Globalization and Prosperity Lab — UC San Diego SCHOOL OF SOCIAL SCIENCES Department of Economics

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1 Fundamentals

We consider a global economy with N destination regions $d \in \mathcal{D} := \{1, 2, \cdots, N\}$ for trade flows and I industries $i \in \mathcal{I} := \{0, 1, 2, \cdots, I\}$. The N regions are simultaneously source regions $s \in \mathcal{D}$. Given global input-output relationships, the I industries are simultaneously supply industries i and use industries j. We use subscripts in sequence from left to right so that $sidj$ denotes a trade flow from source region s's supply industry i to destination region d's use industry j.

1.1 Households

The representative household at a destination d is endowed with L_d units of labor and K_d units of capital. Capital does not accumulate, and there is no savings decision. The representative household chooses consumption of final goods C_d^F to maximize utility subject to the household budget constraint:

$$C_d^F = \max_{\{C_{di}^F\}_{i \in \mathcal{I}}} \Pi_{i \in \mathcal{I}} \left[C_{di}^F \right]^{\eta_{di}^F} \text{ with } \textstyle\sum_{i \in \mathcal{I}} \eta_{di}^F = 1 \qquad s.t. \quad P_d^F C_d^F \leq w_d L_d + r_d K_d - NX_d, \tag{1}$$

where NX_d are region d's exogenously given net exports (the trade surplus), $w_d L_d$ is labor income and $r_d K_d$ capital income, P_d^F is the aggregate price index of final consumption goods, and η_{di}^F are the household's final consumption expenditure shares. The representative household's static choice is contingent on the observed trade imbalance NX_d. In the absence of a savings choice, a trade surplus is similar to an exogenously mandated cross-country income transfer.

1.2 Production

Output of final products originates from a source region s and a use industry j that combines local labor and capital with globally sourced intermediate inputs from supply industries i. In each market sj, a continuum of firms produces varieties ω. The producer of variety ω operates with a nested Cobb-Douglas technology given by

$$y_{sj}(\omega) = z_{sj}(\omega) \left((L_{sj})^{\beta_{sj}} (K_{sj})^{1-\beta_{sj}} \right)^{\mu_{sj}} \left(\Pi_{i \in \mathcal{I}} (M_{isj})^{\alpha_{isj}^M} \right)^{1-\mu_{sj}} \qquad s.t. \quad \sum_{i \in \mathcal{I}} \alpha_{isj}^M = 1, \tag{2}$$

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where $y_{sj}(\omega)$ denotes output of variety ω by use industry j produced in region s; $z_{sj}(\omega)$ is total factor productivity; L_{sj} is employment; K_{sj} is capital use; β_{sj} is the labor-share in value-added; μ_{sj} is the value-added share in output; M_{isj} is the use of intermediate inputs from supply industry i by industry j; and α^M_{isj} is the cost-share of intermediate inputs from industry i in the total cost of intermediate inputs used in industry j.

1.3 Government

The government in region d collects revenues T_d from taxes and tariffs. Government expenditure goes to subsidies S_d for producers and to government consumption C^G_{di}. Given government revenues T_d and subsidy expenditures S_d from long-term policy decisions (by the legislative branch or by executive order), the executive branch optimally procures C^G_{di} in its day-to-day operations to maximize a government procurement index $\Pi_{i\in\mathcal{I}}[C^G_{di}]^{\eta^G_{di}}$ subject to the government budget constraint:

$$C^G_d = \max_{\{C^G_{di}\}_{i\in\mathcal{I}}} \Pi_{i\in\mathcal{I}} \left[C^G_{di}\right]^{\eta^G_{di}} \text{ with } \sum_{i\in\mathcal{I}} \eta^G_{di} = 1 \qquad s.t. \quad P^G_d C^G_d \leq T_d - S_d, \tag{3}$$

where $T_d - S_d$ is net government revenue (tax and tariff revenues less subsidy expenditures), P^G_d is the aggregate price index of government consumption, and η^G_{di} are the government's final consumption expenditure shares (as reported in the input-output table).

1.4 Assembly of goods from varieties

Final household consumption goods. A local assembler of the final goods Y^F_{dj} for household consumption uses the least expensive deliverable varieties $\bar{\omega}$ within each industry j available at destination d and aggregates the sourced varieties in industry j with the following technology:

$$Y^F_{dj} \equiv \left(\int_{[0,1]} y^F_{dj}(\bar{\omega})^{\frac{\sigma_j-1}{\sigma_j}} d\bar{\omega}\right)^{\frac{\sigma_j}{\sigma_j-1}}, \tag{4}$$

where $y^F_{dj}(\omega)$ is the least expensive deliverable variety $\bar{\omega}$ available at destination d for final goods assembly and $\sigma_j > 1$ is the elasticity of substitution between varieties in use industry j. Under this technology, the price of the final goods satisfies

$$P^F_{dj} = \left(\int_{[0,1]} p^F_{dj}(\bar{\omega})^{-(\sigma_j-1)} d\bar{\omega}\right)^{-\frac{1}{\sigma_j-1}}, \tag{5}$$

where $p^F_{dj}(\bar{\omega})$ is the price of the least expensive available variety $\bar{\omega}$ in industry j delivered at destination d for final household goods assembly.

Intermediate goods. A local assembler of the aggregate intermediate good M_{idj} uses the least expensive deliverable varieties $\bar{\omega}$ within supply industry i available at destination d for use in industry j. The intermediate-goods assembler then aggregates the varieties with the following technology:

$$M_{idj} \equiv \left(\int_{[0,1]} m_{idj}(\bar{\omega})^{\frac{\sigma_j-1}{\sigma_j}} d\bar{\omega}\right)^{\frac{\sigma_j}{\sigma_j-1}}, \tag{6}$$

where $m_{idj}(\bar{\omega})$ denotes the least expensive available variety $\bar{\omega}$ of industry i delivered at destination d for use in industry j and $\sigma_j > 1$ is the common elasticity of substitution between varieties. Given this technology, the price of the aggregate intermediate good in market dj satisfies:

$$P^M_{idj} = \left(\int_{[0,1]} p^M_{idj}(\bar{\omega})^{-(\sigma_j - 1)} d\bar{\omega} \right)^{-\frac{1}{\sigma_j - 1}}, \tag{7}$$

where $p^M_{idj}(\bar{\omega})$ is the price of the least expensive available variety $\bar{\omega}$ of industry i delivered at destination d for use in industry j.

Final government procurement goods. A local assembler specializes in supplying the government with its final goods Y^G_{dj} and uses the least expensive deliverable varieties $\bar{\omega}$ within industry j available at destination d. The assembler aggregates the least expensive available varieties for government procurement with the following technology:

$$Y^G_{dj} \equiv \left(\int_{[0,1]} y^G_{dj}(\bar{\omega})^{\frac{\sigma_j - 1}{\sigma_j}} d\bar{\omega} \right)^{\frac{\sigma_j}{\sigma_j - 1}}, \tag{8}$$

where $y^G_{dj}(\omega)$ is the least expensive deliverable variety $\bar{\omega}$ available to region d for government goods assembly and $\sigma_j > 1$ is the common elasticity of substitution between varieties. Given this technology, the price of the final goods satisfies

$$P^{dj}_G = \left(\int_{[0,1]} p^G_{dj}(\bar{\omega})^{-(\sigma_j - 1)} d\bar{\omega} \right)^{-\frac{1}{\sigma_j - 1}}, \tag{9}$$

where $p^G_{dj}(\bar{\omega})$ is the price of the least expensive available variety $\bar{\omega}$ of industry j delivered at destination d for government goods assembly.

1.5 Global and internal trade

Following Eaton and Kortum (2002), we consider independent productivity draws across industries and regions.

Assumption 1 (Eaton-Kortum Productivity). *We assume that $z_{sj}(\bar{\omega})$ is an iid random variable drawn from a market-specific Fréchet distribution*

$$F_{sj}(z) = \exp\left\{ -A_{sj} \, z^{-\theta_j} \right\}.$$

with source-industry specific location parameter A_{sj} and shape parameter θ_j.

The benchmark market structure is perfect competition.[1]

Assumption 2 (Market Structure). *Product and factor markets are perfectly competitive.*

Under constant returns to scale by (2) and Assumption 2, factory-gate prices equal unit production cost.

