Economics 210C - Macroeconomics — Spring 2018-19

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In-Class Final Exam

1. Deficit finance Consider the following version of the MIU model. The household is endowed with $Y_t = Y$ units of output in each period. The household's problem is

$$\max_{\{C_t, M_t^d, B_t^d\}} \mathbb{E}_t \sum\nolimits_{t=0}^{\infty} \beta^t \left(\ln C_t + \omega \ln \left(\frac{M_t^d}{P_t} \right) \right),$$

$$\text{s.t.} \;\; P_t Y + M_{t-1}^d + R_{t-1}^n B_{t-1}^d = P_t C_t + M_t^d + B_t^d,$$

where $0 < \beta < 1$ and $Y, \omega > 0$.

The government purchases $G_t = G$ units of output in each period, where $G \geq 0$. The government budget constraint is

$$P_t G + R_{t-1}^n B_{t-1} = B_t + M_t - M_{t-1}.$$

a. Set up the Lagrangian for the household's problem, and write down the first order necessary conditions for a solution.

b. Show that if asset markets clear, i.e., $M_t^d = M_t$ and $B_t^d = B_t$, then $C_t = Y - G$ for all t.

c. Suppose the government chooses $M_t = M_{t-1}$ for all t, and attempts to finance its spending exclusively with nominal debt. Show that $G > 0$ is not possible in a steady state equilibrium. Provide intuition for this result.

d. Now suppose that the government chooses $B_t = 0$ for all t, and attempts to finance its spending exclusively through seignorage. Show that in a steady state equilibrium, the government can finance levels of G such that $0 \leq G < \overline{G} < Y$, where $\overline{G}$ is a constant. What happens to steady state inflation as G approaches $\overline{G}$ from below? Provide intuition for your answer.

2. Monopolistic competition with markup shocks Consider the following monopolistically competitive RBC model. There is a continuum of intermediate goods $i \in [0,1]$. Good i is produced using the following technology:

$$Y_{it} = Z_t K_{i,t-1}^{\alpha} L_{it}^{1-\alpha},$$

where $0 < \alpha < 1$. Each intermediate good is supplied by a single firm, and these firms are uniformly distributed on $[0,1]$. The firms are price takers in the labor and capital markets, and price setters in their own intermediate good markets.

Cost minimization by firm i implies that $K_{i,t-1}$ and L_{it} satisfy

$$\phi_t Z_t \alpha K_{i,t-1}^{\alpha-1} L_{it}^{1-\alpha} = R_t,$$

$$\phi_t Z_t (1-\alpha) K_{i,t-1}^{\alpha} L_{it}^{-\alpha} = W_t,$$

where ϕ_t is marginal cost. Minimized total cost satisfies

$$R_t K_{i,t-1} + W_t L_{it} = \phi_t Y_{it}.$$

Final goods firms combine intermediate goods to produce the final good using a constant returns to scale technology. They are price takers in the markets for intermediate and final goods. Let $Y_{it} = D_t(P_{it})$ be the demand function for good i. The price elasticity of D_t is given by $\sigma_t > 1$, which is assumed to be constant for all P_{it}.

The household supplies $L_t = 1$ units of labor inelastically in each period. The household's problem is

$$\max_{\{C_t, K_t\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \ln C_t,$$
$$\text{s.t.} \quad R_t K_{t-1} + W_t + \Pi_t^* + (1-\delta) K_{t-1} = C_t + K_t,$$

where $0 < \beta, \delta < 1$, and Π_t^* denotes the aggregate profits of the intermediate goods firms.

a. Set up firm i's profit maximizing pricing problem. Write down the first order necessary condition in terms of P_{it}, ϕ_t and σ_t. Express the markup $\mu_{it} = P_{it}/\phi_t$ as a function of σ_t, and show that $\mu_{it} > 1$.

b. Set up the Lagrangian for the household's problem, and write down the first order necessary conditions for a solution.

c. A symmetric pricing equilibrium has $P_{it} = P_t^*$ for all i. Let $\mu_t = P_t^*/\phi_t$ denote the associated equilibrium markup, let L_t^* and K_{t-1}^* denote the aggregate demands for factors, and let Y_t^* denote aggregate output of intermediate goods. Show that if labor and capital markets clear, then aggregate factor income and aggregate intermediate good output satisfy

$$R_t K_{t-1} + W_t + \Pi_t^* = P_t^* Y_t^* = P_t^* Z_t K_{t-1}^{\alpha}. \tag{1}$$

Also show that the equilibrium capital rental rate satisfies

$$R_t = \frac{P_t^*}{\mu_t} Z_t \alpha K_{t-1}^{\alpha-1}. \tag{2}$$

d. Suppose that the final good price satisfies

$$P_t = P_t^* = 1, \tag{3}$$

and final good output satisfies $Y_t = Y_t^*$. Substitute (1), (2) and (3) into the necessary conditions from part b to obtain a system of three equations in the endogenous variables C_t, λ_t and K_t, and exogenous variables Z_t and μ_t.

e. Solve the equations from part d for the perfect foresight paths of $\Delta\lambda_t$ and ΔK_{t-1} in terms of the variables λ_t, K_{t-1}, μ_t and Z_t.

f. Convert the equations from part e into a continuous time system that determines $\dot{\lambda}_t$ and $\dot{K}_t$ in terms of the variables λ_t, K_t, μ_t and Z_t.

g. Sketch the $\dot{\lambda}_t = 0$ locus is the K_t-λ_t plane, and show how it shifts when μ_t increases, and when Z_t increases. Also do this for the $\dot{K}_t = 0$ locus.

h. Suppose the economy begins in a steady state, with $\sigma_t = \sigma$. At time t_0, σ_t shifts up to $\sigma' > \sigma$, and then shifts back to σ at time $t_1 > t_0$, remaining at σ for all future t. Assume $Z_t = 1$ for all t. These paths are perfectly anticipated by all agents, and the economy returns to the original steady state in the long run. Discuss the perfect foresight equilibrium transition paths of λ_t and K_t.

i. (Extra credit) Solve for the steady state equilibrium of the discrete time model, and show that the household's steady state lifetime utility is strictly increasing in σ. Is there a tax or subsidy policy that implements the socially optimal allocation, given a fixed value of σ?