MEANINGFUL THEOREMS: NONPARAMETRIC ANALYSIS OF REFERENCE-DEPENDENT PREFERENCES*

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Abstract: This paper derives nonparametric conditions for the existence of reference-dependent preferences that rationalize a price-taking consumer's demand behavior. Unless reference points are modelable and sensitivity is constant, reference-dependent models of consumer demand are flexible enough to fit virtually any data. Assuming modelable reference points and constant sensitivity, we characterize continuous reference-dependent preferences, relaxing Kőszegi and Rabin's (2006; "KR") strong functional-structure assumptions. We use our characterization to re-analyze Farber's (2005, 2008) data on cabdrivers' labor supply. Relaxing KR's assumptions greatly increases a nonparametric measure of predictive success. For many drivers, a relaxed reference-dependent model has greater success than its neoclassical counterpart. (*JEL* C14, C23, D11, D12, J22)

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Kahneman and Tversky's (1979; "KT") analysis of prospect theory (see also Tversky and Kahneman 1991) introduces a model of individual decisions in which people have preferences over gains and losses relative to a reference point. Such reference-dependence alters the domain of preferences from levels of outcomes to changes in outcomes; but it remains consistent with a complete and transitive preference ordering over changes, thus not inherently irrational. Although Kahneman and Tversky focus on changes alone, Kőszegi and Rabin (2006; "KR") and most recent analyses allow preferences to respond to both levels and changes, and such reference-dependence also need not be irrational.

This paper studies the leading microeconomic application of reference-dependent preferences, to consumer demand. Reference-dependent consumer theory is a main focus of Tversky and Kahneman's (1991) and KR's landmark theory papers and is now the basis of many structural econometric studies of consumer demand, finance, housing, and labor supply. Our analysis addresses some issues raised by those theoretical and econometric studies by deriving and applying nonparametric conditions for existence of reference-dependent preferences that rationalize a price-taking consumer's demand behavior.

Most of the econometric studies build on Camerer et al.'s (1997) classic analysis of New York City cabdrivers' labor supply.[1] Their model is analogous to a model of consumer demand for earnings and leisure, taking a driver's earnings per hour as a proxy for the wage. In a neoclassical model of demand, with preferences over levels of earnings and leisure, the elasticity of hours with respect to the wage is positive unless there are very large income effects. Camerer et al., however, estimate a strongly negative elasticity.

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[1] Cabdrivers are of particular interest in labor supply because many choose their own hours, unlike most workers in modern economies. Another important impetus to empirical applications of reference-dependent models is Kahneman, Knetsch, and Thaler's (1990) experimental analysis of the endowment effect, whereby people's reservation prices for goods they own often exceeds their willingness to pay to acquire them. More recent applications include Oettinger (1999), Genesove and Mayer (2001), Fehr and Goette (2007), Post et al. (2008), Pope and Schweitzer (2011), Lien and Zheng (2015), and Meng and Weng (2018).

To explain this anomaly Camerer et al. propose a model in which drivers have daily earnings targets, which they suggest could be proxied by the average daily level of earnings. The targets are analogous to KT's reference points, suggesting that drivers' demand behavior might usefully be viewed as rational, but with preferences over a domain larger than the neoclassical one, including changes in as well as levels of consumption, as suggested by reference-dependence.[2] Experiments suggest most people are loss-averse—more sensitive to changes below their targets (losses) than above them (gains). If drivers have reference-dependent preferences, loss aversion creates kinks in their preferences that, depending on the details, may make their earnings bunch around the targets, so they work less on days with higher wages. A reference-dependent model can then reconcile an observed negative earnings elasticity of hours with a positive neoclassical incentive effect of wages.

Farber (2005, 2008) econometrically analyzes another dataset on New York City cabdrivers. In his data, as in Camerer et al.'s, drivers' hours worked and earnings per hour are negatively correlated. He finds that a model with daily earnings targets treated as latent variables fits significantly better than a neoclassical model. But his estimates of the targets are unstable, which he argues limits reference-dependence's usefulness in modeling labor supply.[3]

In a theory paper inspired by Camerer et al.'s and Farber's analyses, KR propose a more general model of reference-dependent preferences with particular attention to the needs of economic applications. KR assume that utility is additively separable across separate components of neoclassical consumption utility and reference-dependent "gain-loss" utility. Unlike KT and Farber, who take no definite position on how reference points are

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[2] One might still argue that income targeting is irrational because it makes a driver sacrifice earnings in exchange for something less tangible. But we use the term rational in the general sense of choice consistency in some domain—just as needed for our nonparametric analysis.
[3] Farber (2015) studies a much larger dataset on New York City cabdrivers and finds evidence of reference-dependence, but he again concludes "…gain-loss utility and income reference-dependence is not an important factor in the daily labor supply decisions of taxi drivers."

determined, KR close their model by equating a consumer's reference points to his good-by-good rational expectations of consumption outcomes.

Like Camerer et al.'s model, KR's can reconcile a negative overall correlation between hours and earnings per hour with the neoclassical prediction that higher wages anticipated tend to increase labor supply: With perfectly anticipated changes in earnings or hours, gain-loss utility drops out of their model, which then replicates the neoclassical prediction. But with unanticipated changes, loss aversion makes daily earnings tend to bunch around its reference point, which may yield a negative overall correlation.

Crawford and Meng (2011; "CM") adapt KR's model to reconsider Farber's (2005, 2008) econometric analyses, using Farber's data.[4] Instead of limiting drivers to earnings targets, CM allow both hours and earnings targets. Instead of treating targets as latent variables, CM model them via natural sample proxies, in the spirit of Camerer et al.'s daily earnings averages and KR's rational-expectations reference points. Modeling the targets avoids the unstable estimates that made Farber doubt his reference-dependent model and appears to yield a useful reference-dependent model of drivers' behavior.

Although reference-dependent models sometimes allow rationality-based explanations of behavior that is anomalous from a neoclassical point of view, several factors have limited their appeal. Because they expand the domain of preferences, many researchers doubt that they yield any testable implications at all—Samuelson's (1947) "meaningful theorems". Such doubts are exacerbated if reference points are not observed or modeled.

More specifically, the theoretical and empirical studies of reference-dependent consumer behavior we are aware of make strong ancillary

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[4] Thakral and Tô (2021) study New York City cabdrivers using a 2013 dataset comparable to Farber's (2015) dataset, replicating CM's main findings and adding an informative analysis of the dynamics of drivers' reference points. Brandon et al. (2023) study Lyft drivers in four U.S. cities. In the observational part of their study they replicate the main conclusions of previous observational studies. However, in their field experiment random windfalls to drivers yield statistically insignificant effects on their stopping decisions—suggesting that for most drivers, e mental accounting of windfalls differs from that of income earned by driving.

assumptions on functional structure. Tversky and Kahneman (1991), Camerer et al. (1997), KR, Farber (2005, 2008, 2015), CM, and all other such studies assume—in our view naturally—that preferences are additively separable across components of consumption and gain-loss utility. All but sometimes KR assume that preferences have constant sensitivity (Tversky and Kahneman's sign-dependence; KR's A3′).[5] And all assume—again in our view naturally—that utility is continuous across gain-loss regimes.

Less naturally, the theoretical and empirical studies also make further functional-structure assumptions that are not directly supported by theory or evidence. All assume, explicitly or implicitly, that gain-loss utility is determined by good-by-good differences between realized and reference consumption utilities. And all, including KR with constant sensitivity, take the sum of consumption and gain-loss utility that determines consumer demand to be additively separable across goods and impose KR's constant-sensitivity knife-edge restrictions on how that sum's marginal rates of substitution vary across gain-loss regimes (CM, Table 1).[6] Finally, the empirical studies all rely on functional form assumptions.[7] Additive separability across goods is viewed as unacceptable in applications of neoclassical consumer theory. Our analysis shows it is no better justified in reference-dependent consumer theory.