Final household consumption goods. Under these assumptions, delivery prices of varieties used for final goods assembly satisfy

$$p^F_{dj}(\bar{\omega}) = \min_{s \in \mathcal{D}} \left\{ \kappa^F_{sdj} \, \tau_{sdj} \, \varsigma_{sdj} \, p_{sj}(\bar{\omega}) \right\} = \min_{s \in \mathcal{D}} \left\{ \kappa^F_{sdj} \, \tau_{sdj} \, \varsigma_{sdj} \, \frac{c_{sj}}{z_{sj}(\bar{\omega})} \right\}, \tag{10}$$

[1] Bertrand price competition is a straightforward alternative and allows for strictly positive profits in equilibrium because a producer in industry j from country s that ships the least expensive variety to d can charge the price of the second-cheapest producer from anywhere in the world shipping to d (see Eaton and Kortum, 2010, Chapter 4).

where $\kappa^F_{sdj} \geq 1$ measure iceberg trade costs of shipping a variety of the industry j good from source region s to a final goods assembler in region d: κ^F_{sdj} units of a variety of good j need to ship out of source region s for one unit of the variety to arrive at destination d. For internal trade (self trade), when the destination region d is also the source region s, we assume that goods sell at the factoy gate price under $\kappa^F_{ssj} = 1$.

The tariff factors τ_{sdj} are one plus the ad valorem import tariff rates for a variety of the industry j good from source region s. The government in destination region d collects *import tariffs* for goods from other source regions $s \neq d$ and can impose a *local producer tax* when goods ship to local consumers ($s = d$).

The subsidy factors ς_{sdj} reflect net payments of a subsidy $-(\varsigma_{sdj} - 1) = 1 - \varsigma_{sdj}$ per unit of output by the government in source region s to producers in s. This generic specification of ς_{sdj} encompasses production subsidies, input subsidies, and export subsidies ($\varsigma_{sdj} < 1$) as well as export taxes ($\varsigma_{sdj} > 1$). A *production subsidy* reduces unit production cost by a scalar $\varsigma_{sdj} = \varsigma_{s\cdot j} \leq 1$ regardless of the destination region d: if a firm produces $q_{sj}(\bar{\omega})$ units of a variety, the subsidized unit production cost becomes $\varsigma_{s\cdot j}\, p_{sj}(\bar{\omega})q_{sj}(\bar{\omega}) < p_{sj}(\bar{\omega})q_{sj}(\bar{\omega})$, where $p_{sj}(\bar{\omega})$ is the factory gate price. A special production subsidy is a subsidy on factor inputs, such as a subsidy for the installation of capital. Given production technology (2), an *input subsidy* can be transformed into an equivalent production subsidy. For example, an input subsidy of capital $\varsigma^K_{s\cdot j}$ is equivalent to reducing capital use by a factor of $\varsigma^K_{s\cdot j}$ in (2), which in turn is equivalent to a production subsidy of $\varsigma_{s\cdot j} = (\varsigma^K_{s\cdot j})^{(1-\beta_{sj})\mu_{sj}}$.

An *export subsidy* reduces unit production cost by a scalar $\varsigma_{sdj} = \varsigma_{sdj|s\neq d} < 1$ only for destinations d other than the source region s. If the firm is required to charge a single factory-gate price $p_{sj}(\bar{\omega})$ regardless of its customer's location, then the firm receives per-unit revenues $(1/\varsigma_{sdj|s\neq d})p_{sj}(\bar{\omega}) > p_{sj}(\bar{\omega})$ for an exported unit of its product but receives only the factory-gate price $p_{sj}(\bar{\omega})$ for a locally sold product unit at $d = s$. In practice, a firm would then optimally choose destination-specific prices and discriminate customers by their location at home or abroad. Under our maintained Assumption 2 that factory-gate prices are set to equal marginal cost, production subsidies are therefore more realistic than export subsidies. An *export tax* raises unit production cost by a scalar $\varsigma_{sdj} = \varsigma_{sdj|s\neq d} > 1$ only for destinations d other than the source region s, with a similar caveat for realism under Assumption 2.

The factory gate price $p_{sj}(\bar{\omega})$ for a variety has two components: the unit production cost c_{sj} that is common to all producers of a variety of good j in source region s, and the producer-specific productivity parameter $z_{sj}(\bar{\omega})$. The common unit cost component c_{sj} is the cost of the input bundle for any producer in region s's industry j, operating with the constant-returns-to-scale technology (2):

$$c_{sj} = \Theta_{sj} \left((w_s)^{\beta_{sj}} (r_s)^{1-\beta_{si}} \right)^{\mu_{sj}} \left(\Pi_{i\in\mathcal{I}} \left(P^M_{isj} \right)^{\alpha^M_{isj}} \right)^{1-\mu_{sj}}, \tag{11}$$

where Θ_{sj} is a collection of Cobb-Douglas coefficients; w_s is the wage in labor market s; r_s is the rental rate of capital in region s; and P^M_{isj} are prices of aggregate intermediate inputs in region s sourced from industry i for use in industry j.

Intermediate goods. Prices of varieties from supply industry i used for production of intermediate goods in industry j and region d differ only in their trade costs, which are use-industry specific:

$$p^M_{idj}(\bar{\omega}) = \min_{s\in\mathcal{D}} \left\{ \kappa^M_{sidj}\, \tau_{sdj}\, \varsigma_{sdj}\, p_{si}(\bar{\omega}) \right\} = \min_{s\in\mathcal{D}} \left\{ \kappa^M_{sidj}\, \tau_{sdj}\, \varsigma_{sdj}\, \frac{c_{si}}{z_{si}(\bar{\omega})} \right\},$$

where κ^{sidj}_M is the iceberg trade cost of shipping a variety from region s's industry i for use in region d and industry j.

Final government procurement goods. Procurement frequently requires that the government favor domestic producers. We capture this feature in the model by having different trade costs for final consumption goods and government procurement goods:

$$p^G_{dj}(\bar{\omega}) = \min_{s\in\mathcal{D}} \left\{ \kappa^G_{sdj}\, \tau_{sdj}\, \varsigma_{sdj}\, p_{sj}(\bar{\omega}) \right\} = \min_{s\in\mathcal{D}} \left\{ \kappa^G_{sdj}\, \tau_{sdj}\, \varsigma_{sdj}\, \frac{c_{sj}}{z_{sj}(\bar{\omega})} \right\},$$

where $\kappa^G_{sdj} \geq 1$ are iceberg trade costs of shipping a variety of good j good from s to a government procurement assembler in region d. For internal trade (self trade), when the destination region d is also the source region s, we assume that goods sell at the factoy gate price under $\kappa^G_{ddj} = 1$. A home bias in procurement raises the cost of sales to government beyond the cost of sales to private households: $\kappa^G_{sdj} > \kappa^F_{sdj}$. As a government approaches complete home bias in procurement of good j, κ^G_{sdj} becomes arbitrarily large.

Demand. Demand for varieties in the assembly of final household consumption goods, intermediate goods, and final government procurement goods and induce internal and international trade. It follows from our Assumption 1 on Fréchet productivity draws that prices of the final consumption good aggregates P^F_{dj}, intermediate good aggregates P^M_{idj}, and final government procurement good aggregates P^G_{dj} satisfy:

$$P^F_{dj} = \gamma_j \left(\Phi^F_{dj}\right)^{-\frac{1}{\theta^j}}, \tag{12}$$

$$P^M_{idj} = \gamma_j \left(\Phi^M_{idj}\right)^{-\frac{1}{\theta^j}}, \tag{13}$$

$$P^G_{dj} = \gamma_j \left(\Phi^G_{dj}\right)^{-\frac{1}{\theta^j}}, \tag{14}$$

where $\gamma_j \equiv \Gamma([\theta^j + 1 - \nu]/\theta^j)^{-1/(\sigma_j - 1)}$ and mean prices satisfy $\Phi^F_{dj} \equiv \sum_{n \in \mathcal{D}} A_{nj}[\kappa^F_{ndj}\,\tau_{ndj}\,\varsigma_{ndj}\,c_{nj}]^{-\theta^j}$, $\Phi^M_{idj} \equiv \sum_{n \in \mathcal{D}} A_{ni}[\kappa^M_{nidj}\,\tau_{ndj}\,\varsigma_{ndj}\,c_{ni}]^{-\theta^j}$ and $\Phi^G_{dj} \equiv \sum_{n \in \mathcal{D}} A_{nj}[\kappa^G_{ndj}\,\tau_{ndj}\,\varsigma_{ndj}\,c_{nj}]^{-\theta^j}$. Assumption 1 also yields closed form solutions for trade shares of goods being shipped from region s to region d in the assembly of final or intermediate goods:

$$\lambda^F_{sdj} = \frac{A_{sj}[\kappa^F_{sdj}\,\tau_{sdj}\,\varsigma_{sdj}\,c_{sj}]^{-\theta^j}}{\Phi^F_{dj}}, \tag{15}$$

$$\lambda^M_{sidj} = \frac{A_{si}[\kappa^M_{sidj}\,\tau_{sdj}\,\varsigma_{sdj}\,c_{si}]^{-\theta^j}}{\Phi^M_{idj}}, \tag{16}$$

$$\lambda^G_{sdj} = \frac{A_{sj}[\kappa^G_{sdj}\,\tau_{sdj}\,\varsigma_{sdj}\,c_{sj}]^{-\theta^j}}{\Phi^G_{dj}}. \tag{17}$$

