Economics 245 — Fall 2021

International Trade

Problem Set 2

October 14, 2021

Due:	**Tue, November 2, 2021**
Instructor:	Marc-Andreas Muendler
E-mail:	muendler@ucsd.edu

1 Properties of the Pareto and Fréchet Distributions

Consider a Pareto distributed variable $\phi \sim \mathcal{P}(\phi_0, \theta)$, where ϕ_0 is called the location parameter and θ the shape parameter. The Pareto distribution function is $F(Z \leq \phi) = 1 - (\phi_0/\phi)^\theta$.

1. Show that the Pareto density function is $\mu(\phi|\phi_0, \theta) = \theta(\phi_0)^\theta/(\phi)^{\theta+1}$.

2. For any $\phi^* > \phi_0$, show that the conditional distribution function is: $F(\phi|\phi \geq \phi^*) = 1 - (\phi^*/\phi)^\theta$, also a Pareto distribution function.

3. Consider a transformed random variable $A(\phi)^B$ with $A, B > 0$. Show that the transformed variable is Pareto distributed with location parameter $A\,(\phi_0)^B$ and shape parameter θ/B.

4. Show that the mean of a Pareto distributed variable ϕ is $\mathbb{E}\,[\phi|\phi_0, \theta] = \theta\phi_0/(\theta - 1)$ if $\theta > 1$.

5. Consider the Fréchet distribution $G(Z \leq z) = \exp\{-Tz^{-\theta}\}$. Show that the Fréchet distribution approaches the Pareto distribution "in the right tail", that is show that

$$\lim_{z\to\infty} G(Z \leq z) = \lim_{z\to\infty} 1 - Tz^{-\theta}.$$

   (*Hint:* Use L'Hôpital's rule and the fact that $\lim_{z\to\infty} z^{-\theta} = 0$ for $\theta > 0$.)

2 Chaney (AER 2008) and Gravity

There are N countries. A country carries a subscript s if it is the source of exports, and a subscript d if it is a destination ($s, d = 1, \ldots, N$). In each country lives a measure of L_d consumers, who inelastically supply one unit of labor and own the shares of domestic firms. Firms choose to enter their respective home market s and any export destination d. There are source-destination iceberg transportation costs τ_{sd} between countries, and there is a source-destination specific fixed cost of entry F_{sd}.

The L_d representative consumers have identical CES preferences over a continuum of varieties with

$$U_d = \left[\sum_{s=1}^{N} \int_{\omega \in \mathbf{\Omega}_{sd}} c(\omega)^{\frac{\sigma-1}{\sigma}} \mathrm{d}\omega \right]^{\frac{\sigma}{\sigma-1}} \qquad \text{for } \varepsilon = \sigma > 1.$$

$\mathbf{\Omega}$ is the (fixed) measure of available varieties, given a predetermined measure of firms J_s in every country that sells varieties.

Each firm ω's production technology is constant returns to scale but firms from country s differ in productivity ϕ, which they draw from a Pareto distribution $F(\phi) = 1 - (b_s/\phi)^{\theta}$. It will be convenient to call all firms ω with a given productivity level the firms ϕ.

1. Show that consumer demand for a firm ϕ's variety $c_{sd}(\phi)$ is

$$c_{sd}(\phi) = \frac{(p_{sd})^{-\sigma}}{(P_d)^{1-\sigma}} \, y_d L_d$$

for the ideal price index

$$P_d \equiv \left(\int_{\phi \in \mathbf{\Omega}_{sd}} p_{sd}^{1-\sigma} \, \mathrm{d}\phi \right)^{\frac{1}{1-\sigma}}.$$

2. Upon entry, each firm maximizes operational profits

$$\pi_{sd}(\phi) = \left(p_{sd} - \tau_{sd} \frac{w_s}{\phi} \right) \left(\frac{p_{sd}}{P_d} \right)^{-\sigma} \frac{y_d L_d}{P_d} - F_{sd}.$$

Show that optimal price is a constant markup over unit production cost with

$$p_{sd}(\phi) = \eta \, \frac{\tau_{sd} \, w_s}{\phi}, \qquad \eta = \frac{\sigma}{\sigma - 1}.$$

3. Derive a firm ϕ's total sales $p_{sd}(\phi) q_{sd}(\phi)$ and show that optimal operational profits satisfy $\pi_{sd}(\phi) = p_{sd}(\phi) q_{sd}(\phi)/\sigma$. Using results from Question 1 show that total sales $p_{sd}(\phi) q_{sd}(\phi)$ are Pareto distributed.

4. Show that the least productive firm from country s with a productivity

$$\phi^*_{sd} = \left(\frac{\sigma F_{sd}}{y_d L_d} \right)^{\frac{1}{\sigma-1}} \frac{\eta \tau_{sd} w_s}{P_d}.$$

just breaks even in destination market d. Explain the intuition.

