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Natural capital: from metaphor to measurement

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Abstract

Current efforts to value ecosystem services have contributed little to the valuation of natural capital, which suggests that in practice “nature is capital” primarily serves as a metaphor. We provide a theoretically motivated approach for recovering natural capital prices that expands beyond idealized management to encompass current, likely inefficient, management institutions. Theoretically consistent capital valuation requires adjusting for the net marginal productivity of the natural capital asset and price appreciation. We develop a numerical approach, based on polynomial approximation, for approximating the value of capital that integrates estimates from ecological and economic research. Then we demonstrate the approach is capable of recovering the accounting price of capital when it is known a priori. Third, we show that the common approach of valuing natural capital as a simple marginal annuity of ecosystem services suffers from severe biases relative to our full capital-theoretic approach. Finally, we employ the method to value the Gulf of Mexico reef fish complex as a capital asset under real-world management conditions. This generates credible accounting prices for a pound of reef fish. Our approach to valuation reveals the essential interdisciplinarity of natural capital valuation and highlights the critical importance of understanding the feedbacks between the state of natural capital stocks, human behavior impacting these stocks, and the role of institutions in shaping that feedback.

Keywords: natural capital, ecosystem services, bioeconomics, Gulf of Mexico, reef fish, green accounting

Natural capital: from metaphor to measurement

Capital assets store wealth and generate production for future consumption (Hulten 2006). This is certainly a property of ecological structures. Treating natural resources as capital in economic theory goes back at least 200 years to classical economists such as Ricardo and Faustmann (Gaffney 2008), with modern treatment beginning with Hotelling (1931). Economic scholars have developed persuasive arguments and a strong conceptual framework for treating natural stocks as capital (Arrow, et al. 2004; Arrow, et al. 2012; Daily, et al. 2000; Hartwick 1990; Heal 1998; Nordhaus 2006; Weitzman 1976). The National Research Council (2005) states that developing a system of national satellite accounts that include environmental assets is important for understanding economic growth and prosperity. Outside economics, the concept of "natural capital" has gained enthusiastic support among ecologists and other scientists addressing the long-run concerns implicit in questions of conservation and sustainability. Yet, despite strong theoretical support and interdisciplinary buy-in to the *idea* of natural capital, the measurement and incorporation in decision making of the *value* of natural capital has lagged for many for many critical stocks, such as wild populations, biodiversity, wetlands, and forests (Barbier 2011; Devarajan and Fisher 1982; Hartwick 1990; Heal 1998). Markets for natural capital and the flows of ecosystem services they provide are often missing or incomplete, especially for natural stocks with weak exclusion such as common property or public goods (Nordhaus 2006), necessitating the integration of non-market valuation and capital theory to supply decision makers with valid prices. Despite years of progress in the valuation of ecosystem service flows and the application of capital theory to natural resources (Heal 2012) and progress by natural scientists in measuring quantities of natural stocks, the value of natural capital often remains crudely measured at best. The paucity of estimates of the value of natural capital that are grounded in economic capital theory suggests that *in practice* the treatment of nature as capital remains largely metaphorical. This metaphor has considerable conceptual and communicative value (Barbier 2011), but it currently offers little sense of the *in situ* value of resources –

the opportunity cost of their use – to decision makers. Fully delivering on the promise of natural capital requires developing and implementing credible and theoretically grounded techniques for valuing nature as a capital asset. Failing to close the gap between theory and practice may demote natural capital from its current status as a powerful metaphor to an empty buzzword.

The field of bioeconomics routinely models the outcomes of idealized, dynamically efficient markets bound by ecological dynamics as constrained dynamic optimization problems (Clark 2005).[1] Part of identifying the intertemporally optimal allocation is to solve for a differential equation for the adjoint (or co-state) variables associated with each ecological stock. The adjoint variable is a measure of scarcity, measuring the shadow value of the stock *in situ* (Clark 2005; Heal 1998). In theory, this shadow price is a measure of the value of natural capital that could be used for wealth or sustainability accounting.

Shadow prices from optimized bioeconomic models are seldom used to compute measures of wealth or sustainability, guide benefit-cost analysis, or augment national income and product accounts (NIPA) (Arrow, et al. 2012; Heal 2012). This is true despite a fairly rich bioeconomic literature and lack of market prices for most natural capital. As metaphors, bioeconomic models provide resource managers with qualitative guidance about intertemporal tradeoffs in the allocation of resources and aid economists in developing a theory of efficient natural resource use. However, the neglect of bioeconomic shadow values *in practice* suggests that they are not sufficiently pertinent or reliable measures of natural capital value to inform policy or improve wealth or sustainability accounting. One argument for this neglect is that the shadow values from idealized, optimized bioeconomic models are measuring the wrong thing. Institutional and informational constraints matter, and resource management seldom maximizes the discounted economic surplus. Nevertheless, realized shadow values in this 'kakotopia' (Dasgupta 2001) are seldom zero aside from the equilibrium states of resources

[1] Most bioeconomic models use optimal control theory to solve these dynamic optimization problems.

managed under open access institutions (Gordon 1954; Homans and Wilen 1997). In response, a number of scholars have advocated for the use of *accounting prices* that reflect these real-world social and institutional constraints rather than optimized shadow values (Arrow, et al. 2012; Dasgupta and Maler 2000; Hartwick 1990; 2011).[2] The revealed behavior of society conditional on the state of resource stocks under existing institutions, including environmental regulations and cultural norms, implies a value for resource stocks conditional on a criterion for measuring economic welfare. This feedback rule is described as the *economic program* by Dasgupta and Maler (2000). The adjoint equation defines the implicit price path even if the feedback rule given by the economic program does not maximize the net present value of social surplus (Arrow, et al. 2003; Brock and Xepapadeas 2003; Dasgupta and Maler 2000). The adjoint equation reflects society's resource allocation choices associated with any economic program. As long as an adjoint function exists, a price exists for natural capital that reflects, likely inefficient, real-world institutions. The conditions for existence do not rely on the marginal impact of the control function on the Hamiltonian vanishing – the "maximum" part of the "maximum principle" (Leonard and Van Long 1998).

We build on prior theoretical literature (Arrow, et al. 2003; Dasgupta and Maler 2000; Heal 1998) to develop a practical numerical approach for calculating an *accounting price* for a unit of natural capital that is consistent with Jorgenson's (1963) theory of capital in private markets. Our approach helps move natural capital *theory* to *practice*. Our approach is applicable to public natural capital under current management conditions, even with imperfect, non-optimizing institutions (Arrow, et al. 2003; Dasgupta and Maler 2000). Furthermore, our recovered accounting prices are scarcity measures (Batabyal, et al. 2003; Devarajan and Fisher 1982) that decline as resource stocks increase, all else equal. Another desirable feature of our approach is that it allows for natural capital values to be jointly determined with biophysical, human behavioral, and price dynamics, and their associated feedbacks.

[2] We follow Dasgupta and Maler in using the term accounting price. This should be thought of as meaning the appropriate price for wealth or sustainability accounting.

We demonstrate that the numerical methods work well with a simulated example, where the recovered value of natural capital can be compared to known values. We then apply the approach to the Gulf of Mexico (GOM) reef fish stock to demonstrate how it can be implemented through the integration of estimable economic and ecological models whose parameters are recoverable with available techniques and realistic data collection efforts.

1. A theory of natural capital pricing

Let $x(t)$, either a vector or a scalar, represent individual or collective choices related to consumption of market and non-market goods in the economy. Behavioral choices imply, potentially implicit, demand for consumption of the resource. In practice these choices are rarely chosen in a way that maximizes the net present value of the objective function underlying capital valuation (e.g. profits or economic surplus). Instead these choices reflect the shortcomings of ‘kakotopia’ (Dasgupta 2001), with inefficient management institutions, cultural constraints, information shortages, and behavioral departures from rationality all shaping choices. We refer to the totality of these conditioning factors as ‘societal constraints’ and represent them by the notation Ω. Conditional on these ‘societal constraints’, behavior, $x(t)$, can be characterized as a feedback rule of the state of natural capital, $s(t)$.[3] This feedback can occur through direct effects of human capital on decision making as well as indirect effects of changes in the natural capital stock on management policy and hence elements of Ω. Let $M: (s, \Omega) \mapsto x$ be the mapping that describes this feedback rule – the “economic program” structured by the societal constraints Ω. To reduce notation we will treat the dependence on societal constraints as implicit and represent the mapping functionally as $x\big(s(t)\big)$.