1.6 Equilibrium conditions

Goods market clearing. Goods market clearing can be restated in terms of spending and income equalities. Total spending on final goods sourced from industry i of region s reflects the purchases for final consumption by households:

$$E^F_{si} = \sum_{d \in \mathcal{D}} \frac{\lambda^F_{sdi}}{\tau_{sdi}\,\varsigma_{sdi}} \eta^F_{di} \left(w_d L_d + r_d K_d - NX_d\right). \tag{18}$$

Spending on intermediate goods from industry i of region s includes the share of intermediates $(1 - \mu_{di})$ and the cost share α^M_{idj} in the production of varieties:

$$E^M_{si} = \sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{I}} \frac{\lambda^M_{sidj}}{\tau_{sdi}\,\varsigma_{sdi}} (1 - \mu_{dj}) \alpha^M_{idj} \left(E^M_{dj} + E^G_{dj} + E^F_{dj}\right). \tag{19}$$

Spending on government procured goods from industry i of region s equals net revenues (tariff and tax revenues less subsidies) of all governments in the world:

$$E^G_{si} = \sum_{d \in \mathcal{D}} \frac{\lambda^G_{sdi}}{\tau_{sdi}\,\varsigma_{sdi}} \eta^G_{di} \left(T_d - S_d\right), \tag{20}$$

where gross government revenues are given by (24) below and gross subsidies by (25).

Combined, these three equations characterize total spending on goods coming from industry i of region s:

$$E_{si} \equiv E^F_{si} + E^M_{si} + E^G_{si}. \tag{21}$$

Factor market clearing. We use equation (21) to state N market clearing income conditions for labor and capital in terms of total factor incomes:

$$w_s L_s = \sum_{i\in\mathcal{I}} \mu_{si}\beta_{si}E_{si}, \tag{22}$$

$$r_s K_s = \sum_{i\in\mathcal{I}} \mu_{si}(1-\beta_{si})E_{si}. \tag{23}$$

Government budget. The exogenously set tariff factors τ_{sdj} and subsidy factors ς_{sdj} generate government revenues and expenditures. The government runs a balanced budget, so that

$$P^G_d C^G_d = T_d - S_d$$

by (3).

The government collects tariffs from all trade flows. Tariff and tax revenues T_d are a function of spending on final consumption goods, intermediate goods, and government procurement:

$$T_d = \sum_{s\in\mathcal{D}}\sum_{i\in\mathcal{I}}(\tau_{sdi}-1)\left(\lambda^F_{sdi}E^F_{di} + \sum_{j\in\mathcal{I}}\lambda^M_{sidj}E^M_{idj} + \lambda^G_{sdi}E^G_{di}\right) \tag{24}$$

Similarly, government subsidies S_d in region d amount to:

$$S_d = \sum_{n\in\mathcal{D}}\sum_{i\in\mathcal{I}}(1-\varsigma_{dni})\left(\lambda^F_{dni}E^F_{ni} + \sum_{j\in\mathcal{I}}\lambda^M_{dinj}E^M_{inj} + \lambda^G_{dni}E^G_{ni}\right). \tag{25}$$

1.7 Trade flows

The value of a trade flow from source region s and supply industry i to destination d and use industry j is

$$X^M_{sidj} = \lambda^M_{sidj}(1-\mu_{dj})\alpha^M_{idj}\left(E^M_{dj} + E^G_{dj} + E^F_{dj}\right). \tag{26}$$

The value of a trade flow from source region s to destination d from a supply industry i and going to any use, including final demand for household consumption and final procurement for government consumption, is

$$X_{sdi} = \lambda^F_{sdi}\eta^F_{di}\left(w_d L_d + r_d K_d - NX_d\right) + \sum_{k\in\mathcal{I}} X^M_{sidk} + \lambda^G_{sdi}\eta^G_{di}\left(T_d - S_d\right), \tag{27}$$

so gross exports EX_{si} from source region s and gross imports IM_{di} into destination d are

$$EX_{si} = \textstyle\sum_{d\neq s} X_{sdi} \quad \text{and} \quad IM_{di} = \sum_{s\neq d} X_{sdi}.$$

2 Static Equilibrium in Levels and Changes

2.1 Static Equilibrium Equations in Levels

$$c_{sj} = \Theta_{sj} \left((w_s)^{\beta_{sj}} (r_s)^{1-\beta_{si}} \right)^{\mu_{sj}} \left(\Pi_{i \in \mathcal{I}} \left(P^M_{isj} \right)^{\alpha^M_{isj}} \right)^{1-\mu_{sj}} ; \tag{28a}$$

$$P^F_{dj} = \gamma_j \left(\sum_{n \in \mathcal{D}} A_{nj} [\kappa^F_{ndj} \, \tau_{ndj} \, \varsigma_{ndj} \, c_{nj}]^{-\theta^j} \right)^{-1/\theta^j} , \tag{28b}$$

$$P^M_{idj} = \gamma_j \left(\sum_{n \in \mathcal{D}} A_{ni} [\kappa^M_{nidj} \, \tau_{ndj} \, \varsigma_{ndj} \, c_{ni}]^{-\theta^j} \right)^{-1/\theta^j} , \tag{28c}$$

$$P^G_{dj} = \gamma_j \left(\sum_{n \in \mathcal{D}} A_{nj} [\kappa^G_{ndj} \, \tau_{ndj} \, \varsigma_{ndj} \, c_{nj}]^{-\theta^j} \right)^{-1/\theta^j} ; \tag{28d}$$

$$\lambda^F_{sdj} = \frac{A_{sj} [\kappa^F_{sdj} \, \tau_{sdj} \, \varsigma_{sdj} \, c_{sj}]^{-\theta^j}}{\sum_{n \in \mathcal{D}} A_{nj} [\kappa^F_{ndj} \, \tau_{ndj} \, \varsigma_{ndj} \, c_{nj}]^{-\theta^j}} , \tag{28e}$$

$$\lambda^M_{sidj} = \frac{A_{si} [\kappa^M_{sidj} \, \tau_{sdj} \, \varsigma_{sdj} \, c_{si}]^{-\theta^j}}{\sum_{n \in \mathcal{D}} A_{ni} [\kappa^M_{nidj} \, \tau_{ndj} \, \varsigma_{ndj} \, c_{ni}]^{-\theta^j}} , \tag{28f}$$

$$\lambda^G_{sdj} = \frac{A_{sj} [\kappa^G_{sdj} \, \tau_{sdj} \, \varsigma_{sdj} \, c_{sj}]^{-\theta^j}}{\sum_{n \in \mathcal{D}} A_{nj} [\kappa^G_{ndj} \, \tau_{ndj} \, \varsigma_{ndj} \, c_{nj}]^{-\theta^j}} ; \tag{28g}$$

$$E^F_{si} = \sum_{d \in \mathcal{D}} \frac{\lambda^F_{sdj}}{\tau_{sdi} \, \varsigma_{sdi}} \eta^F_{di} \left(w_d L_d + r_d K_d - NX_d \right) , \tag{28h}$$

$$E^M_{si} = \sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{I}} \frac{\lambda^M_{sidj}}{\tau_{sdi} \, \varsigma_{sdi}} (1 - \mu_{dj}) \alpha^M_{idj} \left(E^M_{dj} + E^G_{dj} + E^F_{dj} \right) , \tag{28i}$$

$$E^G_{si} = \sum_{d \in \mathcal{D}} \frac{\lambda^G_{sdj}}{\tau_{sdi} \, \varsigma_{sdi}} \eta^G_{di} \left(T_d - S_d \right) ; \tag{28j}$$

$$w_s L_s = \sum_{i \in \mathcal{I}} \mu_{si} \beta_{si} \left(E^F_{si} + E^M_{si} + E^G_{si} \right) , \tag{28k}$$

$$r_s K_s = \sum_{i \in \mathcal{I}} \mu_{si} (1 - \beta_{si}) \left(E^F_{si} + E^M_{si} + E^G_{si} \right) ; \tag{28l}$$

$$T_d = \sum_{s \in \mathcal{D}} \sum_{i \in \mathcal{I}} (\tau_{sdi} - 1) \left(\lambda^F_{sdi} E^F_{di} + \sum_{j \in \mathcal{I}} \lambda^M_{sidj} E^M_{idj} + \lambda^G_{sdi} E^G_{di} \right) , \tag{28m}$$

$$S_d = \sum_{n \in \mathcal{D}} \sum_{i \in \mathcal{I}} (\varsigma_{dni} - 1) \left(\lambda^F_{dni} E^F_{ni} + \sum_{j \in \mathcal{I}} \lambda^M_{dinj} E^M_{inj} + \lambda^G_{dni} E^G_{ni} \right) . \tag{28n}$$

