Egalitarianism and Resource Conservation in Hunter-Gatherer Societies

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Abstract

Egalitarianism in hunter-gatherer societies takes the form of implicit taxes on renewable resources the proceeds of which are redistributed among all members of the community. It is argued that these taxes represent an evolutionary response to the dynamics of large game hunting. The implicit resource tax raises output and welfare per capita; high tax rates can prevent the emergence of a "feast and famine" pattern of growth. These predictions are found to be consistent with evidence from the !Kung San hunter-gatherer society in southern Africa. Moreover, they shed light on the evolution of other preindustrial societies. It is demonstrated that the decline of Easter Island could have been avoided by implicit resource taxation.

JEL Codes: Q20, Z10
1 Introduction

Many hunter-gatherer societies have an egalitarian structure in the sense that inequality in the distribution of wealth and power across individuals is very small and no member is dependent on particular other members (e.g., household heads or chiefs) to obtain food or other material goods. Rather can everybody obtain these goods as gifts or through borrowing from anyone else in the community. As far as the consumption of renewable resources is concerned, gift-giving frequently takes the form of sharing. For example, large game hunting is typically undertaken by one individual or a small group (usually adult men). Sharing rules prescribe that the prey is redistributed eventually to all members of the community while everybody receives an (approximately) equal share.

Egalitarianism (in the sense just described) coexists with two other phenomena in hunter-gatherer societies. First, work effort levels have been found to be low in many anthropological studies, which has led Sahlin (1972) to label hunter-gatherers as "the original affluent society". Second, available evidence indicates that many hunter-gatherer societies conserved renewable resources in the sense that they avoided their extinction or transformed their physical environment at a much lower pace than agricultural or industrial societies have. Furthermore, it appears that many hunter-gatherer economies followed a time path of slow but steady expansion in population and output over a long time horizon rather than a "feast and famine" pattern.

This paper aims to present an economic interpretation of the relationship between sharing rules (as a particular form of egalitarianism among hunter-gatherers), harvesting effort, and renewable resource conservation. It is argued that sharing rules in hunter-gatherer societies are an evolutionary response to the dynamics of their physical environment. To this end, sharing rules are interpreted as an implicit tax on the harvest of renewable resources the proceeds of which are redistributed equally among all members of the community. The implicit tax lowers the marginal return to resource harvesting, which reduces effort and increases the resource stock at equilibrium. Moreover, the tax can set a hunter-gatherer economy on a time path of slow but steady expansion which converges towards a long-run equilibrium.

The interpretation of sharing rules as an implicit resource tax is useful in two respects. First, it improves our understanding of the role of cultural variables in ensuring sustainability in hunter-gatherer societies. Second, the model provides an alternative explanation for the contrasting growth patterns that have occurred in other preindustrial economies. For example, population and economic activity on Easter Island rose for several centuries but then declined substantially within a short time span before the first Europeans arrived (Brander and Taylor, 1998). In other Pacific island economies,
however, population and output grew monotonously first and then stabilized at a certain level (Brander and Taylor, 1998, p.129). Brander and Taylor (1998) emphasize that the collapse of the Easter Island civilization has been caused by its dependence on a palm tree species with a low intrinsic growth rate. This paper argues that the decline of Easter Island was not inevitable given its resource endowment but could have been avoided if sharing rules had been established in time, as was the case in many hunter-gatherer societies and still is the case in many parts of contemporary Oceania.

The following analysis extends a general equilibrium model by Brander and Taylor (1998) to a resource tax. Theoretical analysis reveals that different tax rates can generate different growth patterns. An economy that converges cyclically to its long-run equilibrium in the absence of resource taxation can always attain a trajectory of monotonic convergence (i.e. avoid a collapse of economic activity) by choice of a sufficiently high tax rate. However, instability may arise under small tax rates as a result of a supercritical Hopf bifurcation. When long-run equilibria are compared, a marginal increase in the tax rate unambiguously increases individual welfare. However, its effect on aggregate (utilitarian) welfare is ambiguous, as it depends on whether the tax raises or lowers the equilibrium population size.

The model is applied to large game hunting in the economy of the !Kung San, a community of hunter-gatherers who lives on the north-western fringe of the Kalahari desert in southern Africa. Simulations demonstrate that, in the absence of sharing rules, the !Kung San economy would have experienced a pattern of rapid increase and decline that bears some resemblance to what occurred on Easter island. However, the tax rate of 84% which is implicit in the high degree of sharing observed by anthropologists transforms this pattern into a time path of steady expansion towards a long-run equilibrium.

As a second step, the model is applied to Easter Island to evaluate the possible impact of sharing rules. A counterfactual simulation shows that the disruption of economic activity on Easter Island could have been avoided if an (explicit or implicit) resource tax of 50% of the resource units harvested had been adopted no later than 350 years before the historic collapse began. This tax rate is in the same order of magnitude as the resource tax of 45% which is implicit in sharing rules that are still in force in the Kingdom of Tonga in contemporary Oceania (Chakraborty 2001a).

Apart from Brander’s and Taylor’s model, this paper builds on two earlier contributions by Smith (1975, 1993), who argues that the extinction of large herding animals such as mammoth, bison, and mastodon in the Pleistocene was the result of overexploitation by Paleolithic hunters. Smith suggests that the disappearance of this important food source triggered a

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1The "ǃ" is pronounced as an (alveolar-palatal) click. Clicks are characteristic of the Khoisan languages, to which the language of the !Kung belongs.
process of learning which resulted in the evolution of institutions that enhanced resource conservation. Sharing rules can be seen as a subset of such institutions (Smith 1975, p.741). The present contribution is a natural extension of Smith’s work, as it formalizes the impact of sharing rules while treating population as endogenous.

The impact of sharing rules is similar to the impact of rules that restrict access to a renewable resource or put restrictions on harvesting technologies, as was described by Ostrom (1990). However, sharing norms are different in that they refer to resource consumption rather than harvesting. The relationship between sharing rules, work effort and resource conservation in hunter-gatherer societies was analyzed informally by Kägi (2001).

Earlier modifications to the Brander-Taylor model included the accumulation of human-made capital (Erickson and Gwily 2000), a parameter that is to reflect different property regimes (Dalton and Coats 2000), and exogenous technological progress along with institutions for population control (Reuveny and Decker 2000). This paper differs from these contributions in that it focuses on implicit resource taxation and derives all its major conclusions analytically. Moreover, it differs from the first two papers in that the results are derived from individual utility maximization.

This paper is organized as follows. Section 2 describes empirical evidence on egalitarianism and natural resource use in hunter-gatherer societies. Section 3 presents the Brander-Taylor model with a resource tax while Section 4 analyzes the existence, stability, and comparative statics of the resulting long-run equilibrium. Section 5 applies the model to the !Kung San economy and to Easter Island. Section 6 concludes.

2 Hunter-Gatherer Societies

In social anthropology, two classes of hunter-gatherer societies are distinguished (Woodburn 1982): delayed-return and immediate-return societies. In immediate-return societies, the time span between the application of labour and the consumption of its product is very short. The prey of a successful hunt or the vegetable foods gathered are consumed on the same day or shortly after without elaborate processing or longer-term storage. In delayed-return societies, in contrast, individuals "hold rights over valued assets of some sort, which either represent a yield, a return for labour applied over time or, if not, are held and managed in a way which resembles and has similar social implications to delayed yields on labour" (Woodburn 1982, p.432). For example, these assets can take the form of sophisticated tools and equipment (e.g. fishing boats or nets), processed and stored food, or wild plants which have been improved by human labour. From an economic perspective, Woodburn's distinction can be reformulated in terms of whether a society employs capital to any significant extent. In other words,
delayed-return hunter-gatherer societies apply elementary forms of capital while the use of capital is much smaller in immediate-return societies.

