\frac{K_{i,t-1}}{L_{it}} = \kappa_t = \left(\frac{R_t}{\phi_t Z_t \alpha}\right)^{\frac{1}{\alpha-1}}.$$

It follows that

$$Y_t^* = \int_0^1 Y_{it} di = \int_0^1 Z_t K_{i,t-1}^{\alpha} L_{it}^{1-\alpha} di$$

$$= Z_t \kappa_{t-1}^{\alpha} \int_0^1 L_{it} di = Z_t \kappa_{t-1}^{\alpha} L_t^* = Z_t \kappa_{t-1}^{\alpha},$$

and

$$K_{t-1} = \int_0^1 K_{i,t-1} di = \int_0^1 \frac{K_{i,t-1}}{L_{it}} L_{it} di \tag{8}$$

$$= \kappa_{t-1} \int_0^1 L_{it} di = \kappa_{t-1} L_t^* = \kappa_{t-1}.$$

Finally, (7) implies, using (6) and (8):

$$R_t = \frac{P_{it}^*}{\mu_{it}} Z_t \alpha K_{t-1}^{\alpha-1}.$$

d. Substituting as indicated gives:

$$\frac{1}{C_t} = \lambda_t, \tag{9}$$

$$\beta \mathbb{E}_t \lambda_{t+1} \left(\frac{1}{\mu_{t+1}} \alpha Z_{t+1} K_t^{\alpha-1} + 1 - \delta \right) = \lambda_t, \tag{10}$$

$$Z_t K_{t-1}^{\alpha} + (1-\delta) K_{t-1} = C_t + K_t. \tag{11}$$

e. Along a perfect foresight path, (10) implies

$$\lambda_{t-1} = \beta \lambda_t \left(\frac{1}{\mu_t} \alpha Z_t K_{t-1}^{\alpha-1} + 1 - \delta \right),$$

and hence

$$\Delta \lambda_t = \lambda_t \left(1 - \beta \left(\frac{1}{\mu_t} \alpha Z_t K_{t-1}^{\alpha-1} + 1 - \delta \right) \right).$$

Substituting (9) into (11) are rearranging gives

$$\Delta K_t = Z_t K_{t-1}^{\alpha} - \delta K_{t-1} - \lambda_t^{-1}.$$

f. For period length Δ, the necessary conditions (9)-(11) are

$$\frac{1}{C_t} = \lambda_t,$$

$$e^{-r\Delta} \mathbb{E}_t \lambda_{t+\Delta} \left(\frac{1}{\mu_{t+\Delta}} \alpha Z_{t+\Delta} K_t^{\alpha-1} \Delta + 1 - \delta \Delta \right) = \lambda_t,$$

$$Z_t K_{t-\Delta}^{\alpha} \Delta + (1 - \delta\Delta) K_{t-\Delta} = C_t \Delta + K_t,$$

where r is determined by $e^{-r} = \beta$. Along a perfect foresight path, we may write

$$\frac{\lambda_{t+\Delta} - \lambda_t}{\Delta} = \lambda_{t+\Delta} \frac{1 - e^{-r\Delta}}{\Delta} - e^{-r\Delta} \lambda_{t+\Delta} \left(\frac{1}{\mu_{t+\Delta}} \alpha Z_{t+\Delta} K_t^{\alpha-1} - \delta \right),$$

$$\frac{K_t - K_{t-\Delta}}{\Delta} = -C_t + Z_t K_{t-\Delta}^{\alpha} - \delta K_{t-\Delta}.$$

Taking the limit as $\Delta \to 0$ gives

$$\dot{\lambda}_t = \lambda_t \left(r - \left(\frac{1}{\mu_t} \alpha Z_t K_t^{\alpha-1} - \delta \right) \right),$$

$$\dot{K}_t = Z_t K_t^{\alpha} - \delta K_t - \lambda_t^{-1}.$$

g. $\dot{\lambda}_t = 0$ holds if and only if

$$K_t = \bar{K}_t = \left(\frac{\alpha Z_t}{(r+\delta)\,\mu_t} \right)^{\frac{1}{1-\alpha}}.$$

Hence

$$\frac{\partial \bar{K}_t}{\partial Z_t} > 0, \quad \frac{\partial \bar{K}_t}{\partial \mu_t} < 0.$$

The vertical line shifts out when Z_t rises, and shifts in when μ_t rises.