2.2 Hat Algebra

According relative changes to equilibrium outcomes, given relative changes in select parameters such as those for tariffs, subsidies and trade costs, are worked out with common hat algebra and implemented in the solution algorithm. For each variable x, we denote $\hat{x} \equiv x'/x$, where x, x' are the equilibrium value of such a variable before and after, respectively, a change in trade costs.

$$\hat{c}_{sj} = \left((\hat{w}_s)^{\beta_{sj}} (\hat{r}_s)^{1-\beta_{si}}\right)^{\mu_{sj}} \left(\Pi_{i\in\mathcal{I}} \left(\hat{P}^M_{isj}\right)^{\alpha^M_{isj}}\right)^{1-\mu_{sj}} ; \tag{29a}$$

$$\hat{P}^F_{dj} = \left(\sum_{s\in\mathcal{D}} \lambda^F_{sdj} \hat{A}_{sj} [\hat{\kappa}^F_{sdj}\, \hat{\tau}_{sdj}\, \hat{\varsigma}_{sdj}\, \hat{c}_{sj}]^{-\theta^j}\right)^{-\frac{1}{\theta_j}}, \tag{29b}$$

$$\hat{P}^M_{idj} = \left(\sum_{s\in\mathcal{D}} \lambda^M_{sidj} \hat{A}_{si} [\hat{\kappa}^M_{nidj}\, \hat{\tau}_{ndj}\, \hat{\varsigma}_{ndj}\, \hat{c}_{ni}]^{-\theta^j}\right)^{-\frac{1}{\theta_j}}, \tag{29c}$$

$$\hat{P}^G_{dj} = \left(\sum_{s\in\mathcal{D}} \lambda^G_{sdj} \hat{A}_{nj} [\hat{\kappa}^G_{ndj}\, \hat{\tau}_{ndj}\, \hat{\varsigma}_{ndj}\, \hat{c}_{nj}]^{-\theta^j}\right)^{-\frac{1}{\theta_j}} ; \tag{29d}$$

$$\hat{\lambda}^F_{sdj} = \frac{\hat{A}_{sj} [\hat{\kappa}^F_{sdj}\, \hat{\tau}_{sdj}\, \hat{\varsigma}_{sdj}\, \hat{c}_{sj}]^{-\theta^j}}{\sum_{n\in\mathcal{D}} \lambda^F_{ndj} \hat{A}_{nj} [\hat{\kappa}^F_{ndj}\, \hat{\tau}_{ndj}\, \hat{\varsigma}_{ndj}\, \hat{c}_{nj}]^{-\theta^j}}, \tag{29e}$$

$$\hat{\lambda}^M_{sidj} = \frac{\hat{A}_{si} [\hat{\kappa}^M_{sidj}\, \hat{\tau}_{sdj}\, \hat{\varsigma}_{sdj}\, \hat{c}_{si}]^{-\theta^j}}{\sum_{n\in\mathcal{D}} \lambda^M_{sidj} \hat{A}_{ni} [\hat{\kappa}^M_{nidj}\, \hat{\tau}_{ndj}\, \hat{\varsigma}_{ndj}\, \hat{c}_{ni}]^{-\theta^j}}, \tag{29f}$$

$$\hat{\lambda}^G_{sdj} = \frac{\hat{A}_{sj} [\hat{\kappa}^G_{sdj}\, \hat{\tau}_{sdj}\, \hat{\varsigma}_{sdj}\, \hat{c}_{sj}]^{-\theta^j}}{\sum_{n\in\mathcal{D}} \lambda^G_{ndj} \hat{A}_{nj} [\hat{\kappa}^F_{ndj}\, \hat{\tau}_{ndj}\, \hat{\varsigma}_{ndj}\, \hat{c}_{nj}]^{-\theta^j}}, \tag{29g}$$

$$\hat{E}^F_{si} = \sum_{d\in\mathcal{D}} \frac{\lambda^F_{sdj}}{\tau_{sdi}\,\varsigma_{sdi}} \eta^F_{di} \left(\frac{w_d L_d}{E^F_{si}} \hat{w}_d \hat{L}_d + \frac{r_d K_d}{E^F_{si}} \hat{r}_d \hat{K}_d - \frac{NX_d}{E^F_{si}} \hat{NX}_d\right), \tag{29h}$$

$$\hat{E}^M_{si} = \sum_{d\in\mathcal{D}} \sum_{j\in\mathcal{I}} \frac{\lambda^M_{sidj}}{\tau_{sdi}\,\varsigma_{sdi}} (1-\mu_{dj}) \alpha^M_{idj} \left(\frac{E^M_{dj}}{E^M_{si}} \hat{E}^M_{dj} + \frac{E^G_{dj}}{E^M_{si}} \hat{E}^G_{dj} + \frac{E^F_{dj}}{E^M_{si}} \hat{E}^F_{dj}\right), \tag{29i}$$

$$\hat{E}^G_{si} = \sum_{d\in\mathcal{D}} \frac{\lambda^G_{sdj}}{\tau_{sdi}\,\varsigma_{sdi}} \eta^G_{di} \left(\frac{T_d}{E^G_{si}} \hat{T}_d - \frac{S_d}{E^G_{si}} \hat{S}_d\right) ; \tag{29j}$$

$$\hat{w}_s \hat{L}_s = \sum_{i\in\mathcal{I}} \mu_{si} \beta_{si} \left(\frac{E^F_{si}}{w_s L_s} \hat{E}^F_{si} + \frac{E^M_{si}}{w_s L_s} \hat{E}^M_{si} + \frac{E^G_{si}}{w_s L_s} \hat{E}^G_{si}\right), \tag{29k}$$

$$\hat{r}_s \hat{K}_s = \sum_{i\in\mathcal{I}} \mu_{si} (1-\beta_{si}) \left(\frac{E^F_{si}}{r_s K_s} \hat{E}^F_{si} + \frac{E^M_{si}}{r_s K_s} \hat{E}^M_{si} + \frac{E^G_{si}}{r_s K_s} \hat{E}^G_{si}\right) ; \tag{29l}$$

$$\hat{T}_d = \sum_{s\in\mathcal{D}} \sum_{i\in\mathcal{I}} (\tau_{sdi} - 1) \left(\frac{\lambda^F_{sdi} E^F_{di}}{T_d} \hat{\lambda}^F_{sdi} \hat{E}^F_{di} + \sum_{j\in\mathcal{I}} \frac{\lambda^M_{sidj} E^M_{idj}}{T_d} \hat{\lambda}^M_{sidj} \hat{E}^M_{idj} + \frac{\lambda^G_{sdi} E^G_{di}}{T_d} \hat{\lambda}^G_{sdi} \hat{E}^G_{di}\right), \tag{29m}$$

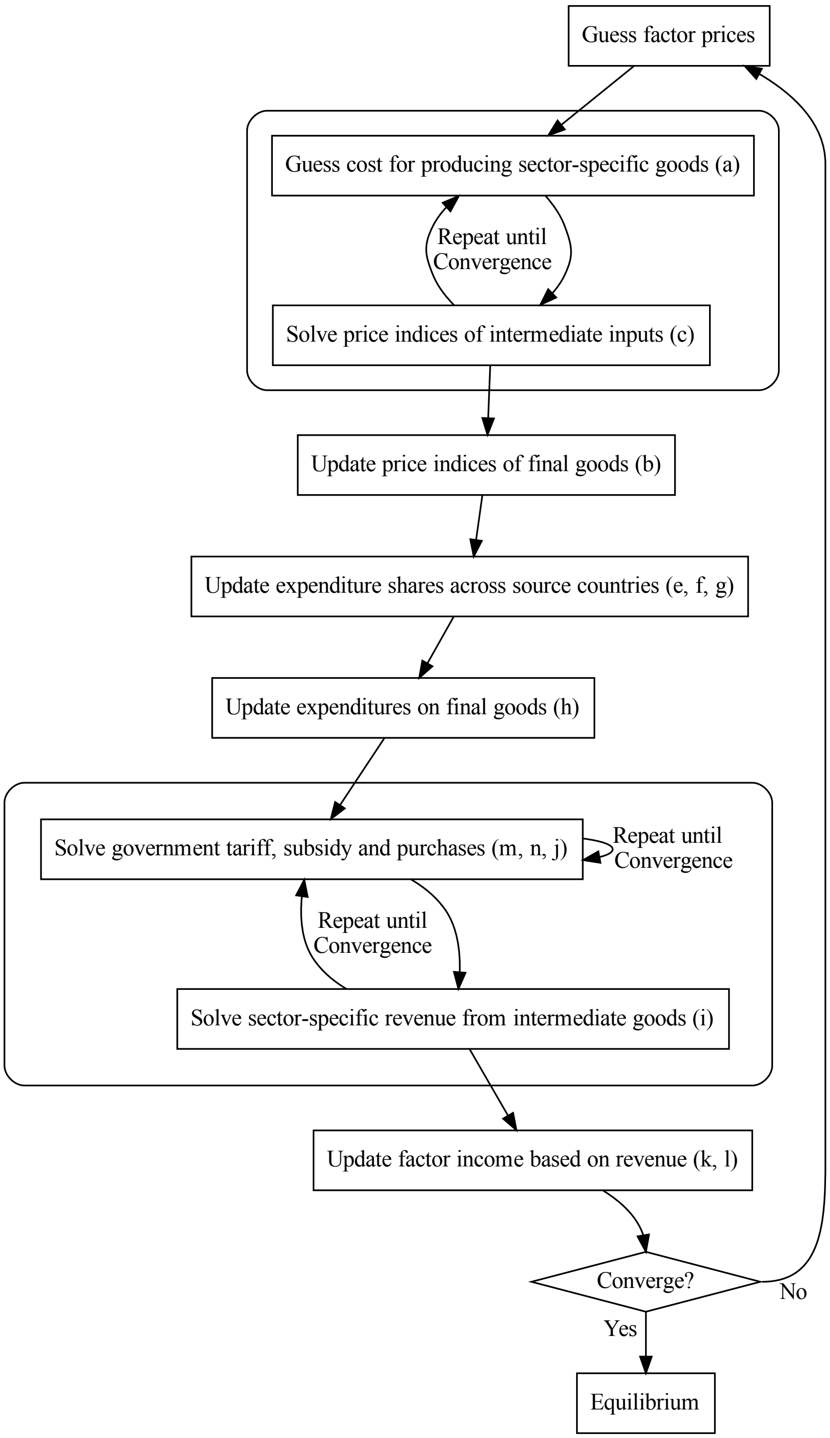
$$\hat{S}_d = \sum_{n\in\mathcal{D}} \sum_{i\in\mathcal{I}} (\varsigma_{dni} - 1) \left(\frac{\lambda^F_{dni} E^F_{ni}}{S_d} \hat{\lambda}^F_{dni} \hat{E}^F_{ni} + \sum_{j\in\mathcal{I}} \frac{\lambda^M_{dinj} E^M_{inj}}{S_d} \hat{\lambda}^M_{dinj} \hat{E}^M_{inj} + \frac{\lambda^G_{dni} E^G_{ni}}{S_d} \hat{\lambda}^G_{dni} \hat{E}^G_{ni}\right). \tag{29n}$$