Egalitarianism is strongest in immediate-return societies while delayed-return societies may exhibit marked inequality. The following description confines itself to immediate-return societies; it is based on Woodburn’s (1982) overview and on case studies of the !Kung San in southern Africa (Marshall 1960, 1961, 1976; Lee 1979), the Batek in the Malaysian rainforests (Endicott 1988), and the Nayaka in southern India (Bird-David 1990).

Effort levels are low in these societies (Barnard and Woodburn 1988, pp.11-12; Kägi 2001). For example, Lee (1979, p.256) reports that adult members of the Dobe !Kung (a subgroup of the !Kung San) hunt or gather for only 2.4 days per week on average. Sahlin’s (1972) contains a summary of anthropological case studies on effort levels. Important institutions that serve to maintain egalitarianism are sharing, direct access to natural resources, and the flexibility of social groupings (Woodburn 1982).

Sharing. In immediate-return societies, sharing extends to large game and (to a lesser extent) to small game, vegetable foods, and craft objects. Earlier interpretations viewed sharing as a primitive form of exchange (Mauss 1954) or simply as a substitute for storage in settings where storage was considered physically impossible. However, field studies revealed that individuals may give away and obtain identical goods from others in the sharing process (Endicott 1988, p.116). Furthermore, some food donors remain on balance donors. For example, hunting yields are generally concentrated among a small group of hunters, who remain net donors of meat during their lifetime (Lee 1979, pp.242-244). Finally, storage technologies exist in many immediate-return societies, as meat or fish can be dried and seeds can be conserved (Marshall 1976, p.124). However, these societies choose not to employ these technologies on any significant scale.

More recent anthropological contributions see sharing as an institution which promotes egalitarianism in immediate-return societies (Woodburn 1982). Sharing serves to prevent the accumulation of wealth in the hands of few individuals who could then use their wealth to dominate others. In contrast, recent economic interpretations of renewable resource sharing have focused on its impact as a resource tax or as informal insurance (Bender, Kägi and Mohr 2001; Kägi 2001; Chakraborty 2001a, 2001b).

Sharing takes place according to a set of sophisticated rules. For example, large game is shared in several rounds. In each round, the recipients of the previous round redistribute part of their receipts to a wider circle of individuals, which results in every member of the community finally obtaining some meat. Marshall (1961, pp.237-239) provides the following account of the meat distribution among the Nyae Nyae !Kung, a hunter-gatherer
community in Namibia:

"When the kill is made the hunters have the prerogative of eating the liver on the spot and may eat more of the meat until their hunger is satisfied. (...) They then carry the animal to the band (...)
The first distribution of the animal is made in large portions usually to five or six persons. They are the owner of the arrow [that killed the animal, R.C.], the giver of the arrow (if the arrow was not one the owner had made himself), and the hunters. The meat, always uncooked in the first distribution, is given on the bone, unless the animal is large and the meat has been cut into strips at the kill.

In a second distribution the several persons who got meat in the first distribution cut up their shares and distribute them further. This meat also is given uncooked. The amounts depend on the number of persons involved, but should be as much as the giver can manage. In the second distribution close kinship is the factor which sets the pattern of the giving. Certain obligations are compulsory. A man's first obligation at this point, we were told, is to give to his wife's parents. He must give to them the best he has in as generous portions as he can, while still fulfilling other primary obligations, which are to his own parents, his spouse, and offspring. He keeps a portion for himself at this time and from it would give to his siblings, to his wife's siblings, if they are present, and to other kin, affines, and friends who are there, possibly only in small quantities by then.

Everyone who receives meat gives again, in another wave of sharing, to his or her parents, parents-in-law, spouses, offspring, siblings, and others. The meat may be cooked and the quantities small. Visitors, even though they are not close kin or affines, are given meat by the people whom they are visiting. (...) It ends in everybody getting some meat."

The members of hunter-gatherer communities consider sharing as a social obligation rather than as a measure of resource conservation. Sharing is enforced by the threats of ostracism and exclusion from the benefits of sharing in the future. Monitoring is easy as the killed animal is carried to the camp of the sub-group ("band") by a group of people, which never goes undetected. As dwellings are not fixed, the meat is cooked at open fires. Consequently, it is impossible for any individual to prepare and eat meat in secrecy.

Large game is never owned by the successful hunter(s). Instead, ownership is allocated by other rules. Among the !Kung San and the Batek,
ownership of the prey is allocated to the owner of the arrow (Batek: the blowpipe) that killed the animal. However, ownership predominantly implies the obligation to share the meat according to the prevailing rules; it does not include the right to sell or consume the prey at the owner’s will.

Plants gathered or artifacts made are owned by the gatherer or producer. Vegetable foods are generally gathered by everybody and, therefore, are shared to a lesser extent. However, an individual who is eating vegetable food would not deny to share the food with his or her neighbour if the latter is without food (Lee 1979, pp.200-201). The sharing of crafts objects frequently occurs in the form of borrowing. In some societies (e.g. the Nayaka), the borrower is obliged to ask the owner for the permission to use the object, which is usually granted (Bird-David 1990, p.193). Moreover, crafts are frequently given as gifts, which may involve a dimension of reciprocity. Social pressure to give away artifacts as gifts rises as artifacts accumulate in the hands of a single individual. Sharing may then take the form of "demand sharing" (Barnard and Woodburn 1988, p.12).

Access to natural resources and flexible groupings. Access to natural resources is open in immediate-return hunter-gatherer societies. This is brought about by two sets of institutions. First, every member of a group has the right of access to its physical environment for gathering and hunting (Barnard and Woodburn 1988, pp.15-16; Endicott 1988, p.114; Marshall 1960, pp.331-334). As far as outsiders are concerned, restrictions exist in some communities. For example, gathering territories are allocated to particular sub-groups among the !Kung San (Marshall 1961). However, sub-groups allow each other to gather food in their territories during times of scarcity (e.g. during the dry season). Access to hunting grounds is generally open to outsiders. Permission needs to be asked in cases where restrictions exist, which is usually granted, however.

The second factor which contributes to open access is the flexibility of groupings. Individuals can choose to join another sub-group without incurring high transaction or relocation costs, which further contributes to the access to natural resources being effectively open. One of the combined effects of sharing, flexible groupings and unrestricted access to natural resources is to promote egalitarianism (Woodburn 1982, p.445).