We make the common assumption that economic program is time autonomous, an assumption that describes many, but not all, economic programs. This assumption is not completely innocuous;

[3] We treat $s(t)$ as a scalar in this paper for transparency. However, it is conceptually straightforward to extend the theory and numerical methods we develop to the case with multiple state variables (capital stocks).

calendar date, phenology of stocks, exogenous technological or ecological changes, or other non-autonomous dynamics could be important. Nevertheless, time autonomy covers a large number of cases. Furthermore, when the economic program is non-autonomous, the model of the bioeconomic system can often be mapped into an autonomous system through a redefinition of state variables (Arrow, et al. 2003), and Asheim (2000) provides a blueprint for incorporating non-autonomous dynamics.

With the economic program defined, the state of natural capital changes according to

[1] $\dot{s} = G(s) - f\big(s, x(s)\big).$

G is an ecological growth function, which is zero for nonrenewable resources, and f is an anthropogenic impact function for total environmental extraction or degradation of natural capital.

Let $W\big(s(t), x(t)\big)$ be an index of the net benefits accruing to society (or the subset of society that is counted for purposes of capital valuation) at time t. We assume that $W(s, x)$ provides a stable monetary index of social welfare at different levels of s and x so that discounting at a constant exponential rate is justified (Dasgupta, et al. 1999). At time t, the present value of net benefits is

[2] $V\big(s(t)\big) = \int_t^\infty e^{-\delta(\tau - t)} W\big(s(\tau), x(s(\tau))\big)\, d\tau$

where $s(\tau)$ follows Eq [1]. Substitution of the autonomous economic program into W, the assumption of an infinite time horizon, and the assumption that resource dynamics [1] are also autonomous, enables V to be expressed solely as a function of $s(t)$. We maintain the intermediate function $x(s)$ for exposition. However, by substituting through mapping M it is possible to redefine W over the single variable s so that $W^*\big(s(\tau)\big) \equiv W\big(s(\tau), x(s(\tau))\big)$. From now on, at the risk of a slight abuse of notation, we assume the partial derivative of W includes the feedback through the economic program: $W_S\big(s(\tau), x(s(\tau))\big) = \frac{\partial W}{\partial s} + \frac{\partial W}{\partial x}\frac{dx}{ds} = W_S^*$, where the asterisk indicates the elimination of x.[4] δ is the

[4] A similar substitution of the economic program into f yields $f\big(s, x(s)\big) = f^*(s)$ so that $f_S = \frac{\partial f}{\partial s} + \frac{\partial f}{\partial x}\frac{dx}{ds} = f_S^*$.

social discount rate (Dasgupta, et al. 1999). The accounting price of the resource at time t for a given a set of societal constraints, Ω, is

[3] $$p(s(t)) \equiv \partial V\big(s(t)\big)/\partial s(t).$$

Definition [3] states the accounting price of the stock is the change in the net present value to society from holding more natural capital stock *in situ*.

Following Dasgupta and Maler (2000) and Arrow, et al. (2003) we assume V is differentiable and differentiate $V\big(s(t)\big)$, using its definition on the RHS of Eq [2], with respect to t. We suppress the dependence of s of t where doing so does not cause confusion. The result can be expressed as

[4] $$\frac{dV}{dt} = \delta V - W\big(s, x(s)\big).$$

Eq [4] states that the change in the present value of the net benefits with respect to time is equal to the present value of the net benefits multiplied by the discount rate – the equivalent perpetuity – less the current dividend or ecosystem service flow.

The time derivative of Eq [4] can also be expressed as

[5] $$\frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = p(s(t))\,\frac{ds}{dt}.$$

Eq [5] states that the time rate of change of the present value of benefits is equal solely to the effect from changes in the natural capital stock. In the second equality in Eq [5] we substitute in the definition of the natural capital price from [3]. Since Eqs [4] and [5] are identical to one another, it immediately follows that

[6] $$\delta V = W\big(s, x(s)\big) + p(s(t))\dot{s} = H(s, x, p) = H^{*}(s, p)$$

$H(s, x, p)$ is the current value Hamiltonian (CVH), composed of the flow of current benefits (dividends), $W\big(s, x(s)\big)$, and the value of increments to the stock (capital gains), $p\dot{s}$.[5] The substitution of the economic program allows the CVH to be expressed purely as a function of s and p. Note that this CVH is evaluated using a non-optimized economic program. Nevertheless, Eq [6] demonstrates that the CVH is

[5] (Asheim 2000) suggests a generalization for non-autonomous systems.

the current return on the present value of net benefits: $\delta V\big(s(t)\big) = H^*(s(t), p(s(t)))$. The Hamilton-Jacobi-Bellman equation holds unequivocally in the case of non-optimized, autonomous systems (Dasgupta 2009).

Rearranging Eq [6] yields

[7] $$V(s) = \frac{W\big(s, x(s)\big) + p(s(t))\dot{s}}{\delta}.$$

Now differentiate Eq [7] with respect to s

[8] $$\frac{\partial V}{\partial s} = p = \frac{1}{\delta}\big(W_s\big(s, x(s)\big) + p_s\dot{s} + p\dot{s}_s\big) = \frac{1}{\delta}\big(W_s\big(s, x(s)\big) + \dot{p} + p\dot{s}_s\big)$$

The final equality follows directly from Eq [3] since the time-autonomous nature of the problem makes $\dot{p} = p_s\dot{s}$. Rearranging Eq [8] yields

[9] $$\dot{p} = \delta p - W_s\big(s, x(s)\big) - p(G_s(s) - f_s(s, x(s)))$$

Eq [9] is an equation of motion for the price of the natural asset given the value function [2] conditional on the economic program. Eq [9] is the adjoint equation associated with maximization of the current value Hamiltonian when behavior $x(t)$, and hence management institutions Ω, are chosen optimally. However, this derivation is more general and shows that the equation of motion for the natural capital price, Eq [9], holds for any time autonomous economic program – optimization is not required. However, the actual evolution of the price of natural capital is coupled with the system dynamics (include the policy choices) and the initial conditions of the system.

The net present value (NPV) approach is the standard rule for pricing capital in the deterministic case (Dixit and Pindyck 1994). In the context of natural capital assets that may be held unchanging and in perpetuity, the NPV rule is operationalized as an annuity by dividing the marginal benefit from ecosystem service flows resulting from a marginal stock increase by the discount rate (Barbier 2011). Eq [9] can be viewed in light of the NPV rule by rearranging it as follows, where subscripts represent partial derivatives:

[10] $$p = \frac{W_s(s,x(s)) + \dot{p}}{\delta - \left(G_s(s) - f_s(s,x(s))\right)}.$$

Equation [10] generalizes the NPV rule by modifying the numerator by changes in asset value and modifying the discount rate in the denominator by the rate of physical change in the quantity of the natural capital.

Eq [10] corresponds to Jorgenson's (1963) equation (p. 249) for the value of invested capital. Jorgenson's equation, expressed using our notation and setting his "rate of direct taxation" to zero, is

$$W_s(s, x(s)) = p\left(-\dot{s}_s + \delta - \frac{\dot{p}}{p}\right).$$

Jorgenson assumes that the marginal change in production with respect to capital can be multiplied by a constant marginal price per unit output to give the current marginal benefit from an increase in capital stock. Our use of $W_s(s, x(s))$ generalizes this to the non-market case where the "production" associated with natural capital may not have a constant marginal price. Given that Jorgenson develops his model for private capital, he refers to the discount rate, δ, as "the rate of interest." He defines the net marginal productivity of the stock, $-\dot{s}_s$, as the "rate of replacement", which is the (constant) rate of capital depreciation in his model of physical, and hence non-renewable, capital. Finally, the accounting price of the stock, p, is the "price of capital goods" in Jorgensen's framework. Aside from our exclusion of taxation and slightly different nomenclature, our result is identical to Jorgenson's after an algebraic manipulation. Thus Eq [10] provides a rigorous, capital-theoretic basis for deriving an accounting price for capital conditional on economic programs, which may not arise from dynamic optimization.