3 The Algorithm

The steps for solving the static problem are summarized in Figure 1 below. For some steps that require iterations to find solutions, a criterion for convergence is placed and the program does not continue to the next step until the criterion is met.

Internal consistency requires, among other implications, that expenditures on final goods (for consumption, intermediate use and government procurement) match factor incomes after adjusting for the trade imbalance. Government tariff revenues must match the sum of tariffs applied on all trade flows,

Figure 1: Solution Algorithm for Static Equilibrium



Note: Letters in parentheses refer to the corresponding equations for hat algebra.

government subsidy expenditures must equal the sum of subsidies on all production values. Value-added used to compute μ must be consistent with the trade flows and exclude tariffs paid at each destination region and subsidies paid in each source region. Factor incomes must be consistent with aggregate trade flows using the factor income shares.

A simple way to verify that all the input data are internally consistent is to solve for the counterfactual outcomes without any shock. If the algorithm reaches equilibrium prices, which are all equal to one, in the first iteration then the necessary conditions are satisfied.

4 Data Preparation and Calibration

In our model setup we assume that an initial equilibrium holds before any counterfactual scenario occurs. An essential component of the calibration task is therefore to specify parameters and quantities that are consistent with the equilibrium conditions for the initial states. For counterfactual scenarios, we also need a set of industry-specific trade elasticities. Below, we discuss details of data preparation and calibration. Table 1 provides a summary of the minimum data requirements. We denote with I the number of industries and with N the number of countries.

For comprehensive international trade and domestic production data covering primary industries, manufacturing and services we use the International Trade and Production Database for Estimation (ITPD-E) Release 1 (May 2020) by Borchert et al. (2020). ITPD-E Release 1 covers 243 countries and 170 industries (following the ISIC Revision 4 classification but using FAOSTAT data by the UN Food and Agriculture Organization for more detailed agricultural information and using the WTO-UNCTAD-ITC Annual Trade in Services Database together with the UN'Trade in Services Database for services information) over the period 2000 to 2016. We use the three-year aggregate of 2014-2016 as our data benchmark.

To account for input-output relationships (supply chains) across countries and activities, we use the World Input-Output Database (WIOD) 2016 Release by Timmer et al. (2015) including variables reported in the WIOD Socio-Economic Accounts (SEA). WIOD covers 28 EU countries and 15 other major economies and 56 sectors (using ISIC Revision 4) over the period 2000 to 2014. We use data for the year 2014 to extract shares of supply industries by source country in use industries by destination (under Cobb-Douglas production) as well as expenditure shares of supply industries in (Cobb-Douglas) household and government consumption.

For trade elasticity estimation we require observable trade cost components and use tariffs. We obtain tariff data from the the World Integrated Trade Solution (WITS) data by the World Bank. WITS offers import tariffs imposed by countries for merchandise trade and is in turn based on a combination of the UNCTAD Trade Analysis Information System (TRAINS), reporting tariffs for more than 160 countries, and the World Trade Organization's Integrated Data Base reporting Most Favored Nation (MFN) applied and, if available, preferential tariffs, both at a respective country's most detailed commodity level for national tariffs. We use the reference year 2014.

4.1 Production Technology and Trade Shares

We consider 43 countries plus an aggregate of the rest of the world for mutual consistency between ITPD-E and WIOD. To calibrate the model for 170 industries at the ITPD-E level in each of the 44 regions (43 countries and the ROW as in WIOD) with potential trade among each industry-region pair, we need to construct an input-output matrix with a size of $170 \times 44 \times 170 \times 44$ that covers the volumes of intermediate input across all possible combinations of producer-user pairs at the industry-country level for both the producer and the user. This matrix allows us to construct parameters for production technology, such as the shares of value added and intermediate input use in gross output for each industry-country pair. It also allows us to obtain the initial levels of gross output, intermediate use, and bilateral trade shares across all possible trade partners. An analogous matrix of size $170 \times 44 \times 44$ gives the volumes of final use of each industry-specific goods from all possible source countries for all user countries. From this matrix, we obtain expenditure shares on final goods across industries and trade shares for final goods produced in all possible source countries.

Table 1: Data Requirements for Static Equilibrium

Notation	Dimensions	Description	Sources
θ	I	Industry-specific trade elasticity (in absolute value)	ITPD-E & WITS
η^F	$I \times N$	Consumer expenditure shares on final goods	WIOD & ITPD-E
η^G	$I \times N$	Government procurement shares on final goods	WIOD & ITPD-E
μ	$I \times N$	Shares of value added in gross output	WIOD & ITPD-E
β	$I \times N$	Wage-bill shares of labor input in value added	SEA & ITPD-E
α	$I \times I \times N$	Shares of industry-specific intermediate inputs in gross output	WIOD & ITPD-E
λ_{F0}	$N \times I \times N$	Initial trade shares of final consumption goods by source	WIOD & ITPD-E
λ_{M0}	$I \times N \times I \times N$	Initial trade shares of intermediate inputs by source	WIOD & ITPD-E
λ_{G0}	$N \times I \times N$	Initial trade shares of government procurement goods by source	WIOD & ITPD-E
wL_0	N	Initial region-level labor income	SEA & ITPD-E
rK_0	N	Initial region-level capital income	SEA & ITPD-E
income_0	N	Initial region-level aggregate income (less trade surplus)	WIOD & ITPD-E
M_0	$I \times N$	Initial expenditure levels of intermediate goods by industry	WIOD & ITPD-E
T_0	N	Initial government tariff revenues	
S_0	N	Initial government subsidy expenditures	
L	N	Exogenous region-level total labor endowments	SEA & ITPD-E
K	N	Exogenous region-level total capital endowments	SEA & ITPD-E
NX	N	Exogenous trade surpluses	WIOD & ITPD-E

Notes: I and N denote the numbers of industries and regions. ITPD-E stands for the International Trade and Production Database for Estimation Release 1 (May 2020) by Borchert et al. (2020); WIOD for the World Input-Output Database (WIOD) by Timmer et al. (2015); SEA for variables in the WIOD Socio-Economic Accounts by Timmer et al. (2015); and WITS for the the World Integrated Trade Solution (WITS) data by the World Bank.

Since the WIOD data only contain input-output relations among the more aggregated 38 industries and the ITPD-E data do not contain any information on the user industry, neither the WIOD data nor the ITPD-E data alone contain the complete information. We therefore apply a variant of the Wolsky (1984) disaggregation method to infer a consistent input-output structure for the 170 ITPD-E industries that map into 38 matching aggregates of the 56 sectoral activities in WIOD. This procedure results in one possible decomposition of the input-output trade volumes given data availability. To ensure consistency of the input-output relations with basic accounting identities, we start from the prepared WIOD data, which offer an input-output matrix of size $38 \times 44 \times 43 \times 44$ for 38 ITPD-E matching aggregates of the 56 sectoral activities in WIOD, 44 regions and 43 uses (38 ITPD-E matching activity aggregates plus 5 types of final uses). We then disaggregate each trade volume based on proportionality assumptions that ensure consistency with the relative magnitude implied by the ITPD-E data within each corresponding WIOD-level trade volume.