3 The Model

This section extends the model of Brander and Taylor (1998) to a resource tax. The dynamics of the natural resource stock \( S \) is assumed to follow a logistic growth function. With the intrinsic growth rate \( r \), carrying capacity

\[ \text{This is true within the boundaries set by the gender division of labour. For example, women do not hunt large game in the societies mentioned above.} \]
3 THE MODEL

$K$, and the harvest rate $H$, the change $\dot{S}$ in the resource stock per unit of time is

$$\dot{S} = rS \left(1 - \frac{S}{K}\right) - H$$  \hspace{1cm} (1)

The first term on the right-hand side of (1) represents the own rate of growth of the resource, i.e. the rate at which the resource regenerates if it is undisturbed by human intervention. It is assumed that the harvest rate supplied by producers $H^P$ rises as the resource stock or the amount of labour $L_H$ allocated to resource harvesting increases:

$$H^P = \alpha SL_H$$  \hspace{1cm} (2)

$\alpha > 0$ is a parameter. The resource is harvested under open access, which implies that resource rents are zero. It is assumed that a resource tax is levied on each unit of the resource which is harvested. The marginal tax rate is assumed to be constant. As perfect competition prevails in the goods market, the supply price $p^P$ of one resource unit harvested must equal its marginal (= average) labour cost, $w/\alpha S$ at the prevailing wage rate $w$, plus the tax rate $t$:

$$p^P = \frac{w}{\alpha S} + t$$  \hspace{1cm} (3)

Apart from the resource harvest, the economy produces a manufactured good (denoted $M$), which can be interpreted as an index of agricultural or artisanal goods and reproductive or cultural activities. Good $M$ is produced with a fixed coefficient technology that uses only labour. One unit of labour is assumed to produce one unit of $M$. As both labour and goods markets are assumed to be perfectly competitive, the price of $M$ equals one, and so does the wage rate $w$ if manufactures are produced. Equation (3) implies that the tax rate $t$ is measured in units of the manufactured good per unit of the resource harvested. Hence, the marginal tax rate is constant only in terms of the manufactured good. Measured in terms of renewable resource units, it varies as the resource price varies.

Labour is allocated between resource harvesting and the production of manufactures according to the following labourforce constraint:

$$\frac{H^P}{\alpha S} + M = L$$  \hspace{1cm} (4)

The labourforce $L$ is assumed to be equal to the population size. The allocation of labour is determined by utility maximization of a representative
consumer who is endowed with one unit of labour. As the resource is under open access, the consumer maximizes instantaneous rather than discounted utility. Her utility function is assumed to have the following form:

\[ u = h^\beta m^{1-\beta} \quad \text{with} \quad 0 < \beta < 1 \]  

(5)

\( h \) and \( m \) are the individual consumption of the resource harvest and the manufactured good, respectively. The proceeds from the resource tax are distributed equally among all consumers. The individual consumer’s budget constraint is then

\[ ph + m = w + t \frac{H^P}{L} \]  

(6)

Utility maximization yields the following aggregate demand quantities of the two goods (details see Full Mathematical Workings):

\[ H^D = \frac{L\beta}{p} \left( w + t \frac{H^P}{L} \right) \]  

(7)

\[ M^D = L(1-\beta) \left( w + t \frac{H^P}{L} \right) \]  

(8)

From (7), the demand price \( p^D \) for the resource can be calculated as

\[ p^D = L\beta \left( \frac{w}{H^D} + t \frac{H^P}{LH^D} \right) \]  

(9)

At short-run equilibrium (where the resource stock and the population are considered as given), the demand price must equal the supply price, \( p^D = p^P = p \), and the quantity demanded must equal the quantity supplied, \( H^P = H^D = H \). Setting (3) and (9) equal yields

\[ \frac{w}{\alpha S} + t = \frac{L\beta w}{H} + \beta t \]

\[ H = \frac{L\beta \alpha S}{1 + \frac{\alpha S}{w}(1-\beta)t} \]  

(10)

Equation (10) shows that the equilibrium harvest with resource taxation is lower than without taxation for any positive tax rate if the resource stock is given. According to (4), the output of manufactures is

\[ M = L - \frac{H}{\alpha S} = L \left( 1 - \frac{\beta}{1 + \frac{\alpha S}{w}(1-\beta)t} \right) \]  

(11)
The equilibrium output of manufactures is higher with resource taxation than without taxation for any positive tax rate. As expected, the tax shifts the composition of output towards manufactures. Inserting (10) into (1) gives the change in the level of the resource stock per unit of time:

\[ \dot{S} = rS(1 - \frac{S}{K}) - \frac{L\beta\alpha S}{1 + \frac{\alpha S}{w}(1 - \beta)t} \]  \hspace{1cm} (12)

The population growth rate positively depends on the difference between the birth rate \( b \) and the death rate \( d \), which is assumed to be negative, \( b - d < 0 \). It also positively depends on the resource harvest per capita \( H/L \). \( \phi > 0 \) is a parameter that measures the responsiveness of population growth to resource harvest income per capita. The change in population per unit of time is then

\[ \dot{L} = L \left( b - d + \phi \frac{H}{L} \right) \]

\[ \dot{L} = L \left( b - d + \phi \frac{\beta\alpha S}{1 + \frac{\alpha S}{w}(1 - \beta)t} \right) \]  \hspace{1cm} (13)

4 Long-run Equilibrium

4.1 Existence

As in the Brander-Taylor model, two obvious solutions for a long-run equilibrium are \( S = L = 0 \) and \( S = K, L = 0 \). The existence of an interior solution is analyzed by setting \( \dot{S} = \dot{L} = 0 \) in (12) and (13):

\[ \dot{S} = 0 \Rightarrow \frac{r(1 - \frac{S}{K}) - \frac{L\beta\alpha}{1 + \frac{\alpha S}{w}(1 - \beta)t}}{} = 0 \text{ for } S > 0 \]  \hspace{1cm} (14)

\[ \dot{L} = 0 \Rightarrow (b - d) + \phi \frac{\beta\alpha S}{1 + \frac{\alpha S}{w}(1 - \beta)t} = 0 \text{ for } L > 0 \]  \hspace{1cm} (15)

Solving (15) for \( S \) and (14) for \( L \) yields

\[ S^* = \frac{d - b}{\phi\beta\alpha - (d - b)\frac{\alpha S}{w}(1 - \beta)t} \]  \hspace{1cm} (16)

\[ L^* = \frac{r}{\beta\alpha} \left( 1 - \frac{S^*}{K} \right) \left[ 1 + \frac{\alpha S^*}{w}(1 - \beta)t \right] \]  \hspace{1cm} (17)

It is assumed that the tax rate is less than \( t^S = \phi\alpha w / [(d - b)(1 - \beta)] \), which ensures that the steady state resource stock in (16) is positive. This
is plausible because the entire harvest is taxed away if the tax rate is equal to $t^S$. To see this, recall that the tax rate $t$ in (3) was fixed in terms of manufactured goods per unit of resource harvest. Measured in resource units per unit of resource harvest, the tax rate is $T = t/p = t/(t + w/\alpha S)$. A tax of $T = 100\%$ implies $1/S \to 0$, which implies at equilibrium that

$$\frac{1}{S^*} = \frac{\phi \beta \alpha - (d - b) \frac{\alpha}{w} (1 - \beta) t}{d - b} = 0$$

(18)

$$t = \frac{\phi \beta w}{(d - b)(1 - \beta)} = t^S$$

(19)

From (1), the equilibrium resource stock cannot exceed the carrying capacity $K$. Furthermore, inspection of (17) shows that the resource stock must be strictly less than $K$ if the steady state population is to be positive. This imposes a restriction on the tax rate, as can be seen from (16):

$$\frac{d - b}{\phi \beta \alpha - (d - b) \frac{\alpha}{w} (1 - \beta) t} < K$$

(20)

$$t < \frac{\phi \beta w}{(d - b)(1 - \beta)} - \frac{w}{\alpha K (1 - \beta)} := t^K$$

(21)

If the tax rate is raised beyond $t^K$, the resource harvest per capita is reduced in the short run. According to (13), this causes population growth to decline, which tends to raise the growth rate of the resource in (12). As the resource stock is already at its maximum, the resource harvest per capita cannot further increase. As a result, the population growth rate continues to be negative until the population level reaches zero.

As $t^K < t^S$, the constraint $t < t^K$ is binding and ensures that both the steady state resource stock and the steady state population level are positive.