Eq [10] says the price of a natural capital asset equals the marginal ecosystem service flow, $W_s(s, x(s))$, adjusted by anticipated price (scarcity) changes (i.e. capital gains or losses) divided by a discount rate adjusted for the overall effect on natural capital growth from adding a little more natural capital. Since the economic program is substituted in prior to taking derivatives, the stock changes include both natural (e.g. through density dependence) and human-induced changes through behavioral feedbacks. The adjustments to the discount rate in the denominator of Eq [10] are the net rate of natural capital productivity (NRNCP). This adjustment term creates a wedge between the rate of return to holding natural capital and δ. If $G_s(s) > 0$, then the effective discount rate, the denominator, adjusts downward to reflect the productivity of the renewable natural capital. If increases in natural capital increase anthropogenic degradation, $f_s(s, x(s)) > 0$, then the discount rate is adjusted upward. In all but special cases, marginal annuity estimates (Barbier 2011), e.g., $W_s(s, x(s))/\delta$, can be misleading because they ignore both $\dot{p}$ and the NRNCP.

2. From theory to measurement

Most terms on the right-hand side (RHS) of [10] can be measured and frequently are the subject of, often decoupled, research programs in natural science and economics. For natural resources that are extracted and traded commercially $W_s(s, x(s))$, the marginal static net benefit from an increase in the stock, is the effect of the change in the stock on economic surplus measures (e.g., net revenues in a commercial fishery with a competitive output market). Many natural capital stocks provide benefits, ecosystem services, to society that market transactions do not directly reveal, e.g., wild populations provide recreational hunting, fishing and viewing opportunities as well as non-use values, and wetlands provide flood control and water filtration – showing the vital importance of non-market valuation techniques (Freeman 2003) for quantifying $W_s(s, x(s))$. Nevertheless, [10] also shows that improving measurements of $W_s(s, x(s))$ is only one part of better valuing natural capital.

The estimation of $G(s)$ and $G_s(s)$ are important aspects of ecology, while $f(s, x(s))$ is addressed in multiple disciplines and interdisciplinary work (Ludwig, et al. 2001; Smith 2008). $G(s)$ is the ecological production function, and $G_s(s)$ is the marginal productivity of natural capital in the absence of human impact. The function f is a production function for anthropogenic impacts on the stock of natural capital; $f_s(s, x(s))$ is the marginal product of increased natural capital stocks on how humans impact the stock of natural capital. In commercial extraction cases $x(s)$ is directly targeted at resource consumption, making the specification of $f\big(s, x(s)\big)$ conceptually straight forward, but perhaps empirically challenging (Tsoa, et al. 1985; Zhang and Smith 2011). In most cases, the human behavior embodied in $x(s)$, is not directly focused on extracting resources. Instead, consumption of the resource is a factor of production for the final good or service (e.g. catch of fish as an input to a recreational fishing trip) or a by-product (e.g., reductions in air quality from visiting a national park) (Bockstael and McConnell 1981; McConnell and Bockstael 2006). The explicit capital-theoretic foundation for valuation in Eq [10] reveals why good biophysical science matters for valuation. Valuing natural capital requires interdisciplinary research beyond that required for the valuation of ecosystem services alone.

Both $W\big(s, x(s)\big)$ and $f\big(s, x(s)\big)$ are functions of the economic program $M: (s, \Omega) \mapsto x$. The derivatives of ecosystem service flows and human impacts to changes in natural capital in [10] must therefore include the feedbacks of human adaptation to the change. This implies that measuring human behavioral responses to changes in natural capital given management institutions is a critical aspect of natural capital valuation. In particular, failing to incorporate behavioral adaptation into the marginal valuation of ecosystem service flows may lead to misguided estimation of the value of natural capital.

The remaining terms on the RHS of Eq [10], are the discount rate and the capital gains term. Appropriate discount rates for natural resources are hotly debated (e.g., Abbott and Fenichel 2014; Carpenter, et al. 2007; Dasgupta, et al. 1999; Dietz and Stern 2008; Heal 1998; Sterner and Persson 2008). Many natural capital stocks are probably best viewed as potentially long-lived public capital – therefore suggesting that natural capital should be discounted similarly to other long-lived public investments with similar risk profiles (e.g., infrastructure). In the absence of strong agreement on an appropriate discount rate, a pragmatic, consistent approach is to employ rates dictated by public accounting authorities such as the Office of Management and Budget (OMB).[6]

The capital gains term $\dot{p}$ is the final numerator term on the RHS of Eq [10]. If this term were recoverable from data, then Eq [10] would provide a formula for computing the value of natural capital. However, this term is not directly recoverable from data because natural capital prices are not observable. This fact has been a barrier to measuring the value of natural capital. We provide a solution to this problem. The structure of Eq [10], and the observable terms, make it possible to use function approximation (Miranda and Fackler 2002) to approximate $\dot{p}$.

2.1 The numerical procedure

The $\dot{p}$ approximation procedure is as follows.

First, the expression for the autonomous economic program $x(s(t))$ must be found conditional on the societal constraints, Ω. In the case of an optimized system, this feedback rule may be derived from the optimization itself. In applications it may be estimated or calibrated from data. The economic program is then substituted into both W and f so that they are both functions of $s(t)$ alone. Therefore, Eq [10] can be expressed as $p = \frac{W_S^*(s(t))+\dot{p}}{\delta-\left(G_S(s(t))-f_S^*(s(t))\right)}$.

Second, let

$$\dot{p} = \frac{dp\left(s(t),x(s(t))\right)}{dt} \approx \mu\left(s(t)\right) = \sum_{n=0}^{N-1} \beta_n \phi_n(s(t))$$

where μ is an approximating function chosen by the analyst. μ is specified as a linear combination of N basis functions (Miranda and Fackler 2002) a set of functions of $s(t)$ that span a particular function space. In practice polynomials of increasing degree are commonly employed as basis functions. The

[6] If natural capital were privately owned, then a private discount rate might be more appropriate. However, when natural capital is privately owned, it may also be possible to observe market prices as lower bounds on the value of natural capital.

coefficients of the linear combination, the β_n, determine the weighting of the basis functions. They are chosen as part of the algorithm below to maximize the quality of the approximation of $\dot{p}$. We use Chebyshev polynomials in the following applications. Chebyshev polynomials have gained popularity in computational economics, and more generally in computational functional approximations, because they are orthogonal over the defined approximation interval (Miranda and Fackler 2002; Press, et al. 2007; Vlassenbroeck and Van Dooren 1988). The orthogonality property is desirable because it distributes the error between the approximating and unknown true function evenly, and this feature makes is generally the best polynomial specification for functional approximation (Press, et al. 2007).

Third, choose a number of distinct evaluation points (i.e., approximation nodes) for s. These can be selected in a number of ways, but at least N points are needed to provide solutions for the β coefficients of a $N-1$ degree Chebyshev approximation.[7] Once the approximation nodes for s are chosen, evaluate the expressions W_S^*, G_s, and f_S^* in Eq [10], but maintain the dependency on $s(t)$.