We encounter two main data limitations that require additional steps. First, the total final use for 178 industry-region pairs are negative in the WIOD data. About half of the negative values are for usage by the rest of the world (ROW). There are also two cases of zeros for the other regions. The main reason for the existence of negative total final use is the presence of negative inventory investment or fixed-capital investment, which are the only final use categories that contain negative values. While there can be temporary and large non-positive final uses from reductions in inventory and disinvestment, in steady-state non-positive final uses are implausible. We, therefore, replace them by values constructed based on average shares of the final use in the corresponding industries across countries in the same income group. Second, for the ITPD-E data, the coverage for the domestic trade flows appear to be incomplete. There are cases in which all trade flows from an industry-region pair are exports, even for many US industries. For this reason, we cannot entirely rely on the original trade flows reported in the ITPD-E data for the disaggregation process.

Considering the data limitations, we take the following steps to disaggregate the WIOD input-output table:

- First, we construct two sets of measures using the ITPD-E data. The first set of measures consist of the share of the gross output of each ITPD-E-level industry-region pair within the corresponding WIOD-level gross output aggregated from the ITPD-E-level values. The second set of measures consist of the shares of the trade volume of each bilateral trade within the corresponding WIOD-level trade volume. Both sets of measures are based on the relative magnitude implied by the ITPD-E data when combining them based on the industry classification. In some cases where none of the trade flows within a WIOD-level industry-region pair are positive, we impute values based on the average shares of the relevant counterparts within countries with the same income level.

- Next, with these shares computed based on the ITPD-E data, we decompose the trade volumes in the WIOD data. We handle domestic flows and cross-region flows separately because of the limitation with the ITPD-E data. For the cross-region flows, we assume that the flow from industry i in region s to industry j in region d satisfy two relations simultaneously. One based on the relative magnitude within WIOD-level source and another based on the relative magnitude within WIOD-level destination. The shares of the flows to the same destination region d within a WIOD-level source satisfy the relative shares of trade flows obtained from the ITPD-E data. The shares of the flows from the same ITPD-level source within a WIOD-level destination satisfy the relative shares of the implied gross output of the user industries based on the ITPD-E data. These two sets of restrictions yield a unique solution to each trade volume from (i, s) to (j, d) given a WIOD-level trade volume.
- For the domestic shares, we proceed in a similar fashion. The difference is that we use shares of ITPD-E-level gross output to restrict the relations for both the source and the destination because of the incomplete coverage of domestic trade from ITPD-E data.

4.2 Factor Income

Value added from production equals the income of labor and capital adopted for production. To determine how that income is partitioned between labor and capital, we utilize the readily available information on factor income from the Socio-Economic Accounts (SEA) accompanying the WIOD data. However, since the industry classification of this dataset matches the 38 industries in WIOD, it does not provide information for each individual industry covered by the 170 ITPD-E industries. Because of this data limitation, we again disaggregate the SEA data based on a proportionality assumption. Specifically, we assume that ITPD-E-level industries nested in the same WIOD industry have the same labor income shares. We then determine the levels of labor income and capital income for each of the 170 industry by combining the labor/capital income shares obtained from the SEA data with the value added data implied by the constructed input-output matrix.

4.3 Estimation of Trade Elasticities

Our model gives rise to aggregate gravity equations for bilateral trade flows. Consequently, we can follow the approach discussed in (Head and Mayer, 2014) and estimate sectoral trade elasticities from observed variation in bilateral trade flows and tariffs across country pairs.

To do so, we adopt the following parametrization of the bilateral iceberg trade costs κ_{sdj}:

$$\log(\kappa_{sdj}) = \beta \mathbf{X}_{sd} + \delta_{sj}^{IMP} + \delta_{dj}^{EXP},$$

where $\mathbf{X}_{sd}$ is a vector of symmetric, bilateral country characteristics capturing non-tariff barriers to trade: Dummy variables indicating whether two countries share a common border, a common language, are both members of the GATT 1995 trade agreement, share a common currency, share a common legal system, as well as the distance between both countries. δ_{sj}^{IMP} and δ_{dj}^{EXP} are industry specific exporter and importer fixed effects.

Using this parametrization of iceberg trade costs and taking logs on both sides of (14) yields the following empirical specification:

$$\log\left(\frac{\lambda^F_{sdj}}{\lambda^F_{ddj}}\right) = \theta^j \log\tau_{sdj} + \alpha^j \mathbf{X}_{sd} + FE^{IMP}_{sj} + FE^{EXP}_{dj} + \epsilon_{sdj}.$$

In this estimation equation, FE^{IMP}_{sj} and FE^{EXP}_{dj} are industry-specific exporter and importer fixed effects, and ϵ_{sdj} is an idiosyncratic error term. Using data on trade flows and tariffs to estimate these gravity equations, we recover the trade elasticity θ^j for all non-service industries, as well as the vector of structural parameters β.

To obtain trade elasticity estimates for industries where trade is not subject to tariffs, we run similar regressions to obtain estimates for α^j for every service sector j. Combined with the previously obtained estimate for β, these estimates allow recovering θ^j for all service industries.

In the sample, we do not observe sufficient variation in tariff for all industries. In some cases, the resulting estimates are too extreme. For this reason, for estimates obtained for the 170 ITPD-E-level industries, we replace some of them by corresponding estimates obtained for the 38 industries from WIOD, when we believe that the estimates for the narrower industry definition ar'e not reliable. We list the estimated trade elasticities by industry in Table 2 on page 18.

5 Examples of Simulated Outcomes

Trade and industrial policies, as well as government procurement policies, affect global economic outcomes. For example, we can simulate the consequences of changes in tariffs at a destination d in the τ_{sdj} matrix by source regions d and industries j; we can assess changes in production or factor input subsidies in a source region s altering the subsidy matrix ς_{sdj} by industries j (and any destination d); we can study changes to procurement that make goods from specific sources s and industries j more expensive for the government at destination d using the κ^G_{sdj} matrix. We briefly discuss in the next Section an application of our simulation model to the U.S.-China Trade War of 2018-2019, which resulted in changes to tariffs τ_{sdj}.

There is a number of outcomes that our simulation model is designed to predict. We present the construction of select candidate measures for outcomes. We denote changes between the initial equilibrium prior to policy changes (no prime) and the new equilibrium after policy changes (with prime) with $\Delta x \equiv x' - x$ for any equilibrium price or quantity x. When we compute outcomes for a subset of industries in a region d, we denote the subset of industries with $\bar{\mathcal{J}}_d$, where $\bar{\mathcal{J}}_d \subset \mathcal{I} := \{0, 1, 2, \cdots, I\}$.

A. Relative change in real aggregate income:

$$\frac{\left(w'_d L_d + r'_d K_d - NX_d\right) / P^{F\prime}_d}{\left(w_d L_d + r_d K_d - NX_d\right) / P^F_d} - 1;$$

B. Relative change in real labor income and real capital returns:

$$\frac{w'_d / P^{F\prime}_d}{w_d / P^F_d} - 1 \quad \text{and} \quad \frac{r'_d / P^{F\prime}_d}{r_d / P^F_d} - 1;$$

C. 1. Employment reallocation flows between industries relative to the endowments:

$$\frac{\sum_{i\in\mathcal{I}} \mathbf{1}_{\Delta L_{si}>0}|\Delta L_{si}|}{L_s} = \sum_{i\in\mathcal{I}} \frac{L_{si}}{L_s} \frac{\mathbf{1}_{\Delta L_{si}>0}\Delta L_{si}}{L_{si}} \quad \text{and} \quad \frac{\sum_{i\in\mathcal{I}} \mathbf{1}_{\Delta L_{si}<0}|\Delta L_{si}|}{L_s|} = \sum_{i\in\mathcal{I}} \frac{L_{si}}{L_s} \frac{\mathbf{1}_{\Delta L_{si}<0}|\Delta L_{si}|}{L_{si}},$$

where $|\Delta L_{si}| = |L'_{si} - L_{si}|$ denotes the absolute value in employment change and $\mathbf{1}_{\Delta L_{si}>0}$ is an indicator for positive employment change, L_s is region s's labor endowment, and $\mathcal{I}$ is the universe of industries, while $\sum_{i\in\mathcal{I}} \Delta L_{si} = 0$ for given endowments (as a benchmark we offer the typical annual changes to these measures in the data for specific regions); to compute employment L'_{si} after policy changes, we express value added in region s's industry i as $\mu_{si}E_{si}$ and note that the Cobb-Douglas share of labor in value added is β_{si}, so that $w'_s L'_{si} / (\mu_{si} E'_{si}) = w_s L_{si} / (\mu_{si} E_{si}) = \beta_{si}$; it then follows that $L'_{si} = \left[(E'_{si}/w'_s)/(E_{si}/w_s)\right] L_{si}$;