McFadden (1985) remarks that using econometrics to flesh out the theory in this way "interposes an untidy veil between econometric analysis and the propositions of economic theory." Are the empirical successes or failures of reference-dependent models due to reference-dependence, or are they artifacts of assumptions on functional structure or form—assumptions maintained, without theoretical justification or testing, in all previous work in this area?

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[5] As explained below, a reference point divides commodity space into gain-loss regimes, such as "earnings loss, hours gain" in labor supply. With constant sensitivity preferences over consumption bundles must be the same throughout a regime but may vary across regimes.
[6] With constant sensitivity the last two restrictions follow from KR's assumption about how gain-loss utility relates to the differences between realized and reference consumption utilities.
[7] Farber (2005, 2008, 2015), CM, Thakral and Tô (2021), and Brandon et al. (2023) all limit reliance on such functional form assumptions in various ways, but none eliminate it.

This paper begins to lift the veil by deriving nonparametric conditions for the existence of reference-dependent preferences that rationalize a price-taking consumer's demand behavior. Like KR we assume preferences are additively separable across consumption and gain-loss utility and we assume rationality, but we expand the domain of preferences in the disciplined way suggested by reference-dependence to include changes in as well as levels of consumption.[8]

We first show that unless reference points are precisely modelable or observable (henceforth "modelable" for short) and sensitivity is constant, reference-dependent models of consumer demand are flexible enough to fit any data, with a minor qualification for modelable reference points. Our results for these cases identify a grain of truth in the common belief that allowing reference-dependence destroys the parsimony of neoclassical consumer theory. They also suggest that analyses that, like Farber's, treat targets as latent variables may be as strongly influenced by the constraints they impose in estimating the targets as by reference-dependence per se.

Next, assuming modelable reference points and constant sensitivity, we characterize continuous reference-dependent preferences. Our characterization derives, from continuity, KR's functional-structure assumption that gain-loss utility is determined, additively separably across goods, by good-by-good differences between realized and reference consumption utilities. But it also relaxes KR's assumption that the sum of consumption and gain-loss utility that determines a consumer's demand is additively separable across goods, and with it KR's restrictions on how its marginal rates of substitution vary across gain-loss regimes. It thereby allows us to assess the extent to which empirical

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[8] By contrast, Farber (2008, p. 1070) retains the viewpoint of neoclassical preferences over levels of consumption alone and concludes that most of his drivers are irrational: "This [earnings-targeting] is clearly nonoptimal from a neoclassical perspective, since it implies quitting early on days when it is easy to make money and working longer on days when it is harder to make money. Utility would be higher by allocating time in precisely the opposite manner." There are other nonparametric theoretical analyses of reference-dependent models, including Gul and Pesendorfer (2006); Abdellaoui, Bleichrodt, and Paraschiv (2007); Ok, Ortoleva, and Riella (2015); and Freeman (2017, 2019). All but Gul and Pesendorfer (2006) and Freeman (2017), discussed in footnote 11, focus on different aspects of the problem.

analyses that model reference points and assume constant sensitivity identify effects of reference-dependence, or are artifacts of their ancillary assumptions.

Our characterization would allow a more general parametric econometric re-analysis of Farber's (2005, 2008) data, using sample proxies like CM's for KR's rational-expectations model of the targets. Here, instead, we choose to illustrate our characterization's empirical potential by using it to re-analyze Farber's data nonparametrically, again using sample proxies for the targets.

Section I introduces our model of reference-dependent preferences.

Section II presents our theoretical analysis, in several subsections. Our Propositions 1 and 2 show that unless reference points are modelable and sensitivity is constant, a reference-dependent model of consumer behavior has no useful refutable implications. Proposition 3 characterizes continuous reference-dependent preferences with constant sensitivity, relaxing KR's assumptions in that case of additive separability across goods and their restrictions on how marginal rates of substitution vary across gain-loss regimes. Proposition 3 shows how to estimate the relaxed model structurally. Propositions 4 and 5 then show how to estimate the model nonparametrically.

Section III illustrates Proposition 5's methods for recovering rationalizing reference-dependent preferences when they exist, using Farber's (2005, 2008) data to reconsider his and CM's econometric analyses. We estimate driver by driver, as in most nonparametric demand analyses. We control for models' varying flexibility using Beatty and Crawford's (2011, pp. 2786-87) proximity-based variant of Selten and Krischker's (1983) and Selten's (1991) nonparametric measure of predictive success, which judges flexibility by the likelihood that random data would fit a model. Using the proximity-based measure, we strongly reject KR's assumption of additive separability across goods. Relaxing additive separability and KR's restrictions on marginal rates of substitution, for many drivers our reference-dependent model fits more than enough better than its neoclassical counterpart to justify its greater flexibility.

Section IV is the conclusion.

I. Reference-dependent Preferences

We consider reference-dependent preferences in settings with a finite number of consumer demand observations for a single consumer—or equivalently for a pooled group of consumers assumed to have homogeneous preferences—but we will speak of a single consumer. We index goods $k = 1,\ldots, K$ and observations $t = 1,\ldots, T$. We assume that the consumer is a price-taker, choosing among consumption bundles $\boldsymbol{q} \in \mathbb{R}_+^K$ with linear budget constraints. His preferences are represented by a family of utility functions $u(\boldsymbol{q}, \boldsymbol{r})$, parameterized by an exogenous reference point $\boldsymbol{r} \in \mathbb{R}_+^K$, conformable to a *K*–good consumption bundle as in Tversky and Kahneman (1991) and CM.[9] If reference points are unmodelable, the data are prices and quantities $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\ldots,T}$, with hypothetical reference points $\{\boldsymbol{r}_t\}_{t=1,\ldots,T}$. If reference points are modelable (or observable), the data are prices, quantities, and reference points $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}$. We sometimes denote goods by scalars indexed by superscripts, so for $k = 1,\ldots, K$, $\boldsymbol{q} \equiv (q^1, \ldots, q^K)$ and observation $t = 1,\ldots, T$, $\boldsymbol{q_t} \equiv (q_t^1, \ldots, q_t^K)$, with analogous notation for $\boldsymbol{p}$, $\boldsymbol{p_t}$, $\boldsymbol{r}$, and $\boldsymbol{r_t}$.

To describe preferences that respond positively to changes in consumption relative to the reference point as well as to levels, we take the consumer's utility function $u(\boldsymbol{q}, \boldsymbol{r})$ to be strictly increasing in $\boldsymbol{q}$ and strictly decreasing in $\boldsymbol{r}$. Our specification is then as flexible as a general strictly increasing function of levels $\boldsymbol{q}$ and changes $\boldsymbol{q} - \boldsymbol{r}$. It nests the neoclassical case where preferences respond only to levels; KT's and Tversky and Kahneman's (1991) case where they respond only to changes; and cases like Camerer et al.'s (1997), Farber's (2005, 2008), KR's, and CM's where preferences respond to both. As in those papers we take $u(\boldsymbol{q}, \boldsymbol{r})$ to be continuous in $\boldsymbol{q}$ and $\boldsymbol{r}$; and we assume that preferences have consumption utility and gain-loss utility components that

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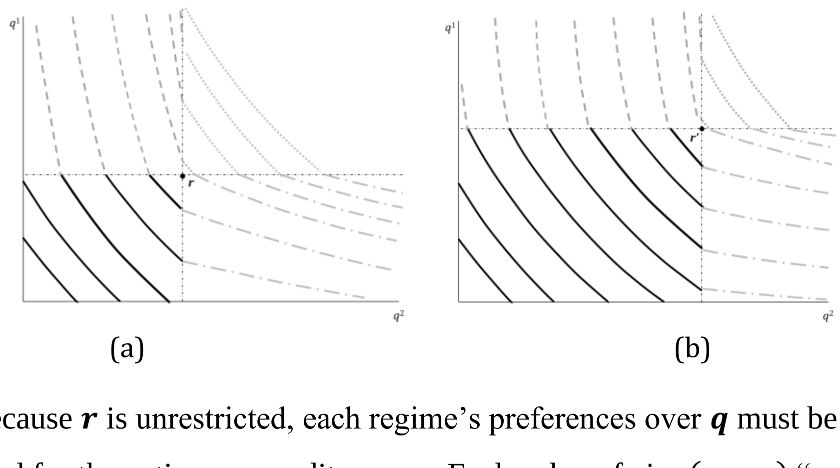
[9] In KR's theoretical model, which makes no allowance for errors, only probabilistic targets make possible the unanticipated changes in outcomes that allow expectations-based reference-dependence to have any effect. CM take advantage of sampling variation to simplify KR's probabilistic targets to point expectations, as we do here.

enter $u(\boldsymbol{q}, \boldsymbol{r})$ additively separably, with the consumption utility function the same for all gain-loss regimes and independent of the reference point.