Fourth, substitute $\mu\big(s(t)\big)$ for $\dot{p}$ into Eq [10] to yield

[11] $$p\big(s(t)\big) = \frac{W_S^*(s(t))) + \mu(s(t))}{\delta - \Big(G_S\big(s(t)\big) - f_S^*\big(s(t)\big)\Big)}.$$

Now differentiate [11] with respect to t, remembering to use the equation of motion for $ds(t)/dt$, e.g., Eq[1]. After differentiating, the right-hand side yields a long expression that is solely a function of the parameters of functions W_S^*, G_s, and f_S^*; δ; stock size, s; and the parameters of the approximating function, $\beta_0, \dots, \beta_{N-1}$. Also, upon differentiating Eq [11] with respect to time, by definition $\frac{dp}{dt} = \dot{p} \approx \mu\big(s(t)\big)$. Therefore

[12] $$\mu\big(s(t)\big) - \frac{d\left(\frac{W_S^*(s(t))) + \mu(s(t))}{\delta - \left(G_S\left(s(t)\right) - f_S^*\left(s(t)\right)\right)}\right)}{dt} \approx 0$$

[7] Traditionally, the approximation coefficients are determined for a $N-1$ degree approximation by evaluating at the "zeros" (roots) of an N-degree Chebyshev polynomial. This procedure generates closed form solutions for the coefficients (Press, et al. 2007).

Assuming the parameters of functions W_s^*, G_s, and f_s^* have been previously estimated reliably, then for any point $s = s(t)$, the remaining unknown quantities are the coefficients of the approximating polynomial, $\beta_0, \ldots, \beta_{N-1}$. For an approximating function with N coefficients, if we choose N values for s, then Eq [12] yields a system of N equations in N unknowns that can be solved exactly for $\beta_0, \beta_1, \ldots, \beta_{N-1}$. Alternatively, we can evaluate the left hand side of [12] at more than N values for s, resulting in an over-determined system so that [12] cannot be satisfied with exact equality at all evaluation nodes. In this case, we solve for the coefficients of the approximating polynomial function that minimizes the sum of squared residuals in the system. Increasing the degree of the approximating polynomial provides an arbitrarily close approximation to $\dot{p}$ (Miranda and Fackler 2002). Once the approximation coefficients are determined $\mu(s)$ provides a value for the missing quantity $\dot{p}$ at any $s(t)$ that can be used to calculate p using Eq [10], providing the final term needed to consistently price natural capital conditional on existing management institutions.

3. Applications

The capital theoretic framework expressed in Eq [10] combined with our numeric approximating method for recovering $\dot{p}$ can be used in practice to recover accounting prices. To show the validity of our numerical approximation method, we first apply it to cases where the accounting prices are known and can be compared to our recovered values. Generally speaking, accounting prices are only known for optimized systems. Therefore, our first application is a version of the classic nonlinear commercial fisheries problem (Clark 2005). We use the known solution for this case to simulate the time path of natural capital, human behavior, and the adjoint. We then apply our approximation approach and compare the results to the known adjoint variable values (a second known example is provided in the appendix). Second, we demonstrate how real-world data can be used to apply the approach to the GOM reef fish complex. Our application to the GOM reef fish complex provides a credible price function that has properties consistent with theory.

3.1 A known example

We model harvest as $h(s,x) = qs^{\frac{1}{2}}x^{\frac{1}{2}}$, where q is a catchability coefficient and x is a measure of effort. This generalized Schaefer specification of harvest has the attractive feature of diminishing returns to fish stock and effort. Assume

[13] $W = mh - cx$

[14] $F = \dot{s} = rs(1 - s/k) - h,$

where the market price is $m = 10$, cost of effort is $c = 10$, the stock follows logistic growth with an intrinsic growth rate $r = 0.5$ and carrying capacity $k = 1{,}000$. We assume $\delta = 0.05$. Formally, the social planner's problem, replicating the outcome of a dynamically efficient market, is expressed as

$$\max_{x(t)} \int_0^\infty e^{-\delta}\left(mqs^{\frac{1}{2}}x^{\frac{1}{2}} - cx\right)dt \qquad \text{s.t. } \dot{s} = rs\left(1 - \frac{s}{k}\right) - qs^{\frac{1}{2}}x^{\frac{1}{2}}$$

The current value Hamiltonian for this problem is

$$H = mqs^{\frac{1}{2}}x^{\frac{1}{2}} - cx + p\left(rs\left(1 - \frac{s}{k}\right) - qs^{\frac{1}{2}}x^{\frac{1}{2}}\right)$$

Where p is the adjoint variable. The first order necessary and adjoint conditions that are necessary for an optimal solution to the infinite horizon problem are

$$\frac{\partial H}{\partial x} = 0 \text{ and } \dot{p} = \delta p - \frac{\partial H}{\partial s}$$

The infinite horizon nature of the problem and joint concavity of the Hamiltonian ensure that setting $\dot{s}$ and $\dot{p}$ equal to zero and using $\partial H/\partial x = 0$ results in a system of three equations in three unknowns whose solution is the long run optimal equilibrium (Fig 1a). This system can be linearized around this equilibrium. The eigenvalues associated with the Jacobian matrix are real and of mixed signs, indicating a conditionally stable saddle point. The eigenvector associate with the negative eigenvalue is followed to perturb the system from equilibrium to initiate numerical simulations backwards in time (Fig 1). The solution to the problem in state-control space, which is the economic program for this example, is illustrated in Fig 1a by the saddle path leading to the optimal long run equilibrium. All numerical work was conducted in Mathematica 9.0 (Wolfram Research).

Having solved this standard bioeconomic model and simulated its solution, we sample 160 points, evenly spaced in time, along the simulated saddle paths to collect "data" and record the actual numerical value of the adjoint variable at each of these points. The black dots in Fig 1b are actual values of the adjoint variable. We sampled 160 values of $h(x(s))$, W_s, $F_s(s, x(s))$ for points along the saddle path (Fig 1), and using a 15$^{\text{th}}$ order Chebyshev polynomial for μ, we minimize the sum of square deviations associated with Eq [12]. The solid curve in Fig 1b is the approximate value of natural capital using Eq [11]. The real and recovered values hardly differ, with a mean approximation error of 0.007%

Consider alternative approximation approaches. If only $\dot{p}$ were ignored, thereby implicitly assuming $\dot{p} = 0$, then the general pattern of accounting prices overestimates the adjoint variable values for stocks below the steady state and underestimates for stocks greater than the steady state (gray dot-dashed curve Fig 1b). Adjusting for the NRNCP but omitting the influence of capital gains has fairly small errors for stock sizes close to the steady state but yields larger errors otherwise, particularly at low stock levels. Treating the value of natural capital as a marginal shift in an annuity, and using W_s/δ as the value of the natural capital, results in estimated accounting prices almost an order of magnitude greater than the true value due to the substantial negative value of the NRNCP (Fig 1b). Unlike omitting capital gains, the effects of failing to correct the rate of discount for the net rate of natural capital productivity do not dissipate near bioeconomic equilibrium.

3.2 A non-optimized system: The Gulf of Mexico reef fish complex

The GOM reef fish stock is a substantial regional resource that consists of 62 jointly managed species. Six species make up most of the catch: gag grouper, gray triggerfish, greater amberjack, red grouper, vermilion snapper, and red snapper. Reef fish are caught using a range of gears, most commonly handline. The fishery is a mixed species fishery with imperfect selectivity across species. Historically, many species in the reef fish complex were overfished. In 1984, a fishery management plan (FMP) was adopted to rebuild stocks "on the basis of maximum sustainable yield (MSY) as modified by relevant economic, social, or ecological factors."[8] The FMP was enforced using a complex set of regulations involving entry restrictions, catch per trip limits, season closures, and spatial closures based on information from stock assessments. Some of these restrictions were loosened in 2007 and 2010 with the staged introduction of individual tradable quotas, ITQs, for key reef fish species.