C. 2. Capital reallocation flows between industries relative to the endowments:

$$\frac{\sum_{i\in\mathcal{I}} \mathbf{1}_{\Delta K_{si}>0}|\Delta K_{si}|}{K_s} = \sum_{i\in\mathcal{I}} \frac{K_{si}}{K_s}\frac{\mathbf{1}_{\Delta K_{si}>0}\Delta K_{si}}{K_{si}} \quad \text{and} \quad \frac{\sum_{i\in\mathcal{I}} \mathbf{1}_{\Delta K_{si}<0}|\Delta K_{si}}{K_s|} = \sum_{i\in\mathcal{I}} \frac{K_{si}}{K_s}\frac{\mathbf{1}_{\Delta K_{si}<0}|\Delta K_{si}|}{K_{si}},$$

where $|\Delta K_{si}| = |K'_{si} - K_{si}|$ denotes the absolute value in the capital stock change and $\mathbf{1}_{\Delta K_{si}>0}$ is an indicator for positive capital accumulation, K_s is region s's capital endowment, and $\mathcal{I}$ is the universe of industries, while $\sum_{i\in\mathcal{I}} \Delta K_{si} = 0$ for given endowments (as a benchmark we offer the typical annual changes to these measures in the data for specific regions); to compute the capital stock K'_{si} after policy changes, we express value added in region s's industry i as $\mu_{si}E_{si}$ and note that the Cobb-Douglas share of labor in value added is $1-\beta_{si}$, so that $w'_s K'_{si}/(\mu_{si}E'_{si}) = w_s K_{si}/(\mu_{si}E_{si}) = 1-\beta_{si}$; it then follows that $K'_{si} = \left[(E'_{si}/w'_s)/(E_{si}/w_s)\right] K_{si}$;

D. Total value of imports in industries in which imports rise more than p% because of a policy change:

$$\textstyle\sum_{k\in\bar{\mathcal{J}}_d} IM'_{dk},$$

where imports IM_{di} follow from (27) and the subset of industries in region d with an import increase by more than p% is defined as $\bar{\mathcal{J}}_d := \{k \text{ s.t. } \Delta IM'_{dk}/IM_{dk} \geq 1 + p/100\}$;

E. Total value of imports in industries where more than p% of all intermediate goods sourcing is from abroad after a policy change:

$$\textstyle\sum_{k\in\bar{\mathcal{J}}_d} IM'_{dk},$$

where imports IM_{di} follow from (27), the subset of industries in region d with more than p% of all sourcing originating abroad is defined as $\bar{\mathcal{J}}_d := \{k \text{ s.t. } \texttt{crit}'_{dk}\}$ with criterion

$$\texttt{crit}'_{dk} := \frac{\sum_{s\neq d}\sum_j X^{M\prime}_{skdj}}{\sum_s \sum_j X^{M\prime}_{skdj}} \geq 1 + p/100,$$

and X^M_{skdj} are intermediate trade flows (26) to a use industry j (but not final consumption or procurement) in region d from source region s and supply industry k;

F. Total value of imports in industries where intermediate goods sourcing from any one region abroad exceeds p% after a policy change:

$$\textstyle\sum_{k\in\bar{\mathcal{J}}_d} IM'_{dk},$$

where imports IM_{di} follow from (27), the subset of industries in region d with at least one dominant source country that ships more than p% of all sourcing after the policy change is defined as $\bar{\mathcal{J}}_d := \{k \text{ s.t. } \texttt{crit}'_{dk}\}$ with criterion

$$\texttt{crit}'_{dk} := \max_{s\in\mathcal{D}} \frac{\sum_j X^{M\prime}_{skdj}}{\sum_n \sum_j X^{M\prime}_{nkdj}} \geq 1 + p/100,$$

and X^M_{skdj} are intermediate trade flows (26) to a use industry j (but not final consumption or procurement) in region d from source region s and supply industry k;

G. Total value of imports in industries where intermediate goods sourcing from country A exceeds p% after a policy change:

$$\textstyle\sum_{k\in\bar{\mathcal{J}}_d} IM'_{dk},$$

where imports IM_{di} follow from (27), the subset of industries in region d with China as the dominant source country shipping more than p% of all sourcing after the policy change is defined as $\bar{\mathcal{J}}_d := \{k \text{ s.t. } \texttt{crit}'_{dk}\}$ with criterion

$$\texttt{crit}'_{dk} := \frac{\sum_j X^{M\prime}_{s=A,kdj}}{\sum_n \sum_j X^{M\prime}_{nkdj}} \geq 1 + p/100,$$

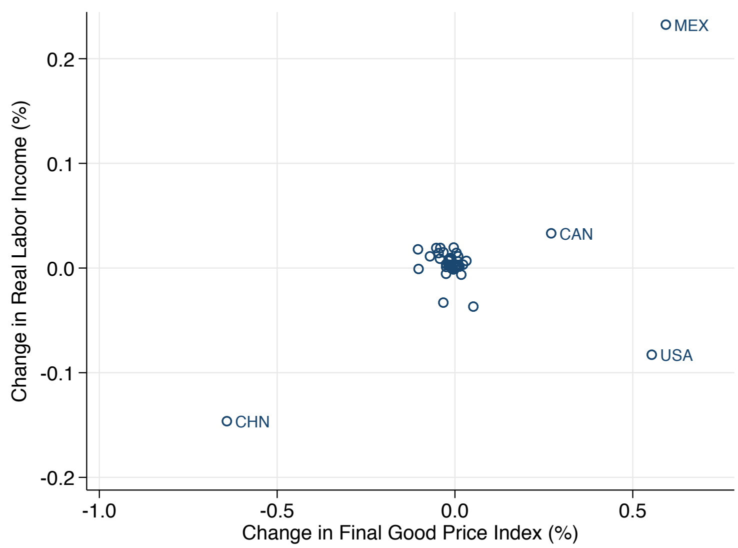
and X^M_{skdj} are intermediate trade flows (26) to a use industry j (but not final consumption or procurement) in region d from source region s and supply industry k.

6 Application: The U.S.-China Trade War

As an illustration for the application of the simulation model, we compare the country-level aggregate outcomes after the rise in tariffs between the United States and China in 2018 with the outcomes in the absence of the U.S.-China trade war. The calibration for the initial equilibrium is based on disaggregated input-output relations that involve 44 countries (including a location for the "rest of the world") and 170 industries based on the 2016 release of the World Input-Output Database (Timmer et al., 2015) and the International Trade and Production Database for Estimation 2020 (Borchert et al., 2020). The changes in tariffs are based on the dataset constructed by Fajgelbaum et al. (2020).

Figure 2 summarizes the changes in real labor income and the final good price indices under the higher tariffs compared with the scenario without the tariff changes. As expected, China and United States are the two economies that are affected the most, with the real labor income being lowered by about 0.1%. Unlike the case for United States where the price index grows by about 0.5%, the price index in China decreases by roughly the same amount. Countries that are not directly facing the tariff changes are not affected much. The exception is that the real labor income in Mexico increases by more than 0.2%. This suggests that the model predicts that the rise in tariffs brings a boost in Mexican export.

Figure 2: Counterfactual Changes after the 2018 Rise in Tariffs between the United States and China



7 Modules

We augment the baseline model in several dimensions to accommodate special cases for simulations.

7.1 CES production, consumption and procurement

To allow for less flexible factor and intermediate input substitution than in the Cobb-Douglas case of Section 1.2, we adopt a CES production function in an extension to the baseline model. Concretely, we modify production function (2) to

$$y_{sj}(\omega) = z_{sj}(\omega) \left((L_{sj})^{\beta_{sj}} (K_{sj})^{1-\beta_{sj}} \right)^{\mu_{sj}} \left[\left(\sum_{i \in \mathcal{I}} (M_{isj})^{\frac{\sigma_{sj}^M - 1}{\sigma_{sj}^M}} \right)^{\frac{\sigma_{sj}^M}{\sigma_{sj}^M - 1}} \right]^{1-\mu_{sj}} \tag{30}$$

and then report results as a function of the choice of elasticity σ_{sj}^M.

Similarly, we modify household consumption (1) and government procurement (3) with a constant elasticity of substitution that is not necessarily unitary.

7.2 Factor immobility

In the short- to medium-run, factor market adjustments can be slow. We therefore consider a specific-factors version of the baseline model, in which both capital and labor are specific to their local industries. The model is the same as specified above, except that factor demand is fixed to industry-specific factor supplies $K_{sj} = \bar{K}_{sj}$, $L_{sj} = \bar{L}_{sj}$, $M_{isj} = \bar{M}_{Isj}$. We solve for the industry-specific factor prices by country.