We call the general case of preferences that can be represented by a utility function $u(\boldsymbol{q}, \boldsymbol{r})$ in the class just described variable sensitivity. An important special case is constant sensitivity (Tversky and Kahneman's 1991 sign-dependence; KR's assumption A3′). Let $\text{sign}(\boldsymbol{q} - \boldsymbol{r})$, the vector whose *k*th component is $\text{sign}(q^k - r^k)$, be the good-by-good sign pattern of gains and losses. A reference point divides commodity space into gain-loss regimes, throughout each of which $\text{sign}(\boldsymbol{q} - \boldsymbol{r})$ remains constant. With constant sensitivity a consumer's preferences over $\boldsymbol{q}$ must be the same and independent of $\boldsymbol{r}$ throughout a given regime but may vary freely across regimes.

DEFINITION 1: [Preferences and utility functions with constant sensitivity.] A reference-dependent utility function $u(\boldsymbol{q}, \boldsymbol{r})$ satisfies constant sensitivity if and only if, for any consumption bundles $\boldsymbol{q}$ and $\boldsymbol{q}^*$ and reference points $\boldsymbol{r}$ and $\boldsymbol{r}^*$ such that $sign(\boldsymbol{q} - \boldsymbol{r}) = sign(\boldsymbol{q}^* - \boldsymbol{r}) = sign(\boldsymbol{q} - \boldsymbol{r}^*) = sign(\boldsymbol{q}^* - \boldsymbol{r}^*)$, $u(\boldsymbol{q}, \boldsymbol{r}) \geq u(\boldsymbol{q}^*, \boldsymbol{r})$ if and only if $u(\boldsymbol{q}, \boldsymbol{r}^*) \geq u(\boldsymbol{q}^*, \boldsymbol{r}^*)$.

Figure 1. A set of regime maps with constant sensitivity and the associated global map for alternative reference points



Because $\boldsymbol{r}$ is unrestricted, each regime's preferences over $\boldsymbol{q}$ must be defined for the entire commodity space: Each value of $\text{sign}(\boldsymbol{q} - \boldsymbol{r})$ "switches on" a different regime's preferences. With two goods, a reference point in the interior of commodity space divides it into four regimes. Figure 1's panels show four regime indifference maps and the associated global indifference maps for reference points $\boldsymbol{r}$ and $\boldsymbol{r}'$. The shift from $\boldsymbol{r}$ to $\boldsymbol{r}'$ does not alter the regime maps, but as $\boldsymbol{r}$ varies, even locally, the shift alters how those maps connect across regimes, and thereby alters the global map.

II. Theoretical Analysis

This section presents our theoretical analysis. Section II.A reviews Afriat's (1967), Diewert's (1973), and Varian's (1982) nonparametric analyses of neoclassical consumer theory, on which our analysis builds.[10]

Assuming that utility is additively separable across consumption and gain-loss utility, two factors determine whether reference-dependent preferences can rationalize choice behavior: whether sensitivity is constant or variable; and whether reference points are unmodelable or precisely modelable or observable (henceforth "modelable" for short). (None of our results depend on the *interpretation* of reference points.) Sections II.B-C show that, unless reference points are modelable *and* sensitivity is constant, the hypothesis of reference-dependent preferences has no useful refutable implications.

Assuming modelable reference points and constant sensitivity, Section II.D then characterizes preferences that are continuous. Finally, Section II.E uses the characterization to derive nonparametric conditions for a rationalization.

A. *Neoclassical Rationalization*

The classic nonparametric analyses of consumer demand in the neoclassical case where preferences respond only to levels of consumption are Afriat (1967), Diewert (1973), and Varian (1982). In the revealed-preference tradition of Samuelson (1948) and Houthakker (1950), they show that a price-

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[10] The benefits of a nonparametric approach to consumer demand are well understood. The theory's testable implications are inequality restrictions on observable, finite data, rather than shape restrictions on objects that are not directly observable. They can be checked directly without estimating econometric models of unobservable objects such as indifference, demand, or labor supply curves. The theory also largely avoids the need for the auxiliary statistical assumptions that structural econometric approaches require for consistent estimation. Measurement error is an exception, but it too can be handled nonparametrically (Varian 1985).

taking consumer's demand behavior can be nonparametrically rationalized by the maximization of a nonsatiated utility function if and only if the data satisfy the Generalized Axiom of Revealed Preference (henceforth "GARP").

DEFINITION 2: [Generalized Axiom of Revealed Preference ("GARP").] $\boldsymbol{q_s} R \boldsymbol{q_t}$ *implies* $\boldsymbol{p}_t \cdot \boldsymbol{q}_t \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_s$*, where R indicates that there is some sequence of observations* $\boldsymbol{q}_h, \boldsymbol{q}_i, \boldsymbol{q}_j, \dots, \boldsymbol{q}_t$ *such that* $\boldsymbol{p}_h \cdot \boldsymbol{q}_h \geq \boldsymbol{p}_h \cdot \boldsymbol{q}_i, \boldsymbol{p}_i \cdot \boldsymbol{q}_i \geq \boldsymbol{p}_i \cdot \boldsymbol{q}_j, \dots, \boldsymbol{p}_s \cdot \boldsymbol{q}_s \geq \boldsymbol{p}_s \cdot \boldsymbol{q}_t$.

AFRIAT'S THEOREM: The following statements are equivalent:
[A] There exists a utility function $u(\boldsymbol{q})$ *that is continuous, non-satiated, and concave, and that rationalizes the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$.
[B] There exist numbers $\{\boldsymbol{U}_t, \lambda_t > 0\}_{t=1,\dots,T}$ *such that*
(1) $\quad U_s \leq U_t + \lambda_t \boldsymbol{p}_t \cdot (\boldsymbol{q}_s - \boldsymbol{q}_t)$ *for all* $s, t \in \{1, \dots, T\}$
[C] The data $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$ *satisfy GARP.*
[D] There exists a non-satiated utility function $u(\boldsymbol{q})$ *that rationalizes the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$.

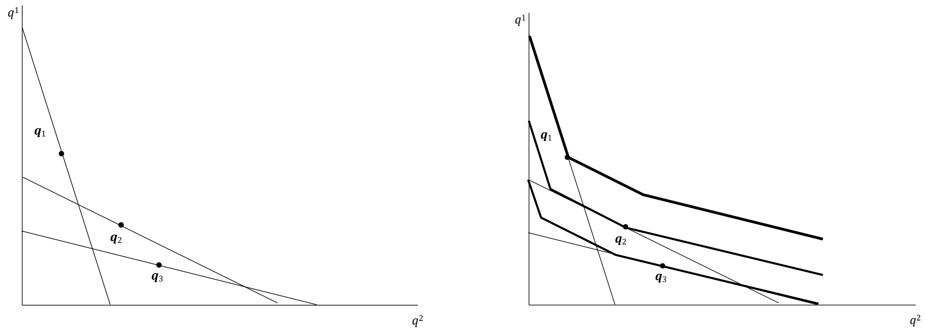
In the proof of Afriat's Theorem (Diewert 1973, Section 3; Varian 1982, Appendix I), which is constructive, for any t [B]'s inequalities (1) can hold with equality for at least one $s \neq t$. This yields a canonical set of rationalizing preferences and an associated utility function, which we call the Afriat preferences and utility function. The Afriat utility function is piecewise linear, continuous, non-satiated, and concave. With finite data the Afriat preferences and utility function are only one of many possibilities for a rationalization (Varian 1982, Fact 4), but they play a central role in Section II.E's analysis.

