Econ 202a Spring 2000
Marc's

Suggested Solutions to the midterm

1 Dornbusch model: Monetary Contraction Abroad

As can be seen from the steady state relationships in the continuous-time version of the model

$$\bar{p} = \bar{m} + (\lambda r^* - \phi \bar{y}),$$

and

$$\bar{e} = \bar{p} + \frac{1}{\delta}[\sigma r^* + (1-\gamma)\bar{y} - u],$$

both the steady state exchange rate $\bar{e}$ and the steady state price level $\bar{p}$ increase to new levels $\bar{e}' > \bar{e}$ and $\bar{p}' > \bar{p}$, when r^* rises. This is the same in the discrete time version where

$$\bar{p} = \bar{m} - \phi\bar{y} + \eta i^*,$$

$$\bar{e} = \bar{p} + \frac{1}{\delta}\bar{y},$$

and i^* rises.

Thus, in figure 1, the steady state moves along the 45^o line out to the Northeast after the monetary contraction abroad. We have found the change in the long-run. The saddle-paths, which correspond to the old and the new steady state, are drawn as falling lines through the respective steady states, but they are irrelevant for the long-run analysis because the economy sleeps at the steady state in the long run.

In the short-run, however, the price level is fixed. Therefore, in figure 1, the economy must initially move to a new position along the horizontal line through $\bar{p}$. Only the exchange rate can adjust. Now the saddle-paths matter

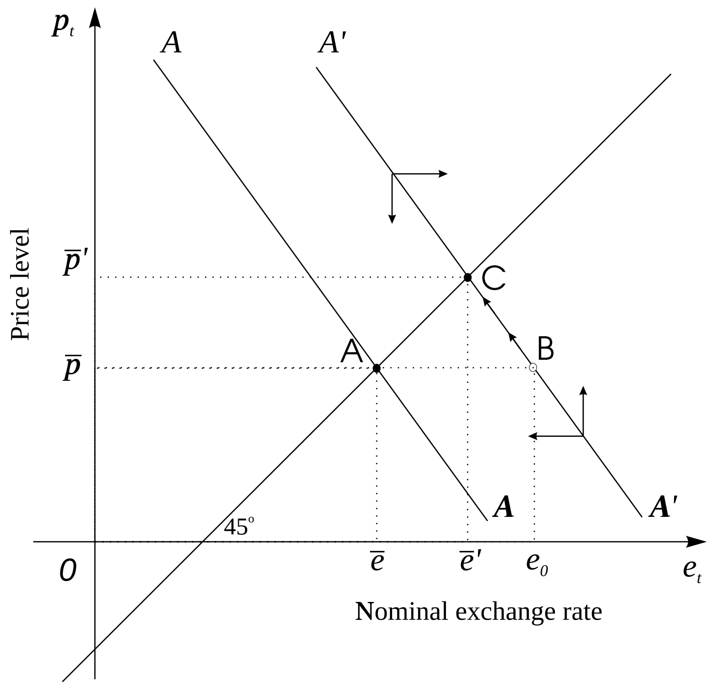

Figure 1: **Phase Diagram for Dornbusch model before and after monetary contraction abroad**

for the analysis. We know that the economy must be on the new saddle-path at the moment when the monetary contraction abroad takes effect (otherwise the economy explodes, an event we excluded as unreasonable). Since the foreign monetary contraction takes effect immediately (in this question), the economy must jump to point B in the figure instantaneously. From then on, it follows the saddle-path to the new steady state $(\bar{e}', \bar{p}')$. The exchange rate depreciates more in the short-run than in the long-run, it overshoots.

2 Rational Expectations Equilibrium

In the question. aggregate demand is given by

$$p_t = m_t - y_t, \tag{1}$$

where m_t follows an AR(1) process

$$m_t = m_{t-1} + \tilde{\epsilon}_t. \tag{2}$$

with $\mathbb{E}[\tilde{\epsilon}_t] = 0$. Aggregate supply is given by

$$y_t = b\left(p_t - \mathbb{E}_{t-1}[p_t]\right) + \tilde{v}_t, \tag{3}$$

with $\mathbb{E}[\tilde{v}_t] = 0$.

1. Plugging (2) and (3) into (1) yields

   $$p_t = m_{t-1} + \tilde{\epsilon}_t - b\left(p_t - \mathbb{E}_{t-1}[p_t]\right) + \tilde{\epsilon}_t - \tilde{v}_t,$$

   and solving out for p_t gives us the answer

   $$p_t = \frac{1}{1+b} m_{t-1} + \frac{1}{1+b}\left(\tilde{\epsilon}_t - \tilde{v}_t\right) + \frac{b}{1+b}\mathbb{E}_{t-1}[p_t]. \tag{4}$$