In the $(S, L)$ plane, the long-run equilibrium solution $(S^*, L^*)$ can be characterized as follows (see Figure 1). For $L \neq 0$, the locus $\dot{L} = 0$ is a straight line parallel to the L-axis at $S = S^*$. To facilitate the analysis of the locus $\dot{S} = 0$, (14) can be rewritten as

$$L = \frac{r}{\beta \alpha} \left(1 - \frac{S}{K}\right) \left[1 + \frac{\alpha S}{w} (1 - \beta) t\right] =: \bar{L}(S)$$

(22)

For $t > 0$, the locus $\dot{S} = 0$ is a parabola which cuts the S-axis at $S_1 = K$ and $S_2 = -w/[(\alpha (1 - \beta) t)]$. The peak value of the parabola is at

$$S_3 = \frac{1}{2} (S_1 + S_2) = \frac{1}{2} \left(K - \frac{w}{(1 - \beta) \alpha t}\right)$$

(23)
Figure 1: Long-run equilibrium with and without an environmental tax

The equilibrium is represented by the intersection of the parabola with the locus $\hat{L} = 0$ (see Figure 1). For a tax rate of zero, which represents the Brander-Taylor case, the locus $\hat{S} = 0$ is a straight line which extends from $(S = K, L = 0)$ to $(S = 0, L = r/(\beta\alpha))$. The locus $\hat{L} = 0$ is closer to the origin than with a positive tax. The equilibrium is represented by the point $[S^*(t = 0), L^*(t = 0)]$ in Figure 1.

$S_3$ declines as the tax rate declines. As $S_3 \to -\infty$ as $t \to 0$, the Brander-Taylor solution can be interpreted as a limiting case where the segment of the parabola in the upper quadrant in Figure 1 degenerates into a straight line.

4.2 Stability

This section considers the local stability of the system (12)-(13) linearized at its equilibrium point.\footnote{It can be shown that the system is globally stable if it is locally stable. See Full Mathematical Workings for details.} It can be shown that the boundary solutions $(S = 0, L = 0)$ and $(S = K, L = 0)$ are unstable saddlepoints (see Full Mathematical Workings for details). As far as the interior solution is concerned, the determinant and the trace of the Jacobian $J = (J_{ik})$ can be calculated as (see Full Mathematical Workings for details)
\[ |J| = \frac{\phi \alpha^2 \beta^2 S^* L^*}{[1 + \frac{\alpha S^*}{w}(1 - \beta)t]^2} > 0 \]  \hspace{1cm} (24)

\[ tr(J) = J_{11} = r(1 - \frac{2S^*}{K}) - \frac{\alpha \beta L^*}{[1 + \frac{\alpha S^*}{w}(1 - \beta)t]^2} \]  \hspace{1cm} (25)

In a 2x2-System, the eigenvalues \( \lambda_{1,2} \) can be expressed as

\[ \lambda_{1,2} = \frac{tr(J) \pm \sqrt{[tr(J)]^2 - 4|J|}}{2} \]  \hspace{1cm} (26)

The determinant \( |J| \) is unambiguously positive. The sign of the trace, however, is ambiguous. The stability properties of the interior solution can therefore not be seen by inspection of (25). Graphically, the possibility of instability arises because the slope of the parabola \( \dot{S} = 0 \) is negative for \( S^* > S_3 \) and positive for \( 0 < S^* < S_3 \), as the following proposition states.

**Proposition 4.1.** Consider an interior solution \( (S^*, L^*) \) of the system (14)-(15). The equilibrium is stable if \( S^* > S_3 \); it is unstable if \( S^* < S_3 \). If \( S^* = S_3 \), the equilibrium is a centre.

**Proof.** Substituting (17) into (25) gives (details see Full Mathematical Workings)

\[ tr(J) = r \frac{- \frac{S^*}{K} + (1 - \frac{2S^*}{K}) \frac{\alpha S^*}{w}(1 - \beta)t}{1 + \frac{\alpha S^*}{w}(1 - \beta)t} \]  \hspace{1cm} (27)

The trace is negative if the numerator of (27) is negative, which is the case if

\[ S^* > \frac{1}{2} \left( K - \frac{w}{\alpha(1 - \beta)t} \right) = S_3 \]

As the determinant is positive, \( S^* > S_3 \) implies in conjunction with (26) that the real parts of all eigenvalues are negative, which implies that the equilibrium is stable. \( S^* < S_3 \) implies that the real parts of all eigenvalues are positive, which implies that the equilibrium is unstable. \( S^* = S_3 \) implies that the trace equals zero. As the determinant is positive and the trace equals zero, the real parts of all eigenvalues are zero while both imaginary parts are nonzero by (26), which gives rise to a centre. \( \square \)
As $S^*$ depends on the tax rate, the question arises about what is the impact of the tax rate on local stability. A necessary condition for instability is $S_3 > S^* > 0$, which implies $S_3 > 0$. As $S_3 \to -\infty$ for $t \to 0$ and $S_3$ rises as the tax rate increases, this implies that instability can arise if $t > w/(\alpha K(1 - \beta))$. However, conditions can be stated for the existence of unstable equilibria that are both necessary and sufficient.

**Proposition 4.2.** An equilibrium which represents an interior solution to the system (12)-(13) has the following properties.

1. It is locally stable for all tax rates $t \in [0, t^K]$ if $-1 < (b - d)/(\alpha \beta K \phi) < -3 + 2\sqrt{2}$.
2. If $-3 + 2\sqrt{2} < (b - d)/(\alpha \beta K \phi) < 0$, two values $t_1$ and $t_2$ of the tax rate exist with $0 < t_1 < t_2 < t^K$ and the following properties.
   (a) The equilibrium is locally stable if $0 \leq t < t_1$ or $t_2 < t < t^K$.
   (b) It is unstable if $t_1 < t < t_2$.
   (c) It gives rise to a Hopf bifurcation with respect to the tax rate at $t = t_1$ and $t = t_2$.
3. If $(b - d)/(\alpha \beta K \phi) = -3 + 2\sqrt{2}$, the system is locally stable for all $t$ with $0 \leq t < t^K$ and $t \neq t^K/2$.

**Proof.** As was established in Proposition 4.1, an equilibrium is stable if $S^* > S_3$. This inequality can be reformulated using (16) and (23), which yield the following condition (details see Full Mathematical Workings):

\[
-K\frac{\alpha}{w}(1 - \beta)(d - b)t^2 + [K\phi\alpha\beta - (d - b)]t - \frac{w\phi\beta}{1 - \beta} < 0 \tag{28}
\]

The left-hand side of inequality (28) is equal to the difference $S_3 - S^*$; graphically, it represents a parabola in $(S_3 - S^*, t)$-space which is open from below. Therefore, inequality (28) is fulfilled for any tax rate if the equation $S_3 - S^* = 0$ has no real solutions. This is the case if its discriminant $D$ is negative:

\[
D = (K\phi\alpha\beta)^2 + (d - b)^2 - 2K\phi\alpha\beta(d - b) - 4K\phi\alpha\beta(d - b) < 0 \tag{29}
\]

With $d - b := z$, $D$ equals zero for

\[
z_{1,2} = K\phi\alpha\beta \frac{6 \pm \sqrt{36 - 4}}{2} = (3 \pm 2\sqrt{2})K\phi\alpha\beta \tag{30}
\]