DEFINITION 3: [Afriat preferences and Afriat utility function.] For data $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\dots,T}$ *that satisfy GARP, or equivalently condition B) of Afriat's Theorem, the Afriat preferences are those represented by the Afriat utility function* $u(\boldsymbol{q}) = min_{t \in \{1,\dots,T\}} \{U_t + \lambda_t \boldsymbol{p}_t \cdot (\boldsymbol{q} - \boldsymbol{q}_t)\}$*, where for any given* t *the*

U_t *and* λ_t *are those that satisfy condition [B] with inequality (1) binding for at least one* $s \neq t$.

Figure 2 illustrates the Afriat preferences for a three-observation dataset that satisfies GARP. Figure 2a shows the observations' budget sets and consumption bundles. Figure 2b shows the associated Afriat indifference map, whose marginal rates of substitution are determined by the budget lines.

Figure 2. Neoclassical Afriat preferences for data that satisfy GARP



(a) (b)

The classic nonparametric analysis of neoclassical consumer demand makes essential use of the rationality assumption, in that the consumer must have a complete and transitive preference ordering over levels of consumption. Even though our analysis of reference-dependent consumer demand is of interest mainly when a neoclassical rationalization is impossible, we can adapt the classic analysis by extending rationality to allow preferences with a domain expanded as suggested by reference-dependence, to include changes as well as levels. Our adaptation requires more than a translation of the classic analyses because levels of and changes in consumption are bundled and priced together and a reference-dependent consumer's choices can change his preferences by altering how consumption relates to the reference point.

B. *Reference-dependent Rationalization with Unmodelable Reference Points*

This section shows that if reference points are unmodelable, the hypothesis of reference-dependent preferences is nonparametrically irrefutable (even if

GARP is violated). Our definition of rationalization then allows a reference point to be chosen hypothetically for each observation—the nonparametric analog of Camerer et al.'s and Farber's treatment of targets as latent variables.

DEFINITION 4: [Rationalization with unmodelable reference points.] Reference-dependent preferences, an associated utility function $u(\boldsymbol{q},\boldsymbol{r})$*, and hypothetical reference points* $\{\boldsymbol{r}_t\}_{t=1,\ldots,T}$*, rationalize the data* $\{\boldsymbol{p}_t,\boldsymbol{q}_t\}_{t=1,\ldots,T}$ *if and only if* $u(\boldsymbol{q}_t,\boldsymbol{r}_t) \geq u(\boldsymbol{q},\boldsymbol{r}_t)$ *for all* $\boldsymbol{q}$ *and* $\boldsymbol{t}$ *such that* $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$*.*

PROPOSITION 1:[11] *[Rationalization with unmodelable reference points via preferences with variable or constant sensitivity.] For any data* $\{\boldsymbol{p}_t,\boldsymbol{q}_t\}_{t=1,\ldots,T}$ *with unmodelable reference points, there exist reference-dependent preferences and an associated utility function* $u(\boldsymbol{q},\boldsymbol{r})$ *that are continuous, increasing in* $\boldsymbol{q}$*, and decreasing in* $\boldsymbol{r}$*, and a sequence of hypothetical reference points* $\{\boldsymbol{r}_t\}_{t=1,\ldots,T}$*, that rationalize the data.*

Proof: Recall that we denote goods by superscripts, so that $\boldsymbol{q} \equiv (q^1, \ldots, q^K)$, $\boldsymbol{q_t} \equiv (q_t^1, \ldots, q_t^K)$, and so on. Let $a^k \equiv min_{t=1,\ldots,T}\{p_t^k\} > 0$ for each *k* and *t* such that $q_t^k \geq r_t^k$; and $a^k \equiv max_{t=1,\ldots,T}\{p_t^k\} > 0$ for each *k* and *t* such that $q_t^k < r_t^k$. Define the utility function $u(\boldsymbol{q},\boldsymbol{r}) \equiv \sum_k a^k q^k + \sum_k a^k (q^k - r^k)$, which is strictly increasing in $\boldsymbol{q}$, strictly decreasing in $\boldsymbol{r}$, and satisfies constant sensitivity and Proposition 1's conditions for continuity. For observation *t*, set $\boldsymbol{r}_t = \boldsymbol{q}_t$ and consider any bundle $\boldsymbol{q} \neq \boldsymbol{q}_t = \boldsymbol{r}_t$ that (without loss of generality given strict monotonicity) exactly satisfies *t*'s budget constraint. For such bundles, $\sum_k p_t^k\,(q^k - q_t^k) = 0$ and, by the definition of the a^k,

[11] We have found no informative results for cases with partial knowledge of reference points. Gul and Pesendorfer (2006) and Freeman (2017) prove results with conclusions like Proposition 1's. However, Gul and Pesendorfer's rationalizing preferences do not satisfy KR's and our assumption of additive separability across consumption and gain-loss utility, and they allow the strength of loss aversion to vary wildly with the cardinality of their (finite) choice set. Freeman's Observation 1 does not restrict preferences, even to be monotonic. By contrast, Proposition 1's rationalizing preferences are credible candidates for an empirical explanation.

(2) $\sum_k (a^k - p_t^k)(q^k - q_t^k) = \sum_k (a^k - p_t^k)(q^k - r_t^k) < 0$ and $\sum_k a^k (q^k - r_t^k) < 0$ and

(3) $u(\boldsymbol{q}, \boldsymbol{r}_t) - u(\boldsymbol{q}_t, \boldsymbol{r}_t) = 2 \sum_k a^k (q^k - q_t^k) = 2 \sum_k a^k (q^k - r_t^k) < 0$,

so $u(\boldsymbol{q}, \boldsymbol{r})$ rationalizes the choice of $\boldsymbol{q}_t$. Similarly for variable sensitivity. ■

Proposition 1's proof hypothesizes a reference point for each observation with $\boldsymbol{r}_t = \boldsymbol{q}_t$ and preferences that, with those reference points, put the observation's consumption bundle at the kink of an approximately Leontief indifference curve. Those preferences satisfy continuity, constant sensitivity, and Farber's, KR's, CM's, and Thakral and Tô's functional form assumptions, so they too are nonparametrically untestable. The rationalization works entirely by varying reference points across observations, showing as directly as possible that the empirical usefulness of reference-dependent consumer theory depends crucially on modeling reference points.

C. *Reference-dependent Rationalization with Modelable Reference Points and Variable Sensitivity*

This section shows that if reference points are modelable and sensitivity is variable, the hypothesis of reference-dependent preferences is refutable only via violations of GARP within subsets of observations that share *exactly* the same reference point. For such subsets, reference-dependent preferences reduce to neoclassical preferences, so in this case reference-dependence still adds nothing that is empirically useful to the neoclassical model. Our variable-sensitivity results are independent of how it varies, as long as it is not constant.

DEFINITION 5: [Rationalization with modelable reference points.] Reference-dependent preferences and an associated utility function $u(\boldsymbol{q}, \boldsymbol{r})$ rationalize the data $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$ with modelable reference points if and only if $u(\boldsymbol{q}_t, \boldsymbol{r}_t) \geq u(\boldsymbol{q}, \boldsymbol{r}_t)$ for all $\boldsymbol{q}$ and $\boldsymbol{t}$ such that $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$.

