

### Consumption Loans Model Exercises

**1. Rate of return dominance** Consider the consumption loans model, with constant nominal balances  $M > 0$ . Suppose there exists a storage technology that pays  $1 + R$  units of the consumption good at period  $t + 1$  for each unit of the good stored at  $t$ . Assume  $R > -1$ . In period  $t$ , each young agent chooses to store  $S_t^d \geq 0$  units of the consumption good, and hold  $M_t^d \geq 0$  units of money.

- a. Derive the budget constraints at periods  $t$  and  $t + 1$  for agents born at  $t$ .
- b. Set up the agents' lifetime utility maximization problem, and write down the first-order conditions for optimal interior choices of  $S_t^d$  and  $M_t^d$ .
- c. Use the first order conditions to solve for the steady state equilibrium inflation rate  $\pi$ .
- d. Show that a monetary equilibrium exists (i.e., agents choose to hold strictly positive nominal balances) if  $R$  is small, but not if  $R$  is large. Is there a monetary equilibrium in which agents choose strictly positive storage?

**2. Government finance via seignorage** Consider a version of the consumption loans model with a government sector. Agents' lifetime utility is given by

$$U(C_{1,t}, C_{2,t+1}) = \ln C_{1,t} + \beta \ln C_{2,t+1},$$

where  $0 < \beta < 1$ . Assume that  $N > 0$  agents are born in each period  $t$  (i.e.,  $n = 0$ ). The government consumes  $NG$  units of the consumption good in each period, where  $G > 0$ .

- a. Derive the symmetric Pareto-optimal allocation of consumption goods across generations (i.e., the allocation satisfies  $C_{1,t} = \omega$  for each  $t$ ).

**b.** Suppose the government supplies  $M > 0$  units of fiat money. In addition, the government finances its purchases in each period by means of a lump-sum tax  $T$  on each old agent. Derive the monetary SSE in this case, and show that the equilibrium allocation coincides with the Pareto-optimal allocation.

**c.** Now suppose that the government finances its purchases by issuing new money; i.e.,  $M_{t+1} - M_t = P_{t+1}NG$  for each  $t$ . Derive the monetary SSE in this case, and show that the equilibrium allocation fails to be Pareto-optimal.