

Exercise on Extensions of RBC Model

For each of the following specifications of the RBC model, complete these steps: (i) Derive necessary conditions for a solution to the social planner's problem; (ii) Restate the necessary conditions in terms of detrended variables, if necessary; (iii) Solve for the non-stochastic steady state, and verify that it is unique; and (iv) Log-linearize the necessary conditions around the steady state. In each case, $\{\varepsilon_t\}$ is a white noise process, and the law of motion for capital is

$$K_t = (1 - \delta)K_{t-1} + Y_t - C_t.$$

a. Preference shocks

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\ln C_t + \chi_t \ln(1 - L_t)),$$

$$Y_t = K_{t-1}^\alpha L_t^{1-\alpha}, \quad \chi_t = \chi_{t-1}^\rho e^{\varepsilon_t}.$$

b. Labor adjustment costs

Assume L_t is chosen in period t .

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \frac{L_{t-1}^{1+1/\eta}}{1 + 1/\eta} \right),$$

$$Y_t = Z_t K_{t-1}^\alpha L_{t-1}^{1-\alpha} - \frac{\gamma}{2} (L_t - L_{t-1})^2, \quad Z_t = Z_{t-1}^\rho e^{\varepsilon_t}, \quad \gamma > 0.$$

c. Habit persistence preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\ln(C_t - \phi C_{t-1}) + \ln(1 - L_t)), \quad \phi > 0,$$

$$Y_t = Z_t K_{t-1}^\alpha L_t^{1-\alpha}, \quad Z_t = Z_{t-1}^\rho e^{\varepsilon_t}.$$

d. Non-logarithmic preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} (1 - L_t)^{\chi(1-\sigma)} - 1}{1 - \sigma}, \quad \sigma \neq 1, \chi > 0,$$

$$Y_t = Z_t K_{t-1}^{\alpha} (X_t L_t)^{1-\alpha}, \quad Z_t = Z_{t-1}^{\rho} e^{\varepsilon_t}, \quad X_t = \mu^t, \quad \mu > 1.$$

e. Population growth

N_t denotes the population at t .

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln \left(\frac{C_t}{N_t} \right) + \ln \left(1 - \frac{L_t}{N_t} \right) \right),$$

$$Y_t = Z_t K_{t-1}^{\alpha} (X_t L_t)^{1-\alpha}, \quad Z_t = Z_{t-1}^{\rho} e^{\varepsilon_t},$$

$$N_t = N_0 \nu^t, \quad X_t = \mu^t, \quad N_0, \nu, \mu > 0.$$