PROPOSITION 2:[12] *[Rationalization with modelable reference points via preferences with variable sensitivity.] For any data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$ *with modelable reference points, there exist reference-dependent preferences and an associated utility function* $u(\boldsymbol{q}, \boldsymbol{r})$ *that for each observation t and reference point* $\boldsymbol{r}_t$*, are continuous and strictly increasing in* $\boldsymbol{q}$ *and that rationalize the data, if and only if every subset of the data whose observations share exactly the same reference point satisfies GARP.*

Proof: Partition the observations into subsets $\tau^j, j = 1, \dots, J$, such that if and only if two observations $\{\boldsymbol{p}_s, \boldsymbol{q}_s, \boldsymbol{r}_s\}$ and $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}$ have the same reference point $\boldsymbol{r}_s = \boldsymbol{r}_t$, they are in the same subset. If there exists a reference-dependent utility function with the stated properties that rationalizes the data, then the data must satisfy GARP within any such subset, by Afriat's Theorem. Conversely, suppose the data within each such subset satisfies GARP. Let $b^k \equiv min_{t=1,\dots,T}\{p_t^k\}$, so that $0 < b^k \leq p_t^k$, and let $\boldsymbol{b} \equiv (b^1, \dots, b^K)$. For any subset $\tau^j$and observation $t \in \tau^j$, let the indicator function $I_{\tau^j}(t) = 1$ if the observation $t \in \tau^j$ and $I_{\tau^j}(t) = 0$ otherwise, and let $u(\boldsymbol{q}, \boldsymbol{r}) \equiv \sum_j I_{\tau^j}(t) U^j(\boldsymbol{q}, \boldsymbol{r}_t)$, where $U^j(\boldsymbol{q}, \boldsymbol{r}_t) \equiv min_{\rho \in \tau^j}\{U_\rho^j + \lambda_\rho^j \boldsymbol{p}_\rho \cdot (\boldsymbol{q} - \boldsymbol{q}_\rho)\} - \boldsymbol{b} \cdot \boldsymbol{r}_t$, which is Definition 3's Afriat utility function for observations in τ^j, with the U_ρ^j and λ_ρ^j taken from τ^j's binding condition B) inequalities (1) in Afriat's Theorem. If τ^j is a singleton subset, the terms in $U^j(\boldsymbol{q}, \boldsymbol{r}_t)$ follow observation t's budget line. If not, those terms follow the minimum of τ^j's observations' budget lines, as in the right-hand panel of Figure 2. Either way, $\boldsymbol{r}_t$ completely determines the $\boldsymbol{p}_\rho$ and $\boldsymbol{q}_\rho$ for all $\rho \in \tau^j$, as required to determine $U^j(\boldsymbol{q}, \boldsymbol{r}_t)$.

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[12] As Proposition 2's proof shows, restricting sensitivity short of assuming that it is constant, for example by assuming diminishing sensitivity, still does not yield refutable implications. Unlike Proposition 1, Proposition 2 does *not* claim that $u(\boldsymbol{q}, \boldsymbol{r})$ is continuous in $\boldsymbol{q}$ and $\boldsymbol{r}$ or decreasing in $\boldsymbol{r}$. A rationalization might require discontinuous preferences if observations with nearby $\boldsymbol{r}$**'s** have very different budget sets. We have not tried to characterize rationalizability via a continuous $u(\boldsymbol{q}, \boldsymbol{r})$. But for data generated by continuous preferences, Proposition 2's rationalizations should converge to a continuous limiting $u(\boldsymbol{q}, \boldsymbol{r})$ as the data become richer.

For each $\boldsymbol{r}_t$, $u(\boldsymbol{q},\boldsymbol{r}_t)$ and $U^j(\boldsymbol{q},\boldsymbol{r}_t)$ are continuous and increasing in $\boldsymbol{q}$. For any subset τ^j and observation $t \in \tau^j$ and any $\boldsymbol{q}$ with $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$, using τ^j's binding condition B) inequalities (1) for the preferences in that subset,

$$(4) \qquad U^j(\boldsymbol{q},\boldsymbol{r}_t) \equiv min_{\rho\in\tau^j}\{U_\rho^j + \lambda_\rho^j \boldsymbol{p}_\rho \cdot (\boldsymbol{q}-\boldsymbol{q}_\rho)\} - \boldsymbol{b}\cdot\boldsymbol{r}_t$$
$$\leq U_t^j + \lambda_t^j \boldsymbol{p}_t \cdot (\boldsymbol{q}_t - \boldsymbol{q}_t) - \boldsymbol{b}\cdot\boldsymbol{r}_t = U_t^j - \boldsymbol{b}\cdot\boldsymbol{r}_t \equiv U^j(\boldsymbol{q}_t,\boldsymbol{r}_t). \blacksquare$$

D. *Characterizing Reference-dependent Preferences That Satisfy Constant Sensitivity and Continuity*

Sections II.B-C's results show that nonparametrically refutable, empirically useful implications of reference-dependence depend on modeling reference points *and* imposing constant sensitivity. To prepare for Section II.E's analysis of rationalization in that case, this section characterizes reference-dependent preferences and utility functions with constant sensitivity and continuity.

Specifically, suppose preferences and an associated utility function $u(\boldsymbol{q},\boldsymbol{r})$ satisfy: additive separability across consumption and gain-loss utility; constant sensitivity; and continuity in $\boldsymbol{q}$ and $\boldsymbol{r}$; with the number of goods $K \geq 2$, with reference-dependence active for all K goods; and, for any $\boldsymbol{r}$, with the induced preferences over $\boldsymbol{q}$ differentiable in the interior of each gain-loss regime, and with marginal rates of substitution that differ across regimes throughout commodity space. Proposition 3 shows that the preferences must then be representable by a utility function $u(\boldsymbol{q},\boldsymbol{r})$ with gain-loss utility functions that are additively separable across regimes, across $\boldsymbol{q}$ and $\boldsymbol{r}$, and across goods within each regime; and—as in KR's functional structure assumptions—whose good-by-good responses to reference points exactly mirror their responses to the components of consumption.

Proposition 3's preferences nest the functional structure assumptions with constant sensitivity maintained in KR's analysis and all previous empirical work, but are more general in two ways, which Section III's analysis shows can be important empirically: They allow the preferences over consumption bundles induced by consumption plus gain-loss utility to vary as freely as possible across gain-loss regimes while preserving continuity, relaxing the knife-edge cross-regime links between marginal rates of substitution implied by KR's model with constant sensitivity (CM, Table 1). And they allow consumption utility, and therefore with nonparametric flexibility the sum of consumption and gain-loss utility, *not* to be additively separable across goods.

Let $G(\boldsymbol{q}, \boldsymbol{r})$ be a vector of binary numbers of length K with kth component 1 if $q^k \geq r^k$ and 0 otherwise. The gain-loss regime indicator $I_{\boldsymbol{g}}(\boldsymbol{q}, \boldsymbol{r}) = 1$ if $\boldsymbol{g} = G(\boldsymbol{q}, \boldsymbol{r})$ and 0 otherwise; and the gain-loss indicators $G_+^k(\boldsymbol{q}, \boldsymbol{r}) = 1$ if $q_t^k \geq r_t^k$ and 0 otherwise and $G_-^k(\boldsymbol{q}, \boldsymbol{r}) = 1$ if $q_t^k < r_t^k$ and 0 otherwise.