We consider the fishery prior to the introduction of ITQs – a time when the most prominent stocks in the complex were generally considered overfished. During this period stocks and catch were relatively stable (Fig 2).[9]

Our objective is to obtain an accounting price for an *in situ* living pound of reef fish vulnerable to fishing gear *conditional on the non-optimized management that persisted in the fishery at this time*. We follow Zhang and Smith (2011) and model the vulnerable reef fish complex as a single stock growing logistically. Zhang and Smith (2011) estimate

[15] $$\dot{s}(t) = 0.3847s(t)\left(1 - \frac{s(t)}{3.59\times10^{8}}\right) - h(s(t), x(t)) \text{ with } h\big(s(t), x(t)\big) = qs(t)x^{\alpha}$$

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[8] http://www.gulfcouncil.org/fishery_management_plans/reef_fish_management.php

[9] The most recent assessment year reports that provide the measurements we need for calibration were used. We aggregated the caught biomass across species as well as reported biomass estimates (Fig 2). The reported biomass estimates do not focus solely on vulnerable stock. All caught fish are vulnerable. The fishery data used to calibrate the parameter y come from NOAA's Southeast Data, Assessment and Review documents (SEDAR), http://www.nmfs.noaa.gov/sfa/hms/SEDAR/SEDAR.htm.

where s is the stock of reef fish in pounds.[10] The second RHS term in Eq [15] is the generalized Schaefer harvest function, where x is fishing effort measured in crew-days. The catchability coefficient, $q = 3.17 \times 10^{-4}$, is calibrated to replicate pre-2005 average catch per unit effort levels observed in National Marine Fisheries Service logbooks from the fishery at the equilibrium catch and effort levels defined below. The technology parameter, $\alpha = 0.544$, is set to Zhang and Smith's estimated value for handline gear. Zhang (2011) estimates the empirical effort response function – the "economic program"

[16] $x(s) = ys^{\gamma}$,

with $\gamma = 0.7882$ conditional on pre-ITQ management. We calibrate the parameter $y = 0.157$ so that the system equilibrates at the mean level of biomass harvest between 1986 and 2004 – implying a stock at 24% of carrying capacity (Fig 3, solid curve). This calibration reflects the judgment that managers were implicitly (whether intentionally or not) targeting sustained yields and stocks significantly below the stated MSY goals of the FMP (or there were substantial "relevant economic, social, or ecological factors"). This judgment is supported by the fairly stable pattern of overall reef fish catch and biomass over these years (Fig 2). This pattern is consistent for many of the commercially valuable individual stocks. The effort response model empirically builds in behavioral responses to changes in the fish stock, and implicitly accounts for contemporary management institutions in a reduced form manner through the values of its parameters. This provides a model of the economic program. W is defined as in Eq [13]. We use National Marine Fisheries Service logbook and trip cost survey data to calibrate $m = \$2.70$/lb of reef fish, and $c = \$153$ per crew-day in 2005. We follow OMB's 2012 appendix C for 30 year plus projects and use an annual discount rate of 0.02. We then contrast our results to OMB's base case discount rate of 0.07. Our qualitative results are not overly sensitive to this range of variation in the discount rate.

[10] We adjust Zhang and Smith's monthly parameter estimates to an annual time scale where necessary.

We choose 500 stock sizes between 500,000 lbs and carrying capacity as the roots of a 500th order Chebyshev polynomial. Next we use a 50th order Chebyshev polynomial to approximate μ. The average deviation per simulated data point is 5.4×10^{-6}. Increasing the polynomial order had no measurable effect on the quality of approximation.[11] A summary of results at the steady state and at 50% of the steady state for the annuity approach, an approximation ignoring price changes, and the complete approximation are shown in Table 1.

The value per pound of *in situ* reef fish conditional on pre-2005 management institutions varies with stock size from $1.80/lb to over $10/lb (Fig 4) – with a value of $3.08 at the steady state. As expected, the accounting price function is downward sloping in stock. Ignoring $\dot{p}$ overestimates the accounting price at low stocks and underestimates it at high stocks (Fig 4). It is interesting to compare these accounting prices to the share trade prices after ITQs were introduced to the red snapper fishery in 2007. The ITQ policy allocated fishermen transferable rights to shares (percentages) of the total allowable catch; the transfer price of these shares should reflect the present value of expected future profit flows (Newell, et al. 2005). Prices for shares of red snapper, one of the more highly valued species of the six that we aggregate over, have steadily increased over successive seasons. The equivalent price per pound transferred was $8.73 in 2007 and $25 in 2011.[12] Theory suggests that ITQs should alter the economic program, including potentially altering the economic (m and c), technological (q and α), and behavioral (y and γ) parameters of the model through changes along the extensive and intensive margins of effort (Smith 2012), and thereby raising the value of the stock. Such share prices suggest that our substantially lower recovered accounting prices – reflecting comparatively poor management institutions – are reasonable. The marginal annuity approach substantially overestimates accounting

[11] A narrower range of stocks, limited to the range of the data, was also tried as approximation points, with little influence on the recovered accounting price over that range.

[12] http://sero.nmfs.noaa.gov/sf/ifq/2011_RS_AnnualReport_Final.pdf

prices per pound, as in the prior example, but also results in an upward sloping accounting price function (in Fig 4 in \$x10), which is driven by the economic program. Specifically, the upward sloping price function associated with the annuity approach results from the slope and curvature of the generalized Schafer production function when combined with the estimated behavioral response (economic program) of fishermen.

Computing an accounting price for reef fish facilitates including the reef fish stock, as a capital asset, in a comprehensive wealth measure or the NIPA, conditional on the current management. For instance, the pre-2005 management equilibrium (24% of carrying capacity) leads to an accounting price of \$3.08/lb using a 2% discount rate and \$2.54/lb using a 7% discount rate. Valuing the entire stock at this marginal price implies a total stock capitalization or the component of "comprehensive wealth" (Arrow, et al. 2012) attributable to the stock of between \$265.5M (2% discount rate) and \$219.1M (7% discount rate).[13] Alternatively, at equilibrium where there is only a dividend flow of ecosystem services but no change in natural capital stocks or prices, the annual net benefits that flow from the fishery are \$24.7M (2% discount rate), which yields a net present value, V, of \$1.24B (2% discount rate) or \$353M (7% discount rate). These provide measures of the value of the fishery that are only valid at equilibrium.

A management action or positive environmental shock that made the stock less scarce and increased the vulnerable stock to the level supporting MSY (0.5k), without altering the nature of the fisher response function, would reduce the accounting price to \$2.33/lb, but would increase the asset value to \$418.9M (assuming a 2% discount rate). This range of capitalization is on par with "small caps"

[13] We are *not* suggesting that such a linear extrapolation from a marginal price is valid as a measure of welfare evaluation for the "value of the stock" (i.e. willingness to pay for preservation of the stock). Rather we are utilizing the recovered accounting price of natural capital in the same way that market prices of capital are employed in NIPA and wealth accounting. We believe the "market" capitalization comparison is useful for context and serves to put natural capital on equal footing with financial capital.

publically traded companies. Such changes should be accounted for in adjusted net or comprehensive investment measures as well as the NIPA. Given the lack of management change – so that stocks will eventually be fished back to their pre-intervention equilibria – realizing MSY represents a temporary increase of $153.4M in natural capital holdings using the capitalization value. Fig 4 suggests that ignoring price changes still gives qualitatively accurate results. However, if capitalized value is used as a metric, which may be the case in wealth accounting, the NIPA, or the measurement of comprehensive wealth (Arrow, et al. 2012), then Table 1 shows that ignoring price changes could lead to qualitatively different conclusions about whether a change in the stock is wealth enhancing.

Given that the application of the model only considered ex-vessel prices and harvesting costs, these capital values only reflect the value of fish capital for the harvesting sector, holding all other human, fishing, or processing capital constant. They are not a measure of consumer surplus in the final product market or processor-related producer surplus. Therefore, our marginal accounting prices, like the ITQ share price, are lower bounds for the marginal social value of natural capital in this system. Nevertheless, our approach can readily be extended to include non-market values conditional on available estimates.

The finding that the current value Hamiltonian is the current rate of return on the value function implied by the economic program, $\delta V\big(s(t)\big) = H^{*}(s(t), p(s(t)))$, suggests that the CVH is an appropriate instantaneous index for comparing the net benefits resulting from alternative economic programs. The current value Hamiltonian captures the idea that the instantaneous flow of benefits in a given period from natural capital depends on both its ecosystem service dividends and wealth accumulation associated with net investments in natural capital, changes in fish stock, multiplied by p.