$D < 0$ if $z_2 < z < z_1$, which implies

\[
D < 0 \iff -3 - 2\sqrt{2} < \frac{b - d}{K\phi\alpha\beta} < -3 + 2\sqrt{2} \tag{31}
\]
From (20), the condition $S^* < K$ can be rewritten as

$$(d - b) - \phi \alpha \beta K + \frac{\alpha}{w} (1 - \beta) K (d - b) t < 0 \quad (32)$$

As the third term in (32) is positive, the sum of the first two terms has to be negative, which implies

$$\phi \alpha \beta K - (d - b) > 0 \quad (33)$$

$$\frac{(b - d)}{\phi \alpha \beta K} > -1 \quad (34)$$

Inserting this result into (31) gives

$$D < 0 \iff -1 < \frac{(b - d)}{K \phi \alpha \beta} < -3 + 2 \sqrt{2} \quad (35)$$

This establishes the first part of Proposition 4.2. Condition (33) implies that the coefficient of $t$ in (28) is positive. As the coefficient of $t^2$ is negative and the constant term $-w \phi \beta / (1 - \beta)$ is negative, too, the equation $S_3 - S^* = 0$ has two positive real solutions if $D > 0$. The solutions are denoted $t_1$ and $t_2$ with $t_1 < t_2$. As $S_3 - S^*$ represents a parabola which is open from below, the system is stable ($S^* > S_3$) if $t < t_1$ or $t > t_2$. In contrast, it is unstable ($S^* < S_3$) if $t_1 < t < t_2$. The difference $t^K - t_2$ can be computed by setting the left-hand side of (28) equal to zero and solving for $t_2$, which yields (details see Full Mathematical Workings)

$$t^K - t_2 = \frac{(b - d + \alpha \beta K \phi - \sqrt{(b - d)^2 + (\alpha \beta K \phi)^2 + 4 \alpha \beta (b - d) K \phi} w)}{2 \alpha (-1 + \beta)(b - d) K} \quad (36)$$

With (33), $b - d + \alpha \beta K \phi > 0$. As $b - d < 0$, the term $\sqrt{()}$ is less than the sum of the first two terms in the numerator, which implies that $t^K > t_2$. At $t = t_1, t_2$, the system has two purely imaginary eigenvalues while the first derivative of its trace with respect to $t$, evaluated at $t = t_1, t_2$ is different from zero, which causes a Hopf bifurcation to arise (details see Full Mathematical Workings). This establishes the second part of Proposition 4.2. If $(b - d) / (\alpha \beta K \phi) = -3 + 2 \sqrt{2}$, the parabola which represents $(S_3 - S^*)$ as a function of $t$ has a repeated zero value, which can be computed from (28) as follows:

$$t = \frac{b - d + K \phi \alpha \beta}{2K \phi^2 (1 - \beta)(d - b)} = \frac{1}{2} t^K \quad (37)$$
Figure 2: Stability under alternative tax rates: an illustration of Proposition 4.2

Graphically, the parabola touches the $t$-axis in $t = t^K/2$ (which results in a centre) but remains below the axis for all other $t$ (which results in a stable focus). This establishes the third part of the Proposition.

Figure 2 shows a graphical illustration in $(S,t)$-space. The function $S = S^*(t)$ is a hyperbola which rises at increasing rates in the interval $[0,t^K]$. The function $S = S_3(t)$, in contrast, increases at decreasing rates in the same interval. The two curves do not intersect if condition (35) holds; in this case, $S^*$ is larger than $S_3$ for any tax rate in the interval. If $D > 0$, a Hopf bifurcation occurs as the tax rate is raised above $t_1$ or lowered below $t_2$. Simulation results presented in Section 5.1 demonstrate that the bifurcation at $t_1$ is supercritical, as it gives rise to a stable limit cycle.

The conditions stated in Proposition 4.2 demonstrate the importance of population dynamics, carrying capacity, labour productivity, and consumer preferences for the stability properties of the model. At a given carrying capacity, instability can arise if the responsiveness of population growth to resource availability increases, labour productivity rises, or consumers value the resource more highly.
4.3 Monotonic or Cyclical Adjustment?

The system (12)-(13), linearized at its equilibrium \((S^*, L^*)\), cyclically adjusts to its equilibrium (i.e. its eigenvalues are complex) if the discriminant of (26) is negative. This condition can be expressed as (see attached Mathematica file for details)

\[
r < \frac{4\alpha \beta K \phi w[(1 - \beta)(d - b)t - \beta \phi w]^2[(-\alpha(1 - \beta)(d - b)K + (b - d + \alpha \beta K \phi)w + \beta \phi w^2] := f(t)}{(d - b)[\alpha(-1 + \beta)^2(d - b)K + (b - d + \alpha \beta K \phi)w + \beta \phi w^2]^2}
\]

(38)

It can be shown that a system which adjusts cyclically to its long-run equilibrium values can always be transformed into a system that adjusts monotonically by choice of a sufficiently high tax rate. In other words:

**Proposition 4.3.** Let \(\Delta(t)\) with \(t \in [0, t^K]\) be the discriminant to the characteristic equation of the system (12)-(13), linearized at its equilibrium \((S^*, L^*)\). For any \(r \leq f(t = 0)\), a tax rate \(t^* \in [0, t^K]\) exists with \(\Delta(t > t^*) > 0\).

**Proof.** Consider the largest value of \(t\) which satisfies the condition \(r = f(t)\) at any chosen value of \(r \leq f(t = 0)\). Denote this value with \(t^*\). A sufficient condition for \(\Delta(t > t^*)\) to be positive is that \(f(t)\) is strictly decreasing in the interval \([t^*, t^K]\).

The numerator of \(f(t)\) is a polynomial in \(t\) of degree three. It has a single zero value in \(t = t^K\) and a repeated zero value in \(t^S = \beta \phi w/[(1 - \beta)(d - b)] > t^K\). As the coefficient of \(t^3\) is negative, the numerator tends to infinity for \(t \to -\infty\). This implies that the numerator is strictly positive in the interval \([-\infty, t^K]\).

The denominator can be written in the form \((d - b)\delta^2\) where \(\delta\) represents the term in square brackets. The denominator has no zero values if the discriminant of \(\delta = 0\) is negative:

\[
(b - d)^2 + 6\alpha \beta (b - d)K \phi + (\alpha \beta K \phi)^2 < 0
\]

(39)

Inequality (39) is identical with inequality (29). Hence, the discriminant is negative if \(-1 < \frac{b-d}{\alpha \beta \phi} < -3 + 2\sqrt{2}\); it is positive if \(-3 + 2\sqrt{2} < \frac{b-d}{\alpha \beta \phi} < 0\).

**Case (a): Discriminant is negative.** In this case, \(f(t)\) is continuous and differentiable for all \(t\). The denominator of \(f(t)\) is a polynomial of degree four and strictly positive for all tax rates. As the numerator is a polynomial of degree three with the coefficient of \(t^3\) being negative, \(f(t)\) converges to zero from above for \(t \to -\infty\); it converges to zero from below for \(t \to \infty\).
By Rolle’s theorem, at least one local extremum must exist in each of the intervals \( (-\infty, t^K], [t^K, t^S], \) and \([t^S, \infty)\).

The first derivative of \( f \) with respect to \( t \), \( f'(t) \), is a fraction the numerator of which is a polynomial of degree four (see attached Mathematica file for details). Hence, \( f'(t) \) cannot possess more than four zero values. Furthermore, \( f(t^S) = f'(t^S) = 0 \). This implies that exactly one local maximum exists in the interval \( (-\infty, t^K] \). If the maximum is at \( t = t^M < 0 \), \( f(t) \) is strictly decreasing in \([0, t^K]\). As \( t^* \in [0, t^K] \), this implies that \( f(t) \) is strictly decreasing in the interval \([t^*, t^K]\) for any value of \( r \leq f(0) \).