PROPOSITION 3:[13] *[Preferences and utility functions with continuity and constant sensitivity.] Suppose there are $K \geq 2$ goods, with reference-dependence active for all K goods, and that a reference-dependent preference ordering and an associated utility function have additively separable consumption utility and gain-loss utility components. Then the ordering satisfies constant sensitivity if and only if an associated utility function $u(\boldsymbol{q}, \boldsymbol{r})$ can be written, for some consumption utility function $U(\cdot)$ and gain-loss regime utility functions $V_{\boldsymbol{g}}(\cdot,\cdot)$ and $v_{\boldsymbol{g}}(\cdot)$, as*

$$(5) \qquad u(\boldsymbol{q}, \boldsymbol{r}) \equiv U(\boldsymbol{q}) + \textstyle\sum_{\boldsymbol{g}} I_{\boldsymbol{g}}(\boldsymbol{q}, \boldsymbol{r}) V_{\boldsymbol{g}}(v_{\boldsymbol{g}}(\boldsymbol{q}), \boldsymbol{r}).$$

Suppose further that the induced preferences over $\mathbf{q}$ *are differentiable in the interior of each regime, with marginal rates of substitution that differ across regimes throughout commodity space. Then the ordering satisfies constant sensitivity and continuity if and only if it is representable by a utility function $u(\boldsymbol{q}, \boldsymbol{r})$ that can be written, for some consumption utility function $U(\cdot)$ and*

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[13] In riskless environments with convex budget sets, if $K = 1$ all monotone preferences are observationally equivalent, so reference-dependence cannot be empirically meaningful. And, as Proposition 3's wording suggests, its assumptions don't tie down the functional structure for goods for which reference-dependence is inactive. As we seek general characterizations, Propositions 4 and 5 take Proposition 3's conclusion (not its assumptions) as their premises.

gain-loss component utility functions $v_+^k(\cdot)$ and $v_-^k(\cdot)$ (with the indicator functions $G_+^k(\cdot,\cdot)$ and $G_-^k(\cdot,\cdot)$ doing the work of $I_{\boldsymbol{g}}(\cdot,\cdot)$), as

(6) $$u(\boldsymbol{q},\boldsymbol{r}) \equiv U(\boldsymbol{q}) + \sum_k [G_+^k(\boldsymbol{q},\boldsymbol{r})\{v_+^k(q^k) - v_+^k(r^k)\} + G_-^k(\boldsymbol{q},\boldsymbol{r})\{v_-^k(q^k) - v_-^k(r^k)\}].$$

Conversely, any combination of induced regime preferences over $\boldsymbol{q}$ is consistent with continuity and constant sensitivity for some gain-loss utility functions.

Proof: The "if" part of each claim is immediate. The "only if" part regarding (5) follows from Definition 1 via the standard characterization of additively separable preferences. To prove the "only if" part regarding (6), note that $u(\boldsymbol{q},\boldsymbol{r})$ in (5) is continuous if and only if

(7) $$V_{\boldsymbol{g}}\big(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}\big) = V_{\boldsymbol{g}'}\big(v_{\boldsymbol{g}'}(\boldsymbol{q}),\boldsymbol{r}\big)$$

for any $\boldsymbol{q}$, $\boldsymbol{r}$, and i with $q^i = r^i$ and any gain-loss regimes $\boldsymbol{g}$ and $\boldsymbol{g}'$ that differ in the gain-loss status of good i. But (7) can hold under those conditions only if each regime's $V_{\boldsymbol{g}}\big(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}\big)$ is additively separable in the components of $\boldsymbol{q}$ and, for component utility functions $v_+^k(\cdot)$ and $v_-^k(\cdot)$, $k = 1, \dots, K$,

(8) $$\sum_{\boldsymbol{g}} I_{\boldsymbol{g}}(\boldsymbol{q},\boldsymbol{r}) V_{\boldsymbol{g}}\big(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}\big) \equiv \sum_k [G_+^k(\boldsymbol{q},\boldsymbol{r})\{v_+^k(q^k) - v_+^k(r^k)\} + G_-^k(\boldsymbol{q},\boldsymbol{r})\{v_-^k(q^k) - v_-^k(r^k)\}].$$

First suppose that (7) is satisfied for some $\boldsymbol{q}$, $\boldsymbol{r}$, and i with $q^i = r^i$. If $\partial V_{\boldsymbol{g}}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r})/\partial q^j \neq 0$, (7) implies that $\partial V_{\boldsymbol{g}'}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r})/\partial q^j \neq 0$ as well. Adding $U(\boldsymbol{q})$ to each side of (7), partially differentiating each side with respect to q^j and then q^i, with $r^i = q^i$, and taking ratios would then show that the marginal rates of substitution between goods *i* and *j* are equal across regimes $\boldsymbol{g}$ and $\boldsymbol{g}'$ for all $q^i = r^i$, a contradiction. Thus with $q^i = r^i$, $\partial V_{\boldsymbol{g}}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r})/\partial q^j \equiv \partial V_{\boldsymbol{g}'}(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r})/\partial q^j \equiv 0$ for any $j \neq i$, and standard characterization results show that for a regime $\boldsymbol{g}$, $V_{\boldsymbol{g}}\big(v_{\boldsymbol{g}}(\boldsymbol{q}),\boldsymbol{r}\big)$ is additively separable across the components of $\boldsymbol{q}$. Given that, changing the gain-loss status of a good *j* with $q^i = r^i$ would violate (7) and therefore continuity, unless for some functions $w_+^k(\cdot)$ and $w_-^k(\cdot)$, *k* = 1,…, *K*,

(9) $\sum_g I_g(\boldsymbol{q},\boldsymbol{r})V_g(v_g(\boldsymbol{q}),\boldsymbol{r}) \equiv \sum_k[G_+^k(\boldsymbol{q},\boldsymbol{r})\, w_+^k(q^k,\boldsymbol{r}) + G_-^k(\boldsymbol{q},\boldsymbol{r})w_-^k(q^k,\boldsymbol{r})]$.

Finally, unless the $w_+^k(\cdot,\cdot)$ and $w_-^k(\cdot,\cdot)$ are also additively separable in $\boldsymbol{r}$, with good-by-good responses to reference points that exactly mirror their good-by-good responses to bundles as in (8) (with $w_+^k(q^k,\boldsymbol{r}) \equiv \{v_+^k(q^k) - v_+^k(r^k)\}$ and $w_-^k(q^k,\boldsymbol{r}) \equiv \{v_-^k(q^k) - v_-^k(r^k)\}$), for some $\boldsymbol{q}$, $\boldsymbol{r}$, and *k*, changing q^k and r^k with $r^k = q^k$would induce different changes in $V_g\big(v_g(\boldsymbol{q}),\boldsymbol{r}\big)$ and $V_{g'}\big(v_{g'}(\boldsymbol{q}),\boldsymbol{r}\big)$, violating (7) and continuity. The contradiction establishes our claim regarding (8) and completes the proof of (6). A similar argument shows that any combination of induced regime preferences over $\boldsymbol{q}$ is consistent with continuity and constant sensitivity for some gain-loss utility functions. ■

Proposition 3’s characterization (6) plays an important role in Section II.E’s conditions for a rationalization with modelable reference points and constant sensitivity. With constant sensitivity a consumer’s induced preferences over $\boldsymbol{q}$ and his optimal choice of $\boldsymbol{q}$ are independent of $\boldsymbol{r}$ within a gain-loss regime, but the maximized value of $u(\boldsymbol{q},\boldsymbol{r})$ still varies with $\boldsymbol{r}$ within a regime. (6)’s terms in $v_+^k(r^k)$ and $v_-^k(r^k)$ ensure continuity of $u(\boldsymbol{q},\boldsymbol{r})$ despite such variations, by subtracting a regime-by-regime “loss cost”. In effect the consumer faces a menu of fixed, exogenous regime charges, which influence his incentive to defect from an observation’s consumption bundle to bundles in other regimes.

E. *Reference-dependent Rationalization with Modelable Reference Points and Constant Sensitivity*

This section uses Proposition 3’s characterization of reference-dependent preferences that satisfy constant sensitivity and continuity to derive conditions that are sufficient for a rationalization.

With modelable reference points, the observations’ consumption bundles can be objectively sorted into gain-loss regimes. By Afriat’s Theorem, GARP for each regime's observations is necessary for a rationalization, because it is required for the existence of preferences that preclude defections from an observation's bundle to affordable bundles within the same regime. But GARP regime-by-regime is not sufficient for a rationalization, for two reasons.

First, the gain-loss regime utility functions that rationalize the consumer's choices within each regime must, for continuity, satisfy Proposition 3's restrictions that their component utility functions must be the same across all regimes. As a result, the regime utility functions that prevent within-regime defections must be derived for all regimes at once, enforcing those restrictions.

Second, the rationalizing gain-loss regime utility functions must also prevent defections from an observation's bundle to affordable bundles in other regimes, in which preferences may differ. This involves Section II.D's loss costs, which are determined by the rationalizing regime utility functions.