It is possible to compare the relative wellbeing generated by two alternative economic programs, perhaps the result of alternative institutional arrangements, by comparing the value of H

using the accounting prices implied by the two programs.[14] To illustrate this approach consider a stylized proposed management change. In our model, a policy could alter parameters related to behavior and the fishing production function. For example, consider a new gear restriction, simulated by reducing α, (e.g., by 10%). The logic of a gear restriction is that it makes a unit of effort less effective, so fewer fish are caught, resulting in a larger stock size. This has the intended biological effect (Fig 3 dashed curve), but making fishing less productive will also likely encourage a behavioral response through a reduction in effort at a given stock level – modeled here as a reduction in γ (e.g., by 6%).[15] In this particular case, together these two effects lead to an even more conservative harvest and larger equilibrium stock size (Fig 5, dotted line).[16] The policy change influences the economic program and reduces p, except at small stock sizes (Fig 5). Fig 5 shows the equilibrium values (Xs on the figure) of the current value Hamiltonian. These values are nothing more than the equilibrium ecosystem service dividends because stocks are not changing. The value of H with the management change (dotted curve) exceeds the value of H without the management change (solid curve). However, simply comparing equilibria could be misleading, since it is possible that the path to the new equilibrium under the alternative economic program is more costly than remaining at the current (calibration) equilibrium. The ability to calculate a valid accounting price conditional on the stock size and economic program enables us to assess the change in the current value Hamiltonian (the annualized difference in the value

[14] Such comparison are regularly made in optimized systems (e.g., Horan, et al. 2011; Rondeau 2001).

[15] Making investments in substitute gear not affected by the regulation could offset the behavioral response and drive up the cost of a unit of effort (Deacon, et al. 2011). Such a response is not included in this illustration. The potential for such a “capital stuffing” response highlights the need to be able to forecast counterfactual economic programs.

[16] We do not mean to suggest that this is a general fishery management result. We simply mean to illustrate how accurately measuring the accounting price could be useful for policy evaluation.

function) at the calibration stock size *under the alternative* economic program (Fig 5, circle). Interestingly, at the calibration stock size W is less under the alternative economic program, and the top panel of Fig 5 shows that p is also reduced. Yet, the change in stock size (investment) is large enough to offset this forbearance.[17] The fact that at all values of the current value Hamiltonian associated with the alternative program are above the value of the baseline current value Hamiltonian suggests that such a change is uniformly welfare enhancing. This result occurs because of the depleted state of the equilibrium fish stock in the baseline economic program. Interestingly, the improvement in the current value Hamiltonian is driven by the reduction in effort at a given stock size, the reduction in γ. Technical change alone, the reduction in α, actually reduces the value of the current value Hamiltonian unless effort is also suppressed by the technical change. These comparisons demonstrate that the method developed here to recover accounting prices is not only useful for valuing natural capital, but also contributes to measuring the relative desirability of current and alternative economic programs.

Discussion

Natural capital may be a large share of the wealth of nations, but in many cases it remains underpriced in markets and unaccounted for in public accounts. What goes unmeasured often goes unvalued – thereby contributing to the poor management of many natural capital stocks. Failure to manage and invest in natural capital means that society is dissipating wealth, reducing future productivity, and potentially jeopardizing the sustainability of wellbeing and wealth of future generations.

The natural capital metaphor is a powerful communication tool. However, if “nature is capital” is to be more than a metaphor, then the valuation approach must integrate economic and natural science

[17] The fact the CVH under the alternative economic program exceeds the CVH under the baseline economic program at a given stock size, may not guarantee that the CVH associated with the alternative economic program will always remain greater than the CVH associated with the baseline program overtime in all cases.

metrics within a rigorous capital-theoretic framework. Many critical policy questions can be framed as decisions to invest or divest natural capital, but investment decisions should be based on the totality of dividends, capital gains, and asset growth. Research is advancing the valuation of the dividends from natural capital, ecosystem services, but this is not the same as valuing natural capital. Treating the value of natural capital as an annuity of marginal ecosystem service dividends is incomplete, and our simulations suggest, potentially highly misleading. Modifying the annuity approach to account for the asset growth associated with natural capital can perform well in the neighborhood of a steady state when capital gains are small, but poorly elsewhere. We have developed and successfully demonstrated a numerical approach to approximate the rate of capital gains implicit in existing management institutions, conditional on 1) the marginal value of ecosystem services, 2) the net marginal productivity of natural capital at different stock levels, and 3) the choice of discount rate. Our numerical approach provides a linchpin for internally consistent valuation of natural capital while also suggesting important implications for future work.

First, substantial improvements in the valuation of natural capital can be made by appropriately adjusting the “discount rate” for the marginal productivity of the natural capital stock. This requires explicitly bringing biophysical science measurements into valuation computations, including the productivity of natural capital in the absence of human impact, and the effect of natural capital on human degradation of the capital stock. Natural capital valuation is fundamentally interdisciplinary – even beyond the level required for the valuation of ecosystem services.

Second, we demonstrate that both the marginal value of ecosystem services and the adjustment to the discount rate require the substitution of an “economic program” predicting the link between human behavior and the natural capital stock. Understanding the feedback between the state of natural capital and human behavior – mediated by markets, regulations, social norms and other institutions that mold this behavior – is imperative to valuation. This necessitates an expansive role for economics in natural capital valuation extending well beyond the "normative" valuation of ecosystem services to also encompass the positive science of understanding human behavioral feedbacks in response to changes in the state of nature and the incentive structures provided by management institutions. While there is ample research considering these behavioral feedbacks for the purposes of avoiding potentially unwelcome feedbacks to natural capital stocks, this literature has largely existed outside the literature on valuation.

Finally, our framework concretely illustrates the necessity of incorporating the two-way feedbacks between natural systems and human behavior under non-idealized economic programs, when valuing natural capital. Valuation without incorporating feedbacks is incomplete. So, just as understanding integrated ecological-economic systems requires valuation; valuation requires understanding feedbacks. The current value of the natural "wealth of nations", and its future rise or fall, hinge critically on the wisdom of the human behavior embodied in the economic program at other values of the capital stock and the effects of this behavior on natural capital.

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Appendix

Second example

As an additional example of the validity of our numerical approach in the case where the adjoint path is known, consider a problem adapted from Conrad and Clark's natural resource text, p.45 (Conrad and Clark 1987). They assume (using our notation) that $W = 20\ln(s) - 0.1x^2$, $\dot{s} = x - 0.1s$. The sole modification we make to Conrad and Clark's (1987) problem is to assume $\delta = 0.02$, rather than zero. This change is necessary to apply the annuity approach. This could be thought of as a problem where human actions enhance rather than degrade a natural capital stock suffering from a constant rate of depreciation. Investment x comes at an increasing marginal cost to current net benefits, and the capital stock provides benefits purely as a function of the stock with no degradation associated with capital services. The solution to the optimal control problem, which follows the same steps as for the example in the main text, is illustrated in Fig A1 by the separatrices or saddle paths that lead to the optimal long run equilibrium. We simulate points along the saddle path (Fig A1), values of $x(s)$, W_s, and $\dot{s}_s(s, x(s))$. We then construct a 18$^{\text{th}}$ order Chebyshev polynomial (Miranda and Fackler 2002), and recover the accounting prices. The recovered accounting prices are illustrated by the black solid curve in Fig A1. The numerical approach to recovering the adjoint variable works well, with most of the approximated prices being off by much less than \$0.01 and a mean deviation of 0.04%. The accounting prices implied by coarser approximations that ignore $\dot{p}$ follow the same pattern as in the main text, overestimating p at low levels of natural capital and underestimating p at high levels of natural capital. Like the examples in the text, the annuity approach values are much greater than the true values.

Table 1. Summary of accounting prices and capitalization value for the GOM reef fish complex at the steady state and 50% of the steady state using a 2% discount rate.