2. Taking conditional expectations $\mathbb{E}_{t-1}$ of both sides of (4) yields

   $$\mathbb{E}_{t-1}[p_t] = \frac{1}{1+b} m_{t-1} + \frac{b}{1+b}\mathbb{E}_{t-1}[p_t],$$

   since $\mathbb{E}_{t-1}[\tilde{\epsilon}_t] = \mathbb{E}_{t-1}[\tilde{v}_t] = 0$. The crucial assumption is that both .$\tilde{\epsilon}_t$ and $\tilde{v}_t$ are not correlated over time. In addition, $\mathbb{E}_{t-1}[m_{t-1}] = m_{t-1}$ because m_{t-1} is known as of $t-1$. Solving out for $\mathbb{E}_{t-1}[p_t]$ immediately yields

   $$\mathbb{E}_{t-1}[p_t] = m_{t-1}. \tag{5}$$

3. In order to solve for equilibrium output, note first that the key determinant for y_t, $p_t - \mathbb{E}_{t-1}[p_t]$, equals

   $$p_t - \mathbb{E}_{t-1}[p_t] = p_t - m_{t-1} = \tilde{\epsilon}_t - y_t$$

   by (5) and (2). Thus,

   $$y_t = b\left(\tilde{\epsilon}_t - y_t\right) + \tilde{v}_t,$$

   and solving out for y_t yields

   $$y_t = \frac{b}{1+b}\tilde{\epsilon}_t + \frac{1}{1+b}\tilde{v}_t. \tag{6}$$

4. Unexpected changes of monetary policy alter the distribution and realizations of $\tilde{\epsilon}_t$. Unexpected changes in monetary policy matter. Expected changes don't.

3 True/False on Stochastic Processes

The autocovariance function of an MA(q)-process *cannot* follow a q th order difference equation with the same coefficients.

To prove a statement wrong, it is sufficient to give one single counter example. We know that the autocovariance function is defined as $\gamma(s) \equiv \mathbb{E}[(y_t - \mathbb{E}[y_t])(y_{t-s} - \mathbb{E}[y_{t-s}])]$. The simple counter-example of an MA(1)-process $y_t = \epsilon_t + \theta\epsilon_{t-1}$ does the job. Since $\mathbb{E}[y_t] = 0$, the autocovariance function at lag zero is

$$\gamma(0) = \mathbb{E}\left[(\epsilon_t + \theta\epsilon_{t-1})^2\right] = \left(1+\theta^2\right)\sigma_\epsilon^2.$$

At the first lag it is

$$\gamma(1) = \mathbb{E}[(\epsilon_t + \theta\epsilon_{t-1})(\epsilon_{t-1} + \theta\epsilon_{t-2})] = \theta\sigma_\epsilon^2.$$

From the second lag on, $\gamma(s) = 0$ $(s \geq 2)$. Clearly, $\gamma(s)$ does not follow any difference equation.

Although it is not necessary to make a general argument, here a possible one. We know from lecture that an MA(q) process has an autocovariance function that dies out (is zero) after the qth lag. Then, however, any difference equation would have to take the value zero after some period, whereas it assigns a non-zero value to the autocovariance before that period. This is impossible for a function in real numbers.

And yet another one. A general MA(q)-process takes the form $y_t = \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} + \ldots + \theta_q\epsilon_{t-q}$. As reported in lecture and shown in section, any finite order MA(q)-process has an AR(∞) representation. We also know from a proof both in lecture and section that the autocovariance function of an AR(p)-process follows a difference equation in exactly the same coefficients. Thus, we can infer: The autocovariance function of any finite MA(q) must either follow a difference equation of infinite order, or none at all. It cannot follow a qth order difference equation for a finite q.

4 True/False on Staggered Price Setting Models

The statement is true: An expected monetary expansion in three periods from today has *no* effect on output in the Fischer model, but *does* have an effect in the Taylor model. The reason is the following. There are two groups of individuals in both models. Only one of the two groups sets prices at a time. Suppose money supply is announced to increase in three periods from now.

When setting prices, the individuals in the Fischer economy form expectations about future periods, and the future behavior of the other half of economic agents. They know that, at time $t+2$, the other half of the individuals will set higher prices for $t+3$ when money supply increases. But they don't worry too much. They simply set their prices for $t+2$ to a higher level but keep their prices for $t+1$ at a different, optimal level. By the mere fact that they can choose two different future prices, there is no spill-over of the future into the present. Even though prices are *predetermined* two periods in advance by half of the individuals, they are not *fixed* across future periods. That is, the price that the group at $t+1$ sets for period $t+2$ can be different from the price that this group sets for $t+3$. Therefore, when the group at t makes its decision about $t+2$, it need not care about anything that is about to happen at $t+3$.

On the other hand, the Taylor model decrees such a spill-over of future events into the present. The reason is that in Taylor's model prices are not only *predetermined* two periods in advance, they are also *fixed* across both future periods. Consider the same expected change in the monetary supply at $t+3$. The group deciding at time $t+1$ will certainly care about money supply at $t+3$ because monetary policy affects price levels and output. So, when the group at $t+1$ chooses its fixed price for the next two periods, the price level at period $t+2$ is affected by increased money supply. The price choice is going to be higher than if there were no change in money supply at $t+3$. This carries through to earlier periods. The group who sets prices at t worries about money supply in $t+3$ now. The t-group rationally expects that the $t+1$-group will increase their price choice due to the increase in money at $t+3$. But that means that the t-group better chooses its next two prices taking into account that future price levels are higher. Thus, the t-group will, in turn, set somewhat higher prices already, and so forth. Announced monetary policy changes far in the future have at least a little effect on the price choice and hence the price level today when prices are not only *predetermined*, but *fixed* in addition as in Taylor's economy.