There is normally a range of rationalizing regime utility functions and choosing among them involves complex trade-offs, as a choice that lowers the gain from defecting *from* bundles in a regime raises the gain from defecting *to* them. Propositions 4 and 5 approach this difficulty in two steps. Proposition 4 derives benchmark conditions for a rationalization, which are necessary and sufficient but not directly applicable because they are conditional on the choice of rationalizing regime utility functions. Proposition 5 then derives directly applicable sufficient conditions based on choosing rationalizing regime utility functions like Definition 3's Afriat utility functions. Those conditions are not necessary because other rationalizing choices normally exist, but they should be asymptotically necessary as explained below.

Let $\Gamma(g;\boldsymbol{r})\}$ be the set of $\boldsymbol{q}$ in regime g for $\boldsymbol{r}$. Let $\Theta\big(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g\big) \equiv \{t \in \{1,\dots,T\}\,\big|\,\boldsymbol{q}_t \in \Gamma(g;\boldsymbol{r}_t)\}$ be the set of t with $\boldsymbol{q}_t$ in regime g for $\boldsymbol{r}_t$.

PROPOSITION 4: [Rationalization with modelable reference points via preferences and utility functions with constant sensitivity.] Suppose that reference-dependent preferences and an associated utility function are defined *over K ≥ 2 goods, that reference-dependence is active for all K goods, that the preferences satisfy constant sensitivity and are continuous, and that the utility function satisfies Proposition 3's (6). Consider data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$ *with modelable reference points. Then the statements [A] and [B] are equivalent:*

[A] There exists a continuous reference-dependent utility function $u(\boldsymbol{q}, \boldsymbol{r})$ *that satisfies constant sensitivity; is strictly increasing in* $\boldsymbol{q}$ *and strictly decreasing in* $\boldsymbol{r}$*; and that rationalizes the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$.

[B] Each gain-loss regime's data satisfy GARP within the regime; and there is some combination of preferences over consumption bundles, with continuous, strictly increasing consumption utility function $U(\cdot)$ *and gain-loss component utility functions* $v_+^k(\cdot)$ *and* $v_-^k(\cdot)$*, such that, for any regime* g *and any pair of observations* $\sigma, \tau \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g)$ *(with the indicator functions* $G_+^k(\cdot,\cdot)$ *and* $G_-^k(\cdot,\cdot)$ *again doing the work of* $I_{\boldsymbol{g}}(\cdot,\cdot)$*),*

$$(10)\qquad U(\boldsymbol{q}_\sigma) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\sigma, \boldsymbol{r}_\tau) v_+^k(q_\sigma^k) + G_-^k(\boldsymbol{q}_\sigma, \boldsymbol{r}_\tau) v_-^k(q_\sigma^k)]$$
$$\leq \mathrm{U}(\boldsymbol{q}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_+^k(q_\tau^k) + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_-^k(q_\tau^k)] + \lambda_\tau \boldsymbol{p}_\tau \cdot (\boldsymbol{q}_\sigma - \boldsymbol{q}_\tau)$$

and for each observation $\{\boldsymbol{p}_\tau, \boldsymbol{q}_\tau, \boldsymbol{r}_\tau\}_{t=1,\dots,T}$ *with* $\tau \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g)$ *and each* $\boldsymbol{q} \in \Gamma(g'; \boldsymbol{r}_\tau)$ *with* $g' \neq g$ *for which* $\boldsymbol{p}_\tau \cdot \boldsymbol{q} \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$*,*

$$(11)\qquad U(\boldsymbol{q}) + \textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_+^k(q^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_-^k(q^k) - v_-^k(r_\tau^k)\}]$$
$$\leq U(\boldsymbol{q}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_+^k(q_\tau^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_-^k(q_\tau^k) - v_-^k(r_\tau^k)\}].$$

Proof: That [B] implies [A] is immediate. To prove that [A] implies [B], take the rationalizing regime preferences represented by $U(\cdot)$ and the $v_+^k(\cdot)$ and $v_-^k(\cdot)$, which satisfy (10). Use Proposition 3 to write the condition preventing defections from the bundle of observation $\tau \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g)$ in regime g to a bundle $\boldsymbol{q} \in \Gamma(g'; \boldsymbol{r}_\tau)$ in regime $g' \neq g$ for $\boldsymbol{r}_\tau$ with $\boldsymbol{p}_\tau \cdot \boldsymbol{q} \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$:

$$u(\boldsymbol{q}, \boldsymbol{r}_\tau) - U(\boldsymbol{r}_\tau) \equiv U(\boldsymbol{q}) + \textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_+^k(q^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}, \boldsymbol{r}_{\tau_\tau})\{v_-^k(q^k) - v_-^k(r_\tau^k)\}] - U(\boldsymbol{r}_\tau)$$
$$(12)\ \equiv \{U(\boldsymbol{q}) + \textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau) v_+^k(q^k) + G_-^k(\boldsymbol{q}, \boldsymbol{r}_\tau) v_-^k(q^k)]\} - \{U(\boldsymbol{r}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}, \boldsymbol{r}_\tau)\{v_+^k(r_\tau^k) + G_-^k(\boldsymbol{q}, \boldsymbol{r}_\tau) v_-^k(r_\tau^k)]\}$$
$$\leq \{U(\boldsymbol{q}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_+^k(q_\tau^k) + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_-^k(q_\tau^k)]\} - \{U(\boldsymbol{r}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_+^k(r_\tau^k) + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) v_-^k(r_\tau^k)]\}$$
$$\equiv U(\boldsymbol{q}_\tau) + \textstyle\sum_k [G_+^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_+^k(q_\tau^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau)\{v_-^k(q_\tau^k) - v_-^k(r_\tau^k)\}] \equiv u(\boldsymbol{q}_\tau, \boldsymbol{r}_\tau) - U(\boldsymbol{r}_\tau).$$

(12)’s central inequality can then be rearranged to yield (11). ■

There are no simple conditions that are necessary and sufficient for a reference-dependent rationalization, as GARP is in the neoclassical case.

Proving Proposition 4 requires linking Section II.D’s loss costs to things that can be estimated from the data, not only at given points but as functions of $\boldsymbol{r}$. The proof shows that this can be done, as in (12).

Figures 3 and 4 illustrate Proposition 4. In each case the entire dataset violates GARP, with observation 1’s consumption bundle chosen in 1’s budget set over observation 2’s bundle, and vice versa. Further, the observations’ reference points put their bundles in different gain-loss regimes, so constant sensitivity allows different preferences for each observation. And finally, each regime’s single observation trivially satisfies GARP within its regime.

Figures 3a-b depict Afriat and non-Afriat rationalizing regime preferences. In each case condition (11) is satisfied, so that a rationalization is possible. By contrast, in Figure 4a Afriat rationalizing regime preferences do not satisfy (11) and Figure 4b shows that there can be no choice of rationalizing regime preferences (Afriat or not) for which (11) is satisfied, so that a rationalization is impossible. For, a rationalization in Figure 4b’s case would require regime preferences that connect a loss-regime indifference curve through observation 1’s bundle to a gain-regime curve that cuts into observation 2’s budget set and stays outside observation 1’s budget set, thus passing northeast of 2’s bundle; and also loss- and gain-regime indifference curves satisfying the analogous conditions interchanging observations 1 and 2. Such curves are plainly inconsistent with optimality of each observation’s consumption bundle.[14]

