Economics 247 — Spring 2020

International Trade

Problem Set 2

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Due:	**Mon, May 11, 2020**
Instructor:	Marc-Andreas Muendler
E-mail:	muendler@ucsd.edu

1 Arkolakis & Muendler (2020) and Product Entry

As in the question before, there are N countries. A country carries a subscript s if it is the source of exports, and a subscript d if it is a destination ($s, d = 1, \ldots, N$). In each country lives a measure of L_d consumers, who inelastically supply one unit of labor and own the shares of domestic firms. Firms choose to enter their respective home market s and any export destination d. There are source-destination iceberg transportation costs τ_{sd} between countries, and there is a source-destination specific fixed cost of entry $F_{sd}(G_{sd})$ that depends on a firm's exporter scope (number of products) G_{sd} at a destination.

The L_d representative consumers have identical CES preferences over a continuum of firms *and* products. The lower tier of the utility aggregate is a firm's unique product mix ("variety")

$$U_d = \left[\sum_{s=1}^{N} \int_{\omega \in \mathbf{\Omega}_{sd}} \left(\int_1^{G_{sd}(\omega)} q_{sd}(g,\omega)^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}g \right)^{\frac{\varepsilon}{\varepsilon-1} \frac{\sigma-1}{\sigma}} \mathrm{d}\omega \right]^{\frac{\sigma}{\sigma-1}} \quad \text{with } \varepsilon \neq \sigma \text{ and } \varepsilon, \sigma > 1.$$

In its product mix, a firm has a continuum of $g \in [1, G_{sd}]$ products at destination market d.

Each firm ω's production technology is constant returns to scale but firms from country s differ in productivity ϕ, which they draw from a Pareto distribution $F(\phi) = 1 - (b_s/\phi)^\theta$. It will be convenient to call all firms ω with a given productivity level the firms ϕ. For each destination market d, a firm chooses its specific exporter scope $G_{sd}(\phi)$. In production, a firm faces constant marginal cost for each product that it adopts.

1. Show that consumer demand for an individual product $q_{sd}(g, \phi)$ is

$$q_{sd}(g,\phi) = (p_{sd}(g))^{-\varepsilon} \, \frac{P_{sd}(\phi; G_{sd})^{\varepsilon-\sigma}}{(P_d)^{1-\sigma}} \, y_d L_d$$

for the ideal price indexes

$$P_d \equiv \left(\sum_{s=1}^{N} \int_{\phi \in \boldsymbol{\Omega}_{sd}} P_{sd}^{1-\sigma} \, \mathrm{d}\phi \right)^{\frac{1}{1-\sigma}} \quad \text{and} \quad P_{sd} \equiv \left(\int_1^{G_{sd}(\phi)} p_{sd}(g, \phi)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}.$$

Hint: The first-order conditions imply for Marshallian demand of a firm's product-mix

$$X_d \equiv \left(\sum_{s=1}^{N} \int_{\phi \in \boldsymbol{\Omega}_{sd}} X_{sd}(\phi; G_{sd})^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}\phi \right)^{\frac{\sigma}{\sigma-1}} \text{ and } X_{sd}(\phi; G_{sd}) \equiv \left(\int_1^{G_{sd}(\phi)} q_{sd}(g, \omega)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}g \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

2. Show that $P_{sd}(\phi; G_{sd})$ strictly decreases in exporter scope G_{sd}.

3. *Cannibalization.* Show that scope diminishes infra-marginal shipments $q_{sd}(g, \phi)$ and infra-marginal scale $p_{sd}(g, \phi) q_{sd}(g, \phi)$ if and only if $\varepsilon > \sigma$.