To determine the directions of motion, differentiate the $\dot{\lambda}_t$ equation at $\dot{\lambda}_t = 0$:

$$\left. \frac{\partial \dot{\lambda}_t}{\partial K_t} \right|_{\dot{\lambda}_t = 0} = Z_t \alpha \bar{K}_t^{\alpha-1} - \delta > \frac{1}{\mu_t} Z_t \alpha \bar{K}_t^{\alpha-1} - \delta = r > 0. \tag{12}$$

It follows that $\dot{\lambda}_t < 0$ at points $K_t < \bar{K}_t$, and $\dot{\lambda}_t > 0$ at points $K_t > \bar{K}_t$.

$\dot{K}_t = 0$ holds if and only if

$$\lambda_t = \bar{\lambda}_t(K_t) = \frac{1}{Z_t K_t^{\alpha} - \delta K_t}.$$

Hence

$$\frac{\partial \bar{\lambda}_t}{\partial Z_t} = -\frac{K_t^{\alpha}}{(Z_t K_t^{\alpha} - \delta K_t)^2},$$

$$\frac{\partial \bar{\lambda}_t}{\partial K_t} = -\frac{\alpha Z_t K_t^{\alpha-1} - \delta}{(Z_t K_t^{\alpha} - \delta K_t)^2}.$$

Given $K_t \leq \bar{K}_t$, it can be shown that (12) implies $Z_t K_t^{\alpha} - \delta K_t > 0$, which gives

$$\frac{\partial \bar{\lambda}_t}{\partial Z_t} < 0, \quad \frac{\partial \bar{\lambda}_t}{\partial K_t} < 0.$$

To determine the directions of motion, differentiate the $\dot{K}_t$ equation:

$$\frac{\partial \dot{K}_t}{\partial \lambda_t} = \lambda_t^{-2} > 0.$$

It follows that $\dot{K}_t < 0$ at points $\lambda_t < \bar{\lambda}_t(K_t)$, and $\dot{K}_t > 0$ at points $\lambda_t > \bar{\lambda}_t(K_t)$.

h. The increase of σ_t from σ to σ' implies a decrease of μ_t from μ to μ'. The $\dot{\lambda}_t = 0$ line shifts out temporarily. λ_t jumps up, then falls as capital increases. Eventually λ_t falls below the original steady state level. Capital hits a peak, and then declines until original saddlepoint path is reached at $t = t_1$. K_t subsequently falls and λ_t rises as the economy returns to the original steady state.

i. In a steady state equilibrium, equations (9)-(11) become

$$\frac{1}{C} = \lambda,$$

$$\beta\left(\frac{1}{\mu}\alpha K^{\alpha-1} + 1 - \delta\right) = 1,$$

$$K^{\alpha} - \delta K = C.$$

Solving for K gives

$$K = \left(\frac{\alpha}{\mu\left(1/\beta - (1-\delta)\right)}\right)^{\frac{1}{1-\alpha}}.$$

Thus,

$$\frac{\partial K}{\partial \mu} < 0.$$

It follows that a rise in σ, which reduces μ, leads to an increase in K. This in turn leads to an increase in C, since C strictly increases with K.

The social optimum is attained when $\mu = 1$:

$$\beta\left(\alpha K^{\alpha-1} + 1 - \delta\right) = 1.$$

This condition can achieved in equilibrium by subsidizing the rental rate at rate $1/\mu$, and financing the payments via a lump sum tax on the household.