Natural capital valuation approach		Natural capital marginal value, p	Stock capitalization value, ps
Marginal annuity approach	Steady state	\$36.55	\$3.156B
	50% of steady state	\$18.94	\$818M
Ignoring price changes	Steady state	\$3.08	\$265.5M
	50% of steady state	\$9.98	\$431M
Full approximation approach	Steady state	\$3.08	\$265.5M
	50% of steady state	\$4.11	\$177M

Figure Legends

Fig 1. The upper panel is the solution to the “known” example fishery problem. The lower panel shows the actual shadow value for each stock size simulated (black dots) and the approximation of the shadow or accounting price (black solid curve). The gray points are the accounting price setting $\dot{p} = 0$ and the gray dashed line is the value based on the annuity approach.

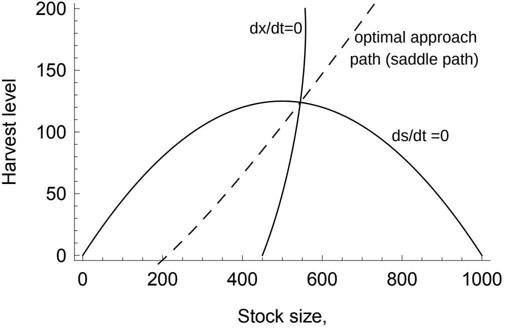
Figure 2. Time series of aggregated biomass (squares) and biomass of harvest (circles) of reef

Fig 3. The logistic growth function for reef fish and the empirical stock-harvest response function (solid black curves), with approach paths decomposing an alternative economic program with labeling relative to base levels of α and γ.

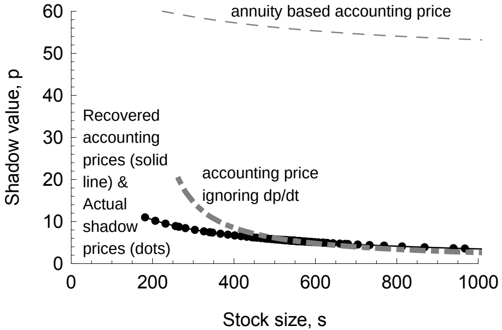
Fig 4. The approximated accounting price (black curve), approximated accounting price ignoring $\dot{p}$ (dot-dashed curve), and the simple annuity price of reef fish (dashed curve).

Fig 5. Accounting prices and the current value Hamiltonian by stock size under alternative economic programs: the base case (solid line), 10% reduction in α (dashed line), 6% reduction in γ (dot-dashed line), and a joint 10% reduction in α and 6% reduction in γ (dotted line). Vertical dotted lines show the calibration equilibrium and the equilibrium that emerges following the management change.

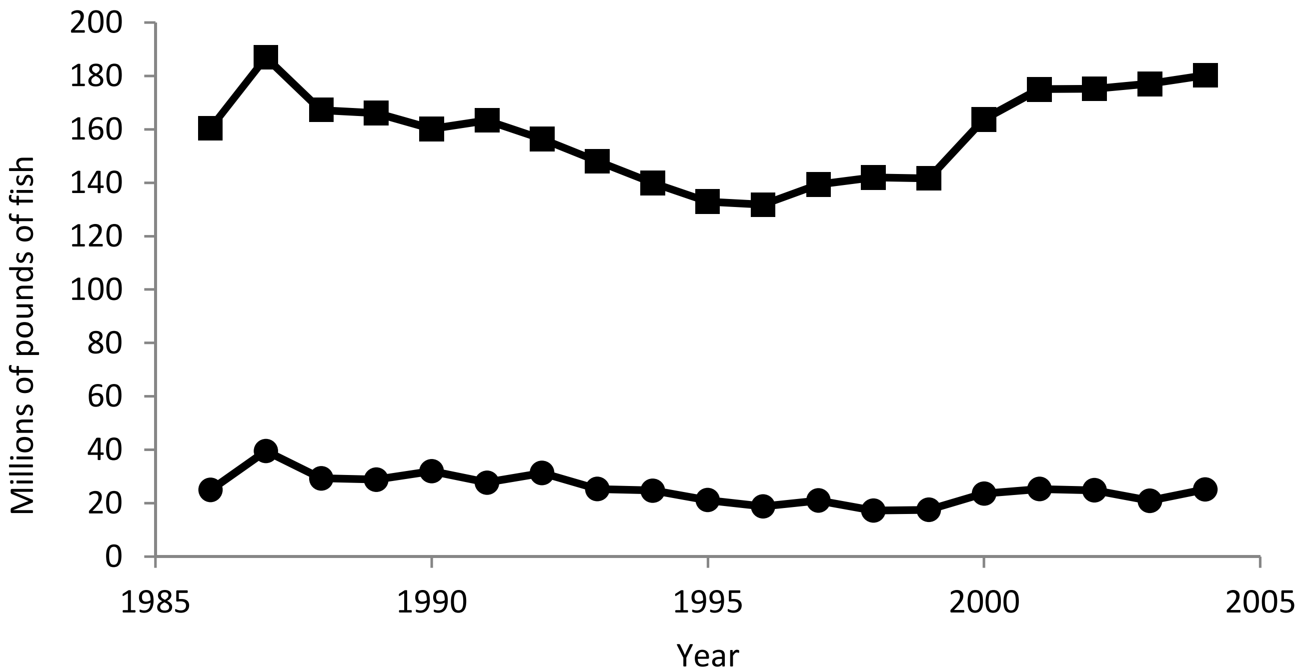
Fig A1. The upper panel is the solution to Conrad and Clark’s (1987) problem. The solid curves are null-clines and the dashed curves are the optimal approach path to the equilibrium. The lower panel shows the actual shadow value for each stock size simulated (points) and the approximation of the shadow or accounting price (black line). The gray dot-dashed curve are the approximated accounting prices if assuming $\dot{p} = 0$, and the gray dashed line is the marginal annuity approximation, W_s/δ.

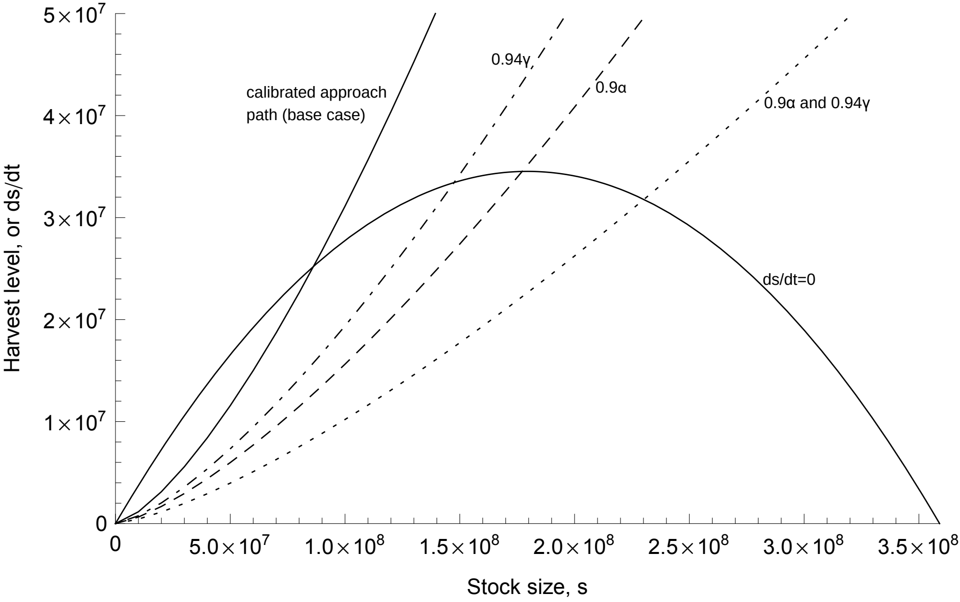


dx/dt=0
optimal approach path (saddle path)
ds/dt =0
Harvest level
0
50
100
150
200
0
200
400
600
800
1000
Stock size,