5. Using results from Question 1, derive aggregate exports

$$X_{sd} = \int_{\phi^*_{sd}}^{\infty} p_{sd}(\phi) q_{sd}(\phi) \mu(\phi | \phi^*_{sd}, \theta) \, \mathrm{d}\phi$$

from source country s to destination d, where $\mu(\phi|\phi^*_{sd}, \theta)$ is the conditional Pareto density for firms from s active at d. State X_{sd} as a function of τ_{sd} and ϕ^*_{sd}. Using the result for ϕ^*_{sd} from above, simplify further.

6. Derive the elasticity of aggregate trade with respect to variable trade costs $\partial \log X_{sd} / \partial \log \tau_{sd}$, using the expression of X_{sd} as a function of τ_{sd} and $\phi^*_{sd}(\tau_{sd})$. Interpret the two terms.

7. Derive the elasticity of aggregate trade with respect to fixed trade costs $\partial \log X_{sd} / \partial \log F_{sd}$.

3 Arkolakis, Ganapati & Muendler (2021) and Product Entry

As in the question before, there are N countries. A country carries a subscript s if it is the source of exports, and a subscript d if it is a destination ($s, d = 1, \ldots, N$). In each country lives a measure of L_d consumers, who inelastically supply one unit of labor and own the shares of domestic firms. Firms choose to enter their respective home market s and any export destination d. There are source-destination iceberg transportation costs τ_{sd} between countries, and there is a source-destination specific fixed cost of entry $F_{sd}(G_{sd})$ that depends on a firm's exporter scope (number of products) G_{sd} at a destination.

The L_d representative consumers have identical CES preferences over a continuum of firms *and* products. The lower tier of the utility aggregate is a firm's unique product mix ("variety")

$$U_d = \left[\sum_{s=1}^{N} \int_{\omega \in \boldsymbol{\Omega}_{sd}} \left(\int_1^{G_{sd}(\omega)} q_{sd}(g, \omega)^{\frac{\varepsilon - 1}{\varepsilon}} \, \mathrm{d}g \right)^{\frac{\varepsilon}{\varepsilon - 1} \frac{\sigma - 1}{\sigma}} \mathrm{d}\omega \right]^{\frac{\sigma}{\sigma - 1}} \quad \text{with } \varepsilon \neq \sigma \text{ and } \varepsilon, \sigma > 1.$$

In its product mix, a firm has a continuum of $g \in [1, G_{sd}]$ products at destination market d.

Each firm ω's production technology is constant returns to scale but firms from country s differ in productivity ϕ, which they draw from a Pareto distribution $F(\phi) = 1 - (b_s/\phi)^\theta$. It will be convenient to call all firms ω with a given productivity level the firms ϕ. For each destination market d, a firm chooses its specific exporter scope $G_{sd}(\phi)$. In production, a firm faces constant marginal cost for each product that it adopts.

1. Show that consumer demand for an individual product $q_{sd}(g, \phi)$ is

$$q_{sd}(g, \phi) = (p_{sd}(g))^{-\varepsilon} \, \frac{P_{sd}(\phi; G_{sd})^{\varepsilon - \sigma}}{(P_d)^{1 - \sigma}} \, y_d L_d$$

for the ideal price indexes

$$P_d \equiv \left(\sum_{s=1}^{N} \int_{\phi \in \mathbf{\Omega}_{sd}} P_{sd}^{1-\sigma} \, \mathrm{d}\phi \right)^{\frac{1}{1-\sigma}} \quad \text{and} \quad P_{sd} \equiv \left(\int_1^{G_{sd}(\phi)} p_{sd}(g,\phi)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}.$$

Hint: The first-order conditions imply for Marshallian demand of a firm's product-mix

$$X_d \equiv \left(\sum_{s=1}^{N} \int_{\phi \in \mathbf{\Omega}_{sd}} X_{sd}(\phi; G_{sd})^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}\phi \right)^{\frac{\sigma}{\sigma-1}} \text{ and } X_{sd}(\phi; G_{sd}) \equiv \left(\int_1^{G_{sd}(\phi)} q_{sd}(g,\omega)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}g \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

2. Show that $P_{sd}(\phi; G_{sd})$ strictly decreases in exporter scope G_{sd}.

3. *Cannibalization.* Show that scope diminishes infra-marginal shipments $q_{sd}(g,\phi)$ and infra-marginal scale $p_{sd}(g,\phi)q_{sd}(g,\phi)$ if and only if $\varepsilon > \sigma$.