If the local maximum is at \( t^M > 0 \), \( f(t) \) rises for \( 0 \leq t < t^M \) and then strictly decreases. In this case, \( t^* \) is greater than \( t^M \) for all values of \( r \leq f(t = 0) \), which can be shown by contradiction. Suppose a value of \( t^* < t^M \) exists. As \( f(t) \) is strictly increasing in the interval \([0, t^M]\), \( f(t^*) > f(0) \). However, \( f(t^*) \) simultaneously has to satisfy the condition \( f(t^*) = r \leq f(0) \), which contradicts the former inequality. This implies that \( f(t) \) is strictly decreasing in the interval \([t^*, t^K]\) for any value of \( r \leq f(0) \).

**Case (b): Discriminant is positive.** In this case, the denominator of \( f(t) \) has two distinct (and repeated) zero values, which are identical with \( t_1 \) and \( t_2 \) from Proposition (4.2) with \( 0 < t_1 < t_2 < t^K \) (see attached Mathematica file for details).

As the numerator is positive for all \( t \in [0, t^K] \) and the denominator is positive except for its two zero values, \( f(t) \) tends to infinity as \( t \) approaches \( t_1 \) or \( t_2 \). This implies that \( f(t) \) is strictly decreasing in \([t_2, t^K]\). As the chosen value of \( r \leq f(0) \) is finite and \( 0 < f(t) < \infty \) \( \forall t \in [t_2, t^K] \), the corresponding value of \( t^* \) has to be in the interval \([t_2, t^K]\). This implies that \( f(t) \) is strictly decreasing in the interval \([t^*, t^K]\) for any value of \( r \leq f(0) \).

The implications of Proposition 4.3 are illustrated in Figure 3. Two cases can be distinguished. In case (a) where \(-1 < (b - d)/(\alpha \beta K \phi) < -3 + 2\sqrt{2} \), \( f(t) \) is continuous in \([0, t^K]\). If \( r < f(t = 0) \), the tax rate has to be raised above \( t^* \) to achieve monotonous adjustment. In case (b) where \(-3 + 2\sqrt{2} < (b - d)/(\alpha \beta K \phi) < 0 \), \( f(t) \) possesses a local minimum at \( t = t^M \) in the interval \([t_1, t_2]\). In this case, the system can behave monotonically even if the tax rate is set below \( t^* \) if \( f(t^M) < f(t = 0) \). However, setting the tax rate between \( t_1 \) and \( t_2 \) will cause instability. If \( r \) is chosen to be greater than \( f(t = 0) \), a small increase in the tax rate above zero can move the system from monotonous to cyclical adjustment, which can be reversed if the tax rate is further increased.

These results show that natural and cultural factors interact in complex ways. In the absence of resource taxes, it is nature who determines the mode of adjustment: economies which are gifted with fast growing resources \((r > f(t = 0)) \) adjust monotonically while those who depend on slow growing resources are forced into cyclical processes that may entail severe declines.
Figure 3: Adjustment processes under alternative tax rates: an illustration of Proposition 4.3
in the population associated with violent social conflicts. Economies who are able to levy resource taxes, in contrast, can overcome the restrictions imposed by their dependence on slow growing resources and set themselves on a trajectory of monotonous adjustment.

4.4 The Impact of the Tax Rate on the Resource Stock, Population, and Welfare

4.4.1 Steady State Resource Stock and Population

As can be seen from (16), the steady state resource stock unambiguously increases as the tax rate rises above zero. As was shown in Section 4.1, the existence of a positive population level at equilibrium requires that \( t \) is less than \( t^K \).

The impact of the tax rate on the steady state population level \( L^* \) in (17) is ambiguous, as it is the result of two contradictory forces. First, as (10) shows, the imposition of a resource tax reduces the harvest level at any given resource stock. This reduces the harvest per capita in the short run, which causes the population to shrink. Second, the reduction in the quantity harvested raises the equilibrium level of the resource stock, which, according to (10), increases the harvest at any given tax rate. This causes the harvest per capita to rise and the population to expand. The overall outcome depends on the relative strength of the two effects, which in turn depends on whether the tax raises the own rate of growth of the resource.

**Proposition 4.4.** Consider a solution \((S^*, L^*)\) of the system (14)-(15). A marginal increase in the tax rate \( t \) raises the steady state population level \( L^* \) if \( S^* < K/2 \). It reduces the steady state population level if \( S^* > K/2 \).

**Proof.** A marginal increase in the tax rate changes the equilibrium population by (details see Full Mathematical Workings)

\[
\frac{dL^*}{dt} = r\left[1 + \frac{wS^* (1 - \beta)t}{\vartheta w}(1 - \beta)S^* \left(1 - \frac{2S^*}{K}\right)\right]
\]

From (40), \( dL^*/dt \) is positive if \( S^* < K/2 \) and negative if \( S^* > K/2 \). \( \square \)

If \( S^* < K/2 \), a marginal increase in the tax rate raises the own rate of growth of the resource. As the aggregate resource harvest cannot exceed the own rate of growth of the resource at equilibrium and the harvest per capita is constant for all tax rates, an increase in the equilibrium resource stock can support a larger population only if it raises the own rate of growth of the resource.
4.4.2 Welfare Effects

This section considers static welfare effects that are caused when a marginal increase in the tax rate moves the economy from one equilibrium to another. The transition process is disregarded.

**Individual welfare.** By inspection of (15), it can be seen that population dynamics fixes the harvest per capita at $H^*/L^* = (d - b)/\phi$ through the condition $\dot{L} = 0$. Hence, an increase in the tax rate cannot increase utility by raising the individual consumption of the resource. However, by increasing the equilibrium resource stock, it reduces the cost of harvesting. As labour is freed to move to the manufactured goods sector, individual consumption of manufactured goods is larger at the new equilibrium.

**Proposition 4.5.** If interior equilibrium solutions to the system (12)-(13) are compared, a marginal increase in the tax rate unambiguously increases individual welfare as measured by the utility function (3).

*Proof.* The effect of a marginal change in the tax rate on individual welfare is (details see Full Mathematical Workings)

$$\frac{du}{dt} = h^\beta (1 - \beta) m^{-\beta} (1 - \beta) h = u(1 - \beta)^2 \frac{h}{m} > 0 \quad (41)$$

□

**A utilitarian welfare function.** A simple way to define a utilitarian welfare function is to multiply individual welfare as defined in Equation (5) with the population size:

$$U = L \cdot u = L h^\beta m^{1-\beta} \quad (42)$$

A marginal increase in the tax rate has two effects on utilitarian welfare. First, it unambiguously raises individual welfare $u$, as was shown in Proposition 4.5. Second, it raises or lowers population size depending on whether $S^*$ is smaller or greater than $K/2$, as was shown in Proposition 4.4. This implies that the impact of a marginal tax increase on utilitarian welfare is ambiguous (see Full Mathematical Workings for a proof).
5 Simulation Results

In the model presented above, the historic evolution of renewable resource dependent preindustrial economies can be interpreted as a process of adjustment of the population level and the resource stock towards their long-run equilibrium values. If \( r < f(t) \) at the prevailing tax rate \( t \), adjustment is cyclical, i.e. a "feast and famine" pattern prevails. If \( r \geq f(t) \), the economy follows a time path of monotonic expansion, provided the initial population is small and the equilibrium is stable. This section applies the model to two cases: the !Kung San hunter-gatherers in Botswana and (counterfactually) to Easter Island. The simulations were undertaken with the software package *Mathematica 3.0.*

5.1 The !Kung San in Southern Africa

One of the most extensively researched hunter-gatherer societies is the community of the !Kung San, who live on the north-western fringe of the Kalahari desert in southern Africa (Marshall 1960, 1961, 1976; Lee 1979; Bieseke, Gordon and Lee 1986). Their area of settlement extends from the northern parts of Botswana and Namibia to the south of Angola. In this section, the model is applied to the Dobe !Kung, a sub-group of the !Kung San who resides in north-western Botswana.