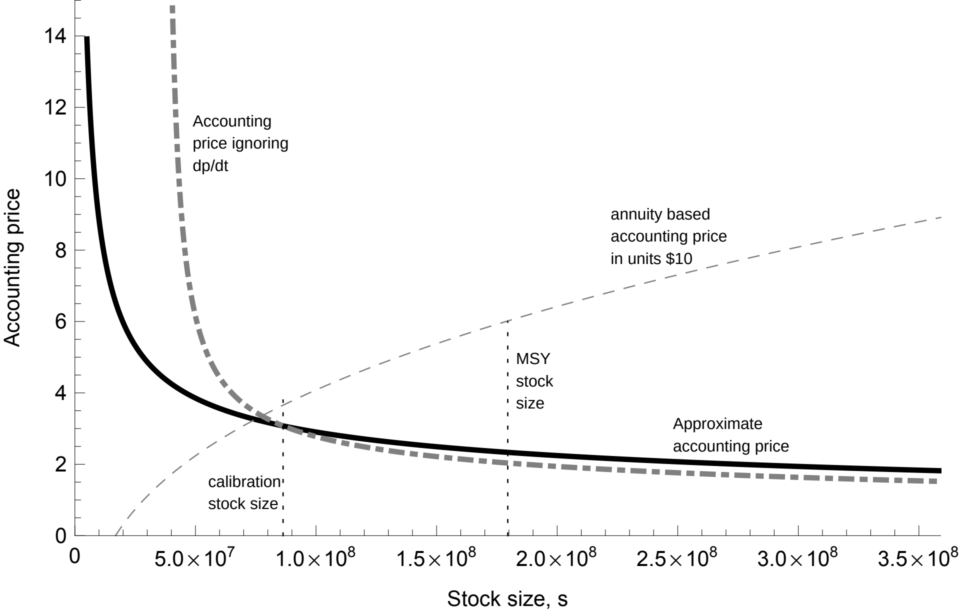


annuity based accounting price
Recovered accounting prices (solid line) & Actual shadow prices (dots)
accounting price ignoring dp/dt
Shadow value, p
0
10
20
30
40
50
60
0
200
400
600
800
1000
Stock size, s

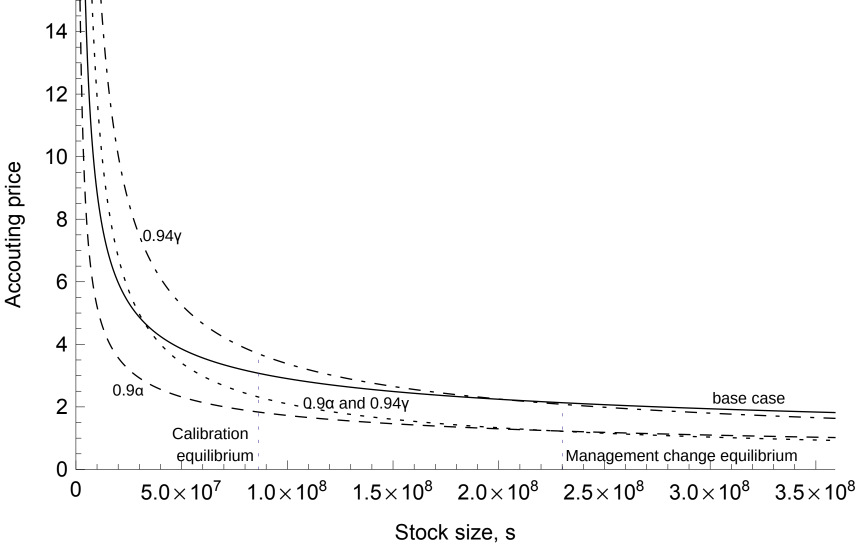

200
180
160
140
120
100
80
60
40
20
0
Millions of pounds of fish
1985
1990
1995
2000
2005
Year

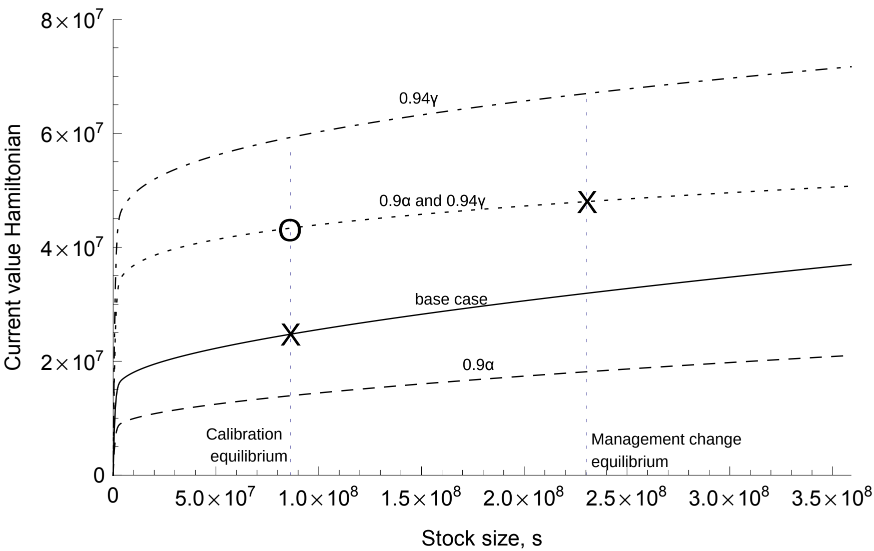


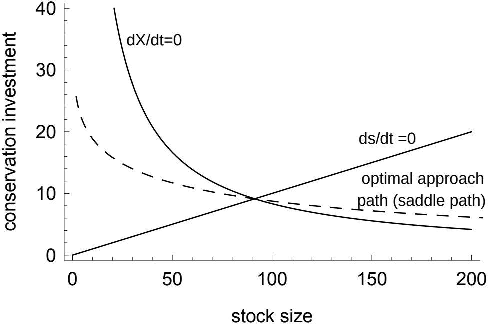
5×10^7
4×10^7
3×10^7
2×10^7
1×10^7
0
Harvest level, or ds/dt
calibrated approach
path (base case)
0.94γ
0.9α
0.9α and 0.94γ
ds/dt=0
0
5.0×10^7
1.0×10^8
1.5×10^8
2.0×10^8
2.5×10^8
3.0×10^8
3.5×10^8
Stock size, s



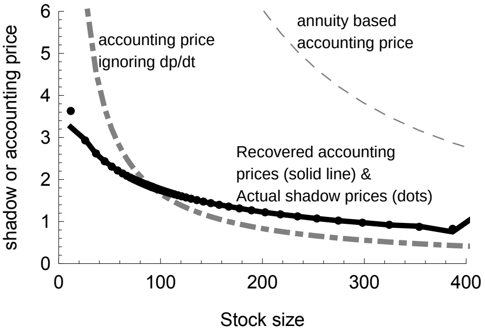
Accounting price
14
12
10
8
6
4
2
0
Accounting price ignoring dp/dt
annuity based accounting price in units $10
MSY stock size
Approximate accounting price
calibration stock size
0
5.0×10^7
1.0×10^8
1.5×10^8
2.0×10^8
2.5×10^8
3.0×10^8
3.5×10^8
Stock size, s


14
12
10
8
6
4
2
0
Accouting price
0.94γ
0.9α
0.9α and 0.94γ
base case
Calibration
equilibrium
Management change equilibrium
0
5.0×10^7
1.0×10^8
1.5×10^8
2.0×10^8
2.5×10^8
3.0×10^8
3.5×10^8
Stock size, s


8×10^7
6×10^7
4×10^7
2×10^7
0
Current value Hamiltonian
0.94γ
0.9α and 0.94γ
O
X
base case
X
0.9α
Calibration
equilibrium
Management change
equilibrium
0
5.0×10^7
1.0×10^8
1.5×10^8
2.0×10^8
2.5×10^8
3.0×10^8
3.5×10^8
Stock size, s



dX/dt=0
ds/dt =0
optimal approach
path (saddle path)
conservation investment
stock size
0
10
20
30
40
0
50
100
150
200



accounting price
ignoring dp/dt
annuity based
accounting price
Recovered accounting
prices (solid line) &
Actual shadow prices (dots)
shadow or accounting price
Stock size
0
1
2
3
4
5
6
0
100
200
300
400