4. Upon entry in market d, each firm maximizes operational profits

$$\pi_{sd}(\phi) = \int_1^{G_{sd}(\phi)} \left(p_{sd}(g) - \frac{\tau_{sd} \, w_s}{\phi} \right) p_{sd}(g)^{-\varepsilon} \frac{(P_{sd})^{\varepsilon-\sigma}}{(P_d)^{-(\sigma-1)}} \, y_d L_d \, \mathrm{d}g - F_{sd}(G_{sd}).$$

Show that optimal price is a constant markup over unit production cost with

$$p_{sd}(g, \phi) = \eta \, \frac{\tau_{sd} \, w_s}{\phi}, \qquad \eta = \frac{\sigma}{\sigma - 1}.$$

Hint: For this purpose, maximize the firm's constrained Lagrangian objective function

$$\max_{P_{sd}, \{p_{sd}(g)\}_{g \in [1, G_{sd}]}} \pi_{sd}(\phi) + \lambda \left(P_{sd} - \left[\int_1^{G_{sd}} p_{sd}(g)^{-(\varepsilon-1)} \mathrm{d}g \right]^{-\frac{1}{\varepsilon-1}} \right).$$

5. Consider G_{sd} as given. Show that optimal product scale is the same for every product g with

$$p_{sd}(g, \phi) \, q_{sd}(g, \phi) = (G_{sd})^{-\frac{\varepsilon-\sigma}{\varepsilon-1}} \; y_d L_d \left(\frac{\phi \, P_d}{\eta \, \tau_{sd} \, w_s} \right)^{\sigma-1}.$$

Hint: Use the fact that $P_{sd}(\phi; G_{sd}) = (G_{sd})^{1/(1-\varepsilon)} p_{sd}(g, \phi)$. Is constant product scale for every product g realistic? How can the firm's optimization problem be extended to generate a product scale distribution within the firm?

6. Using optimal product sales, show that the profit function becomes

$$\pi_{sd}(\phi) = \frac{(G_{sd})^{\bar\sigma}\, y_d L_d}{\sigma} \left(\frac{\phi\, P_d}{\eta\, \tau_{sd}\, w_s}\right)^{\sigma-1} - w_d F_d(G_{sd}) \quad \text{where } \bar\sigma \equiv \frac{\sigma-1}{\varepsilon-1}.$$

From now on, suppose the fixed entry costs take the form $F_d(G_{sd}) = \kappa_d + \gamma_d\,(G_{sd})^{\delta+1}/(\delta+1)$ where $\gamma_d > 0$. Fixed costs are paid in destination market wages w_d.

7. Show that the optimal exporter scope for a firm from s shipping to d is

$$G_{sd}(\phi) = \left[\frac{\bar\sigma\, y_d L_d}{\sigma\, w_d\, \gamma_d} \left(\frac{\phi\, P_d}{\eta\, \tau_{sd}\, w_s}\right)^{\sigma-1}\right]^{\frac{1}{\delta-(\bar\sigma-1)}} \qquad \text{for} \quad G_{sd}(\phi) \geq G_d^* \quad \text{and} \quad \delta > \bar\sigma - 1.$$

What is the intuition for the condition $\delta > \bar\sigma - 1$? Consider the benchmark case of $\bar\sigma = 1$ and relate it to the cannibalization effect. What would optimal scope be if the condition were strictly violated with $\bar\sigma - 1 > \delta$?

8. Applying the zero-profit condition $\pi_{sd}(\phi) = 0$, show that the minimum optimal scope G_d^* of any firm in country d is

$$G_d^* = \left(\frac{(\delta+1)\,\bar\sigma}{\delta-(\bar\sigma-1)}\frac{\kappa_d}{\gamma_d}\right)^{\frac{1}{\delta+1}}.$$

Note that minimum scope is independent of source country characteristics and independent of the destination country's size L_d and per-capita income y_d. Is this realistic?

9. Define the productivity threshold for exporting from s to d

$$\phi_{sd}^* = \left(\frac{y_d L_d}{\sigma}\right)^{-\frac{1}{\sigma-1}} \left(\frac{w_d\,\gamma_d}{\bar\sigma}\right)^{\frac{1}{\sigma-1}\frac{\bar\sigma}{\delta+1}} \left(\frac{(\delta+1)\, w_d\, \kappa_d}{\delta-(\bar\sigma-1)}\right)^{\frac{1}{\sigma-1}\frac{\delta-(\bar\sigma-1)}{\delta+1}} \frac{\eta\, \tau_{sd}\, w_s}{P_d}.$$

Using G_d^* as the optimal exporter scope $G_{sd}(\phi_{sd}^*)$ for a firm at the productivity threshold, verify that the definition is correct.