Two observations suggest that the Dobe !Kung economy experienced a pattern of long-term expansion towards an equilibrium. First, evidence indicates that the !Kung have lived in this area for a very long time (see Thomas and Shaw (1991) for an overview). In the area of the Dobe !Kung, stone tools were found that have been dated to at least 11,000 years Before Present (B.P.) (Lee 1979, p.76). Excavations in other !Kung areas revealed that microlithic technology was used as early as in 19,000 B.P. (Robbins and Campbell 1988). In other parts of the Kalahari, Early Stone Age artifacts were found that date back to 500,000-100,000 years B.P. (Ebert et al. 1976).

Second, animal bones and teeth found at archaeological sites indicate that the composition of prey species around 12,000 B.P. was similar to the pattern identified by social anthropologists in the late 1960s (Yellen et al. 1987). Third, although interaction with pastoralists is documented for the time since 720 A.D. (Denbow 1984, pp.181-182), the !Kung remained a predominantly hunting and gathering society until the 1970s. No evidence has been discovered that indicates the occurrence of a "feast and famine" pattern.

As far as large game is concerned, the most important animals hunted were wildebeest (*connochaetes taurinus*), gemsbok (*oryx gazella*), and kudu (*tragelaphus strepsiceros*) (Lee 1979, p.227). To keep the analysis tractable, only wildebeest hunting is considered in the simulation.\(^4\) The logistic growth

\(^4\)What matters in the model is the responsiveness of human population growth to
model is applicable as an approximation to wildebeest population dynamics because wildebeest is a migratory species that is not limited by non-human predators but by the available plant biomass. As the wildebeest consumes the annual growth of grass in a particular location and then moves on to other areas, the reproductive capacity of the grass is not impaired by their consumption. Spinage and Mathare (1992) applied the logistic growth model to wildebeest populations in the Botswana part of the Kalahari. They estimate an intrinsic growth rate of \( r = 0.217 \) and a carrying capacity of approximately 100,000 animals for their study area of 150,000 \( km^2 \) in the south-west of Botswana, which gives a carrying capacity of 0.66 \( animals/km^2 \). The area inhabited by the Dobe !Kung is estimated by Lee (1979, p.41) as 9,000 \( km^2 \). Applying a unit weight of 123 \( kg/animal \) (Coe, Cumming and Phillipson 1976, p.346) yields a carrying capacity of \( K = 730620 \) \( kg \), which is probably a conservative estimate.

Access to hunting grounds is open to all and sharing takes place according to the rules described in Section 2. Field data indicate that, at an average, one large game is killed per month while a month amounts to 83 hunting (person) days (Lee 1979, pp.227, 260). It is assumed that three quarters of the prey are wildebeest with an average weight of 227 \( kg/animal \) (Lee 1979, p.230). If it is further assumed that the wildebeest stock was in equilibrium and equal to 0.6 \( K \) at the time of the field study, labour productivity in hunting can be computed as \( \alpha = 2.278 \times 10^{-3} \frac{year^{-1}}{ } \) (see Full Mathematical Workings for details). Equation (11) shows that manufactures are produced at any levels of the tax rate and the resource stock; hence, the wage is equal to one. The parameter values are summarized in Table 1. The population size in year zero was assumed to be equal to two individuals while the initial resource stock was assumed to be equal to carrying capacity.

At the prevailing parameter values, the fraction \( (b - d)/(K \phi \alpha \beta) \) is equal to \(-0.167\). Given Proposition 4.2, this implies that instability can arise if the tax rate is between \( t_1 = 0.002 \) and \( t_2 = 0.003 \). Figure 4 shows the graph of function \( f(t) \) defined by Equation (38), which corresponds to case changes in the availability of the resource. If the scarcities of the three species are correlated, this can be reflected in a value of \( \phi \) which is higher than the value one would assume if the biomass of all three species taken together was treated as a single species.


<table>
<thead>
<tr>
<th>Parameter</th>
<th>( r )</th>
<th>( K )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( b )</th>
<th>( d )</th>
<th>( \phi )</th>
<th>( w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dobe !Kung</td>
<td>0.217</td>
<td>730620</td>
<td>0.00171</td>
<td>0.2</td>
<td>2</td>
<td>2.05</td>
<td>0.0012</td>
<td>1</td>
</tr>
<tr>
<td>Easter Island</td>
<td>0.040</td>
<td>12000</td>
<td>0.00001</td>
<td>0.4</td>
<td>2</td>
<td>2.10</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: Values refer to years for the Dobe !Kung but to decades for Easter Island.
Figure 4: Adjustment to long-run equilibrium under alternative tax rates $t$. Note that a logarithmic scale was employed on the vertical axis.

Figure 5: Population time paths: cyclical vs. monotonic adjustment under various tax rates $T$.

Figure 6: Population time paths: Limit cycle vs. monotonic expansion.
(b) in Figure 3 with \( t^K = 0.005 \). At the prevailing intrinsic growth rate of \( r = 0.217 \), the system adjusts cyclically if the tax rate is zero. Adjustment is monotonic, however, if a tax of more than \( t^* = 0.0043 \) units of the composite good per unit of the resource is introduced. Measured as a percentage \( T \) of the resource units harvested at long-run equilibrium, the critical tax rate is \( T^* = 76\% \).6

Figures 5-6 show the evolution of population size over time under various tax rates. At a tax rate of zero \( (T = 0) \), the population reaches a peak of 914 individuals in year 35 and then declines to 350 individuals in year 75. After 300 years of cyclical variation, it stabilizes around its equilibrium value of 529 individuals. A tax rate of \( T = 64\% (t = 0.0035) \) is high enough to ensure stability but lower than the critical tax rate. It dampens the cycles and causes the population to converge to its equilibrium value of 913 individuals. The equilibrium value is now higher than in the case without taxation because the tax raises the own rate of growth of the resource. This is consistent with Proposition 4.4, as the steady state resource stock is greater than half the carrying capacity in the absence of taxation \( (S^*t = 0) = 121,832 \text{ kg} = 0.17 \cdot K < K/2) \). Raising the tax rate to \( T = 79\% \) and beyond eliminates the cycles but reduces the equilibrium population, which eventually falls below the equilibrium value without taxation.

Tax rates between \( t_1 \) and \( t_2 \) generate limit cycles, as Figure 6 shows for an arbitrarily chosen tax rate of \( T = 44\% (t = 0.0023) \). In these cycles, population declines by approximately 30\% during 45 years and then recovers. This is consistent with the emergence of a Hopf bifurcation identified in Proposition 4.2.

The introduction of a tax of \( T = 84\% (t = 0.00484) \) causes a steady expansion of the population towards an equilibrium value of 451, which is close to the number of 457 residents observed by social anthropologists in 1968 (Lee 1979, p.43). The tax rate is consistent with the extent of sharing encountered in the field, given the fact that hunters receive a slightly larger share than all other members of the community because they are allowed to eat parts of the prey immediately after the killing.

