Economics 210C - Macroeconomics Spring 2018-19

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Imperfect Competition and Income Spillovers

Consider the following one-period economy. There is a continuum of households indexed by i or j, uniformly distributed on the unit interval. Household j's utility function is

$$U(C_j, N_j, Y_j) = C_j^{\xi} N_j^{1-\xi} - \chi Y_j,$$

where $0 < \xi < 1$ and $\chi > 0$. C_j is consumption of a composite good, N_j is consumption of an autonomous good, and Y_j is production of good j. The composite good is given by

$$C_j = \left(\int_0^1 C_{ij}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $\sigma > 1$, and C_{ij} is household j's consumption of good i, which is produced by household i. In addition, each household is endowed with $\bar{N} > 0$ units of the autonomous good. Assume that each household i is a price setting monopolist in the market for good i, and a price taker in all other markets.

a. Household j's expenditure minimization problem is

$$\min_{C_{ij},\, i \in [0,1]} \int_0^1 P_i C_{ij} di \quad \text{s.t. (1)}, \tag{2}$$

where P_i is the price of good i, and C_j is treated as a constant. Set up the Lagrangian for problem (2), with P denoting the Lagrange multiplier. Write down the first order necessary conditions, and show that the optimal values of C_i and P are given by

$$C_{ij} = \left(\frac{P_i}{P} \right)^{-\sigma} C_j, \quad P = \left(\int_0^1 P_i^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}.$$

Also show that the solution to problem (2) satisfies

$$\int_0^1 P_i C_{ij} di = PC_j.$$

(Thus we may interpret P as the price of the composite good.)

b. Now consider household i's optimal consumption problem:

$$\max_{C_i, N_i} U\left(C_i, N_i, Y_i\right) \quad \text{s.t.} \quad I_i = PC_i + N_i,$$

where

$$I_i = P_i Y_i + \bar{N},$$

and Y_i is treated as a constant. Solve for the optimal values of C_i and N_i, and show that maximized utility has the form

$$U = V(P) I_i - \chi Y_i,$$

where V is a strictly decreasing function of P.

c. Show that aggregate demand for good i is given by

$$Y_i = \xi \left(\frac{P_i}{P}\right)^{-\sigma} \frac{I}{P}, \tag{3}$$

where

$$I = \int_0^1 P_j Y_j dj + \bar{N}.$$

d. Household i's optimal pricing problem is

$$\max_{P_i} V(P)\left(P_i Y_i + \bar{N}\right) - \chi Y_i,$$

where Y_i is given by (3). Calculate the first order necessary condition for a solution. Show that the optimal P_i is strictly increasing in P, and independent of I. (Thus pricing exhibits strategic complementarity between sellers.) Also evaluate Y_i at the optimal P_i, and show that Y_i is strictly increasing in I. (Thus there is an income spillover across households.)

e. In a symmetric equilibrium, we have $P_i = P$, $Y_i = Y$ and $N_i = \bar{N}$ for all i. Solve for the symmetric equilibrium values of P and Y. Show that an increase in market power, as captured by a decrease in σ, leads to higher P and lower Y. Also show that an increase in $\bar{N}$ has a positive effect on Y, and the effect is smaller when σ is smaller. Explain intuitively the mechanisms underlying these results.