10. Show that a firm's optimal exporter scope can be expressed as

$$G_{sd}(\phi) = G_d^* \left(\frac{\phi}{\phi_{sd}^*}\right)^{\frac{\sigma-1}{\delta-(\bar\sigma-1)}}.$$

11. Show that a firm's optimal total exports are

$$T_{sd}(\phi) \equiv G_{sd}(\phi)\, p_{sdg}(\phi)\, x_{sdg}(\phi) = \frac{(\delta+1)\,\sigma\, w_d\, \kappa_d}{\delta-(\bar\sigma-1)} \left(\frac{\phi}{\phi_{sd}^*}\right)^{(\delta+1)\frac{\sigma-1}{\delta-(\bar\sigma-1)}}.$$

12. Using results from Question **??** show that $G_{sd}(\phi)$ and $T_{sd}(\phi)$ are both Pareto distributed.

2 Dornbusch-Fischer-Samuelson's Ricardian Model with Unbalanced Trade

Consider a version of the Dornbusch-Fischer-Samuelson model of Ricardian trade with transport costs and a non-zero trade balance. There is a continuum of goods indexed with $z \in [0,1]$. There are symmetric iceberg transportation cost so κ melts away and $1/(1-\kappa)$ units of a product need to be made for one unit to arrive abroad.

Consumers have homothetic preferences with consumption basket $C_d \equiv \exp\{\int_0^1 \ln c_d(z)\mathrm{d}z\}$. Using a result from question **??**, demand for a product z is $c(z) = PC/p(z)$ with the ideal price index $P = \exp\{\int_0^1 \ln p(z)\mathrm{d}z\}$.

Labor is only factor of production and makes a product z under unit labor requirements $a(z)$. Define the Home country's comparative advantage in industry z with $A(z) \equiv a^*(z)/a(z)$. Assume without loss of generality that z strictly indexes the industries with the Home's strongest comparative advantage so that $A'(z) < 0$.

1. Using the condition $w\, a(z) \leq w^*\, a^*(z)/(1-\kappa)$ for home production, determine the cutoff industry z^H up to which the home country produces. Similarly, using the condition $w^*\, a^*(z) \leq w\, a(z)/(1-\kappa)$ for foreign production, determine the cutoff industry z^F up to which the foreign country produces. Show that $z^H > z^F$ for $\kappa > 0$ and $A'(z) < 0$.

2. To simplify exposition, consider the functional form $A(z) = \exp\{1-2z\}$. Show that the size of the nontraded sector $z^H - z^F$ can then be expressed as $z^H - z^F = -\log(1-\kappa) > 0$.

3. In equilibrium, global consumption expenditure must equal global income so that $PC + P^*C^* = wL + w^*L^*$ ("market clearing"). Home income equals global expenditure on home produced goods so $wL = z^H PC + z^F P^*C^*$, and a similar expression applies to the foreign country. Define the home trade balance as $TB = wL - PC = -TB^* \neq 0$, that is the excess output over absorption. Make good 1 the numeraire, a foreign produced good, so that $w^* = p^*(1)/a^*(1) = 1/a^*(1)$. Show that the global "market clearing" condition and $TB = wL - PC \neq 0$ imply

$$\frac{w}{w^*} = \left(\frac{\log(1-\kappa)TB}{L^*/a^*(1)} + z^F\right)\frac{L^*/L}{1+\log(1-\kappa)-z^F} \equiv B(z^F).$$

4. Using the cutoff for foreign production $w^*\, a^*(z^F) = w\, a(z^F)/(1-\kappa)$, place conditions on TB so that that this relationship and $B(z^F)$ above result in a unique equilibrium. (*Hint:* Establish monotonicity and limits. Start with $TB = 0$, then generalize.)

5. How does the equilibrium with a non-zero trade balance differ from that derived under a zero trade balance? How does an increase in the home trade balance TB affect the location of industries? How does the increase affect the size of the nontraded sector under $A(z) = \exp\{1 - 2z\}$ and in general? How does the increase affect welfare in the home country?