Two conclusions emerge. First, high implicit tax rates (> 76\%) are required to avert a "feast and famine" pattern. Second, egalitarianism in immediate-return hunter-gatherer societies can be explained as a behavioural adaptation to the dynamics of large game and the human population. Strong cyclical fluctuations of population and output at small or zero tax rates and the emergence of limit cycles at intermediate tax rates may have triggered learning processes which led to the establishment of the high tax rates that were observed in the field. In addition, learning processes

---

6Recall that the marginal tax rate is constant only in terms of the manufactured good. Measured in terms of renewable resource units, it varies as the resource price varies. Measured in terms of resource units, the tax rate can be computed with (3) as \( T := t/p = taS/(w + taS) \). At long-run equilibrium, \( T = T^* \) is evaluated at \( S = S^* \).
may have been triggered by the extinction of very large mammals due to excessive hunting (Smith 1975, 1993) in an earlier phase of the Paleolithic era, i.e. before 12,000 B.P. The discovery of the teeth of the now extinct giant buffalo (*homoeceras bainii*) at an archaeological site in the Dobe area (Yellen 1971) is indicative of this.

### 5.2 Easter Island

Apart from hunter-gatherer economies, the model presented in Section 3 can yield insights into the evolution of agricultural civilizations. Some of these civilizations experienced patterns of rise followed by a catastrophic decline. For example, Brander and Taylor (1998) argue that the collapse of the Easter Island civilization has been caused by its dependence on a palm tree species with a low intrinsic growth rate. Easter Island was settled in about 400 A.D. (see Brander and Taylor 1998, pp.121-122). Its population rose to a maximum of approximately 10,000 inhabitants in 1400 A.D. and then declined dramatically. When James Cook visited the Island in 1774, its population had shrunk to some 3,000 inhabitants. At the time of settlement, the island was covered by a large palm forest, which was cleared by the rising population until it almost vanished in about 1400 A.D. Most of the famous stone statues on the island were carved between 1100 and 1500 A.D. when the population was rising and the resource was still plentiful. Stone carving disappeared after the great population decline.

Simulations by Brander and Taylor (1998) revealed that their model can replicate the historic patterns of growth and decline of the population and the palm forest stock on Easter Island. This section demonstrates that this outcome could have been avoided by implicit taxation. Implicit taxation is relevant in the Easter Island context because sharing rules are still common on many islands in Oceania, which is possibly a consequence of hunter-gatherer traditions. For example, a rule exists on the island of Lofanga in the Kingdom of Tonga that prescribes to each fisherman to share his catch with all other members of the island’s population (Bender, Kägi, and Mohr 2001). The resulting implicit tax rate is approximately 45% (Chakrabarty 2001a).

In order to ensure comparability with the results of Brander and Taylor (1998), the simulations are based on an identical set of values for the relevant parameters, which is summarized in Table 1.\(^7\) As far as initial conditions are concerned, population was assumed to be equal to 40 individuals in year 400 A.D. while the resource stock was assumed to be equal to carrying capacity.

At the prevailing parameter values, the fraction \( (b - d)/(K\phi\alpha\beta) \) was calculated to be equal to -0.52. Given Proposition 4.2, this implies that the system is stable for all tax rates in the interval \([0, t^K = 12.8]\). Figure 7 shows

\(^7\)In this paper, individual values are specified for the birth rate \(b\) and the death rate \(d\) while Brander and Taylor specify only a net birth rate of \(b - d = -0.1\).
Figure 7: Adjustment to long-run equilibrium under alternative tax rates $t$

Figure 8: Population size: time paths under alternative tax rates $T$

Figure 9: The impact of imposing a tax of $T = 50\%$ after 850 A.D. on the time paths of population $L(t)$ and the resource stock $S(t)$
the function \( f(t) \) defined by Equation (38), the shape of which corresponds to case (a) in Figure 3. It can be seen that adjustment is strictly monotonous in the absence of resource taxation if the intrinsic growth rate is greater or equal to 0.71, which confirms the results of Brander and Taylor. With the intrinsic growth rate of \( r = 0.04 \) that prevailed on Easter Island, however, the system adjusts strictly monotonically if a tax of more than \( t^* = 10.9 \) units of the composite good per unit of the resource is introduced. Measured as a percentage \( T \) of the resource units harvested at long-run equilibrium, the tax rate becomes \( T^* = 54\% \), which is in the same order of magnitude as the implicit tax rates found in contemporary Tonga.

Figure 8 shows the time paths of the population size under various tax rates. At a tax rate of zero, the population reaches its peak of 10,161 inhabitants in 1200 A.D. and then drops to 3,325 in 1770 A.D., which roughly corresponds to historic evidence. The introduction of a tax has a threefold impact. First, cycles become less pronounced as the tax rate rises. Although the adjustment process is strictly monotonous only at a tax rate above \( T = 54\% \), the cyclical downswing is small already at a tax rate of 46%; the population peaks in 4380 A.D. at a level of 3,394, which exceeds its equilibrium value of 3,228 by five per cent. With a tax rate of 50\%, the peak value exceeds the equilibrium value by less than one per cent.

Second, the tax delays the adjustment process in the sense that the population reaches its peak later. At a tax rate of zero, the population peaks in 1200 A.D. A tax rate of \( T = 28\% \), however, shifts the peak to 2040 A.D. while a tax rate of 50\% delays it to 6380 A.D. Third, the tax lowers the equilibrium value of the population and raises the equilibrium value of the resource stock. This is consistent with Proposition 4.4, as the steady state resource stock is greater than half the carrying capacity at a tax rate of zero \( (S^*(t = 0) = 6250 > 6000 = K/2) \).

Could the collapse of the Easter Island population have been avoided? Figure 9 illustrates two alternative scenarios for the time paths of the population \( L(t) \) and the resource stock \( S(t) \). Both are identical in the time interval between 400 A.D. (initial settlement) and 850 A.D. It is assumed that resource taxation is absent during this interval. The two scenarios diverge in the following periods. In the base case (dashed curves), resource taxation continues to be absent. In the other scenario, a tax of \( T = 50\% \) is adopted after 850 A.D., i.e. 350 years before the historic collapse started. Although adjustment is not strictly monotonous, the decline in population is small: population peaks in 1860 A.D. at a level of 2,785 and subsequently converges to its equilibrium value of 2,667. The peak level exceeds the equilibrium value by less than five per cent.

It would not have been possible to delay the adoption of the tax significantly. For a delay would require a higher tax rate to dampen the cyclical behaviour of the system. A higher tax rate, however, lowers the equilibrium value of the population, which accentuates the population decline as
measured from the time of the regime change. For example, if a tax of 54% was adopted in 900 A.D., population would shrink from its peak value of 3,188 in 940 A.D. to its equilibrium value of 1,931, which implies that the equilibrium value would be exceeded by 65%.

6 Conclusion

The preceding analysis demonstrated that an egalitarian culture supports renewable resource conservation and can set the economy on a time path of monotonic expansion when it (implicitly) taxes resource harvesting.

A more general result is that institutions which have not consciously been designed to conserve natural resources may have a strong positive impact on conservation. The debate on the management of renewable resources in developing countries has strongly focused on policies that limit access to natural resources to particular social actors during particular periods of time (Ostrom 1990, Madsen 1999, Jeffery and Vira 2001). It appears to be a promising line of both empirical and theoretical research to analyze the impact of institutions that have an indirect impact on natural resource use.

7 References