4. Upon entry in market d, each firm maximizes operational profits

$$\pi_{sd}(\phi) = \int_1^{G_{sd}(\phi)} \left(p_{sd}(g) - \frac{\tau_{sd}\, w_s}{\phi} \right) p_{sd}(g)^{-\varepsilon} \frac{(P_{sd})^{\varepsilon-\sigma}}{(P_d)^{-(\sigma-1)}}\, y_d L_d \, \mathrm{d}g - F_{sd}(G_{sd}).$$

Show that optimal price is a constant markup over unit production cost with

$$p_{sd}(g,\phi) = \eta \frac{\tau_{sd}\, w_s}{\phi}, \qquad \eta = \frac{\sigma}{\sigma - 1}.$$

Hint: For this purpose, maximize the firm's constrained Lagrangian objective function

$$\max_{P_{sd}, \{p_{sd}(g)\}_{g \in [1, G_{sd}]}} \pi_{sd}(\phi) + \lambda \left(P_{sd} - \left[\int_1^{G_{sd}} p_{sd}(g)^{-(\varepsilon-1)} \mathrm{d}g \right]^{-\frac{1}{\varepsilon-1}} \right).$$

5. Consider G_{sd} as given. Show that optimal product scale is the same for every product g with

$$p_{sd}(g,\phi)\, q_{sd}(g,\phi) = (G_{sd})^{-\frac{\varepsilon-\sigma}{\varepsilon-1}}\, y_d L_d \left(\frac{\phi\, P_d}{\eta\, \tau_{sd}\, w_s} \right)^{\sigma-1}.$$

Hint: Use the fact that $P_{sd}(\phi; G_{sd}) = (G_{sd})^{1/(1-\varepsilon)} p_{sd}(g,\phi)$. Is constant product scale for every product g realistic? How can the firm's optimization problem be extended to generate a product scale distribution within the firm?

6. Using optimal product sales, show that the profit function becomes

$$\pi_{sd}(\phi) = \frac{(G_{sd})^{\bar{\sigma}}\, y_d L_d}{\sigma} \left(\frac{\phi\, P_d}{\eta\, \tau_{sd}\, w_s}\right)^{\sigma-1} - w_d F_d(G_{sd}) \quad \text{where } \bar{\sigma} \equiv \frac{\sigma-1}{\varepsilon-1}.$$

From now on, suppose the fixed entry costs take the form $F_d(G_{sd}) = \kappa_d + \gamma_d\,(G_{sd})^{\delta+1}/(\delta+1)$ where $\gamma_d > 0$. Fixed costs are paid in destination market wages w_d.

7. Show that the optimal exporter scope for a firm from s shipping to d is

$$G_{sd}(\phi) = \left[\frac{\bar{\sigma}\, y_d L_d}{\sigma\, w_d\, \gamma_d} \left(\frac{\phi\, P_d}{\eta\, \tau_{sd}\, w_s}\right)^{\sigma-1}\right]^{\frac{1}{\delta-(\bar{\sigma}-1)}} \qquad \text{for} \quad G_{sd}(\phi) \geq G_d^* \quad \text{and} \quad \delta > \bar{\sigma}-1.$$

What is the intuition for the condition $\delta > \bar{\sigma} - 1$? Consider the benchmark case of $\bar{\sigma} = 1$ and relate it to the cannibalization effect. What would optimal scope be if the condition were strictly violated with $\bar{\sigma} - 1 > \delta$?

8. Applying the zero-profit condition $\pi_{sd}(\phi) = 0$, show that the minimum optimal scope G_d^* of any firm in country d is

$$G_d^* = \left(\frac{(\delta+1)\,\bar{\sigma}}{\delta-(\bar{\sigma}-1)} \frac{\kappa_d}{\gamma_d}\right)^{\frac{1}{\delta+1}}.$$

Note that minimum scope is independent of source country characteristics and independent of the destination country's size L_d and per-capita income y_d. Is this realistic?

9. Define the productivity threshold for exporting from s to d

$$\phi_{sd}^* = \left(\frac{y_d L_d}{\sigma}\right)^{-\frac{1}{\sigma-1}} \left(\frac{w_d\,\gamma_d}{\bar{\sigma}}\right)^{\frac{1}{\sigma-1}\frac{\bar{\sigma}}{\delta+1}} \left(\frac{(\delta+1)\, w_d\, \kappa_d}{\delta-(\bar{\sigma}-1)}\right)^{\frac{1}{\sigma-1}\frac{\delta-(\bar{\sigma}-1)}{\delta+1}} \frac{\eta\,\tau_{sd}\, w_s}{P_d}.$$

Using G_d^* as the optimal exporter scope $G_{sd}(\phi_{sd}^*)$ for a firm at the productivity threshold, verify that the definition is correct.

10. Show that a firm's optimal exporter scope can be expressed as

$$G_{sd}(\phi) = G_d^* \left(\frac{\phi}{\phi_{sd}^*}\right)^{\frac{\sigma-1}{\delta-(\bar{\sigma}-1)}}.$$

11. Show that a firm's optimal total exports are

$$T_{sd}(\phi) \equiv G_{sd}(\phi)\, p_{sdg}(\phi)\, x_{sdg}(\phi) = \frac{(\delta+1)\,\sigma\, w_d\, \kappa_d}{\delta-(\bar{\sigma}-1)} \left(\frac{\phi}{\phi_{sd}^*}\right)^{(\delta+1)\frac{\sigma-1}{\delta-(\bar{\sigma}-1)}}.$$

12. Using results from Question 1 show that $G_{sd}(\phi)$ and $T_{sd}(\phi)$ are both Pareto distributed.