7.3 Inoperative extensive margin

To consider the consequences of a trade cost shock at impact in the immediate short run, we follow Dekle, Eaton and Kortum (2008). We assume that, after a shock to trade costs changes prices, customers are locked into their previous sourcing decisions. Therefore, they keep buying each variety from the same supplier as before, so that only the intensive margin adjusts and customers can choose how much to spend on each given variety from its current supplier, not where to source it from. As shown in Dekle, Eaton and Kortum (2008), this essentially amounts to replacing the long-run trade elasticities θ_j with the short-run elasticities $\sigma_j - 1$. According to Boehm, Levchenko and Pandalai-Nayar (2021), the proportion of short-term to long-term elasticities is around one-quarter.

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Table 2: Trade Elasticity Estimates

Industry	Elasticity
1 Wheat	−1.10
2 Rice (raw)	−1.10
3 Corn	−1.36
4 Other cereals	−1.92
5 Cereal products	−1.10
6 Soybeans	−1.10
7 Other oilseeds (excluding peanuts)	−2.56
8 Animal feed ingredients and pet foods	−6.58
9 Raw and refined sugar and sugar crops	−1.10
10 Other sweeteners	−1.10
11 Pulses and legumes, dried, preserved	−1.66
12 Fresh fruit	−7.17
13 Fresh vegetables	−5.66
14 Prepared fruits and fruit juices	−3.35
15 Prepared vegetables	−1.10
16 Nuts	−1.60
17 Live Cattle	−1.10
18 Live Swine	−1.10
19 Eggs	−1.10
20 Other meats, livestock products, and live animals	−2.75
21 Cocoa and cocoa products	−1.10
22 Beverages n.e.c.	−2.36
23 Cotton	−1.10
24 Tobacco leaves and cigarettes	−0.24
25 Spices	−3.25
26 Forestry, logging, fishing and aquaculture	−7.35
27 Mining of hard coal	−4.00
28 Mining of lignite	−4.00
29 Extraction crude petroleum and natural gas	−4.00
30 Mining of iron ores	−4.00
31 Other mining and quarrying	−2.33
32 Electricity production, collection, and distribution	−4.00
33 Gas production and distribution	−4.00
34 Processing/preserving of meat	−4.00
35 Processing/preserving of fish	−8.34
36 Processing/preserving of fruit & vegetables	−3.95
37 Vegetable and animal oils and fats	−4.00
38 Dairy products	−4.00
39 Grain mill products	−4.00
40 Starches and starch products	−7.45
41 Prepared animal feeds	−6.70
42 Bakery products	−2.62
43 Sugar	−4.52
44 Cocoa chocolate and sugar confectionery	−4.00
45 Macaroni noodles & similar products	−4.00
46 Other food products n.e.c.	−4.17
47 Distilling rectifying & blending of spirits	−4.00
48 Wines	−1.78
49 Malt liquors and malt	−4.76
50 Soft drinks; mineral waters	−3.40
51 Tobacco products	−3.29
52 Textile fibre preparation; textile weaving	−3.67
53 Made-up textile articles except apparel	−5.86
54 Carpets and rugs	−5.68
55 Cordage rope twine and netting	−3.80
56 Other textiles n.e.c.	−3.67
57 Knitted and crocheted fabrics and articles	−3.67
58 Wearing apparel except fur apparel	−7.89
59 Dressing & dyeing of fur; processing of fur	−2.63
60 Tanning and dressing of leather	−5.85
61 Luggage handbags etc.; saddlery & harness	−3.76
62 Footwear	−7.50
63 Saw milling and planing of wood	−4.36
64 Veneer sheets plywood particle board etc.	−0.24
65 Builders' carpentry and joinery	−1.91
66 Wooden containers	−4.62
67 Other wood products; articles of cork/straw	−5.15
68 Pulp paper and paperboard	−6.28
69 Corrugated paper and paperboard	−8.41
70 Other articles of paper and paperboard	−6.28
71 Publishing of books and other publications	−0.77
72 Publishing of newspapers journals etc.	−0.77
73 Publishing of recorded media	−0.77
74 Other publishing	−0.77
75 Printing	−8.42
76 Service activities related to printing	−5.04
77 Reproduction of recorded media	−5.04
78 Coke oven products	−4.00
79 Refined petroleum products	−4.00
80 Processing of nuclear fuel	−4.24
81 Basic chemicals except fertilizers	−6.89
82 Fertilizers and nitrogen compounds	−3.65
83 Plastics in primary forms; synthetic rubber	−3.65
84 Pesticides and other agro-chemical products	−3.65
85 Paints varnishes printing ink and mastic	−3.65
86 Pharmaceuticals medicinal chemicals etc.	−3.65

Table 2: Trade Elasticity Estimates (continued)

Industry	Elasticity
87 Soap cleaning & cosmetic preparations	−7.05
88 Other chemical products n.e.c.	−7.46
89 Man-made fibres	−3.65
90 Rubber tyres and tubes	−6.66
91 Other rubber products	−6.52
92 Plastic products	−3.46
93 Glass and glass products	−5.79
94 Pottery china and earthenware	−5.16
95 Refractory ceramic products	−5.09
96 Non-refractory clay; ceramic products	−5.50
97 Cement lime and plaster	−4.24
98 Articles of concrete cement and plaster	−3.97
99 Cutting shaping & finishing of stone	−2.52
100 Other non-metallic mineral products n.e.c.	−7.86
101 Basic iron and steel	−6.33
102 Basic precious and non-ferrous metals	−4.00
103 Casting of iron and steel	−4.00
104 Structural metal products	−6.03
105 Tanks reservoirs and containers of metal	−6.64
106 Steam generators	−1.27
107 Cutlery hand tools and general hardware	−6.42
108 Other fabricated metal products n.e.c.	−6.09
109 Engines & turbines (not for transport equipment)	−2.73
110 Pumps compressors taps and valves	−2.42
111 Bearings gears gearing & driving elements	−1.80
112 Ovens furnaces and furnace burners	−2.73
113 Lifting and handling equipment	−2.73
114 Other general purpose machinery	−4.76
115 Agricultural and forestry machinery	−7.31
116 Machine tools	−2.73
117 Machinery for metallurgy	−2.73
118 Machinery for mining & construction	−2.03
119 Food/beverage/tobacco processing machinery	−6.82
120 Machinery for textile apparel and leather	−2.73
121 Weapons and ammunition	−3.68
122 Other special purpose machinery	−1.62
123 Domestic appliances n.e.c.	−6.89
124 Office accounting and computing machinery	−2.79
125 Electric motors generators and transformers	−1.42
126 Electricity distribution & control apparatus	−3.67
127 Insulated wire and cable	−8.46
128 Accumulators primary cells and batteries	−7.45
129 Lighting equipment and electric lamps	−7.35
130 Other electrical equipment n.e.c.	−4.26
131 Electronic valves tubes etc.	−4.65
132 TV/radio transmitters; line comm. apparatus	−7.14
133 TV and radio receivers and associated goods	−7.19
134 Medical surgical and orthopaedic equipment	−5.88
135 Measuring/testing/navigating appliances etc.	−2.79
136 Optical instruments & photographic equipment	−4.34
137 Watches and clocks	−2.79
138 Motor vehicles	−2.32
139 Automobile bodies trailers & semi-trailers	−6.45
140 Parts/accessories for automobiles	−7.69
141 Building and repairing of ships	−0.58
142 Building/repairing of pleasure/sport. boats	−1.31
143 Railway/tramway locomotives & rolling stock	−0.58
144 Aircraft and spacecraft	−0.58
145 Motorcycles	−3.72
146 Bicycles and invalid carriages	−0.58
147 Other transport equipment n.e.c.	−3.33
148 Furniture	−5.08
149 Jewellery and related articles	−5.42
150 Musical instruments	−0.51
151 Sports goods	−2.93
152 Games and toys	−2.91
153 Other manufacturing n.e.c.	−8.00
154 Manufacturing services on physical inputs owned by others	−8.00
155 Maintenance and repair services n.i.e.	−8.00
156 Transport and warehousing	−4.00
157 Travel	−4.00
158 Construction	−4.00
159 Financial, insurance, and pension services	−4.00
161 Charges for the use of intellectual property n.i.e.	−0.77
162 Telecommunications, computer, and information services	−0.77
163 Other professional, technical and business services	−1.45
164 Heritage and recreational services	−4.00
165 Health services	−4.00

Table 2: Trade Elasticity Estimates (continued)

Industry	Elasticity
166 Education services	−4.00
167 Government goods and services n.i.e.	−4.00
168 Services not allocated	−4.00
169 Wholesale and retail related services	−4.00
170 Other personal services	−4.00
199 Nontraded resource, real estate and household management	−4.00

Note: Industry numbers 160 and 171-198 not assigned.