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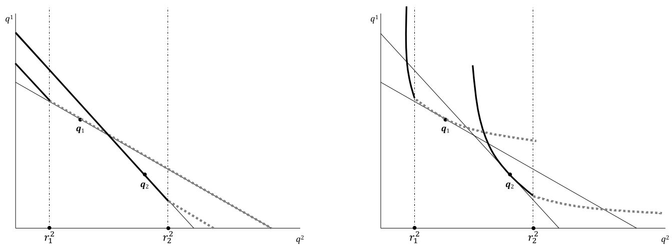
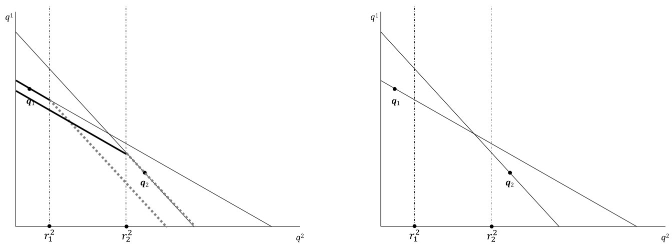
[14] The difference between Figure 3’s and Figure 4’s examples can be understood in terms of the familiar notion of loss aversion. Generalizing Tversky and Kahneman (1991), Online Appendix A’s Definition A1 gives a nonparametric definition of loss aversion with constant sensitivity, requiring that, across gain-loss regimes that differ only in the gain-loss status of a given good, the loss-side marginal rates of substitution between that and any other good are no less favorable to that good than are the gain-side marginal rates of substitution. The change in Afriat preferences across regimes in Figure 3a is consistent with loss aversion, but the change in Figure 4a is not. Appendix A’s Corollary A1 shows that if the rationalizing regime

Figure 3. Rationalizing data that violate GARP via reference-dependent preferences with constant sensitivity
(Solid lines for loss maps, dashed lines for gains maps)



(a) (b)

Figure 4. Failing to rationalize data that violate GARP via reference-dependent preferences with constant sensitivity
(Solid lines for loss maps, dashed lines for gains maps)



(a) (b)

As already noted, Proposition 4's necessary and sufficient conditions for a rationalization are not directly applicable because they are conditional on an unspecified choice of rationalizing regime utility functions. Proposition 5 derives directly applicable sufficient conditions by specifying rationalizing regime utility functions in the style of the regime's Afriat utility functions. Those conditions include inequalities as in Afriat's Theorem, which prevent defections from an observation's consumption bundle to affordable bundles in the same gain-loss regime, while enforcing Proposition 3's restrictions; and also inequalities that prevent defections to affordable bundles in other regimes.

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preferences satisfy loss aversion, Proposition 4's no-cross-regime-defections constraints (11) must be satisfied, so its conditions excluding (11) are then sufficient for a rationalization. Loss aversion is an empirically well-supported assumption known to have important implications, but it has not, to our knowledge, previously been linked to the *existence* of a reference-dependent rationalization. The testability of loss aversion is limited for much the same reason that the convexity of preferences is not nonparametrically testable in the neoclassical case.

PROPOSITION 5: [Sufficient conditions for rationalization with modelable reference points, via reference-dependent preferences and utility function with constant sensitivity and continuity.] The following conditions are sufficient for the existence of continuous reference-dependent preferences and utility function with constant sensitivity $u(\boldsymbol{q},\boldsymbol{r})$ *that rationalize data with modelable reference points* $\{\boldsymbol{p}_t,\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\ldots,T}$*: There exist numbers* U_t, v_{t+}^k, v_{t-}^k, *and* $\lambda_t > 0$ *for each* $k = 1,\ldots,K$ *and* $t = 1,\ldots,T$ *such that:*

[A] For any gain-loss regime g *and any pair of observations* $\sigma,\tau \in \Theta\big(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\ldots,T};g\big)$ *(with the indicator functions* $G_+^k(\cdot,\cdot)$ *and* $G_-^k(\cdot,\cdot)$ *again doing the work of* $I_{\boldsymbol{g}}(\cdot,\cdot)$*),*

$$(13)\qquad U_\sigma + \textstyle\sum_k[G_+^k(\boldsymbol{q}_\sigma,\boldsymbol{r}_\tau)v_{\sigma+}^k + G_-^k(\boldsymbol{q}_\sigma,\boldsymbol{r}_\tau)v_{\sigma-}^k] \leq U_\tau + \sum_k[G_+^k(\boldsymbol{q}_\tau,\boldsymbol{r}_\tau)v_{\tau+}^k + G_-^k(\boldsymbol{q}_\tau,\boldsymbol{r}_\tau)v_{\tau-}^k] + \lambda_\tau\boldsymbol{p}_\tau\cdot(\boldsymbol{q}_\sigma - \boldsymbol{q}_\tau).$$

[B] For observations σ,τ, $q_\sigma^k \geq q_\tau^k$ *for k = 1,..., K,* $U_\sigma \geq U_\tau$*; and for observations* σ,τ *and any k = 1,..., K,* $q_\sigma^k \geq q_\tau^k$, $v_{\sigma+}^k \geq v_{\tau+}^k$, *and* $v_{\sigma-}^k \geq v_{\tau-}^k$.

[C] For any pair of regimes g *and* $g' \neq g$, *observation* $\tau \in \Theta\big(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\ldots,T};g\big)$, *and bundle* $\boldsymbol{q} \in \Gamma(g';\boldsymbol{r}_\tau)$ *for which* $\boldsymbol{p}_\tau\cdot\boldsymbol{q} \leq \boldsymbol{p}_\tau\cdot\boldsymbol{q}_\tau$,

$$(14)\qquad \begin{aligned} &min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\ldots,T};g')}\{U_\rho + \textstyle\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{q}-\boldsymbol{q}_\rho)\}\\ &-min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\ldots,T};g')}\{U_\rho + \textstyle\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{r}_\tau-\boldsymbol{q}_\rho)\}\\ &\leq min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\ldots,T};g)}\{U_\rho + \textstyle\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{q}_\tau-\boldsymbol{q}_\rho)\}\\ &-min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\ldots,T};g)}\{U_\rho + \textstyle\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{r}_\tau-\boldsymbol{q}_\rho)\}. \end{aligned}$$

Proof: Given choices of U_t, v_{t+}^k, v_{t-}^k, and λ_t, $t = 1,\ldots, T$, that satisfy [A] and [B], let $u^g(\boldsymbol{q},\boldsymbol{r})$ denote the rationalizing Afriat regime utility function for regime g, including (6)'s loss costs, which exists regime by regime by Afriat's Theorem. For $\boldsymbol{q} \in \Gamma(g;\boldsymbol{r})$, using (10) as in the proof of Afriat's Theorem:

$$(15)\qquad \begin{aligned} &u^g(\boldsymbol{q},\boldsymbol{r}) - U(\boldsymbol{r}) \equiv U(\boldsymbol{q}) + \sum_k[G_+^k(\boldsymbol{q},\boldsymbol{r})\{v_+^k(q^k) - v_+^k(r^k)\} + G_-^k(\boldsymbol{q},\boldsymbol{r})\{v_-^k(q^k) - v_-^k(r^k)\}] - U(\boldsymbol{r})\\ &\equiv \left\{U(\boldsymbol{q}) + \sum_k[G_+^k(\boldsymbol{q},\boldsymbol{r})v_+^k(q^k) + G_-^k(\boldsymbol{q},\boldsymbol{r})v_-^k(q^k)]\right\} - \left\{U(\boldsymbol{r}) + \sum_k[G_+^k(\boldsymbol{q},\boldsymbol{r})\{v_+^k(r^k) + G_-^k(\boldsymbol{q},\boldsymbol{r})v_-^k(r^k)]\right\}\\ &\equiv min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\ldots,T};g)}\{U_\rho + \textstyle\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r})v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r})v_{\rho-}^k] + \lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{q}-\boldsymbol{q}_\rho)\}\\ &\quad - min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\ldots,T};g)}\{U_\rho + \textstyle\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r})v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r})v_{\rho-}^k] + \lambda_\rho\boldsymbol{p}_\rho\cdot(\boldsymbol{r}-\boldsymbol{q}_\rho)\}. \end{aligned}$$

The rationalizing reference-dependent utility function, including loss costs, is then $u(\boldsymbol{q},\boldsymbol{r}) \equiv U(\boldsymbol{q}) + \sum_g I_{\boldsymbol{g}}(\boldsymbol{q},\boldsymbol{r})\, u^g(\boldsymbol{q},\boldsymbol{r})$. By construction, $u(\boldsymbol{q},\boldsymbol{r})$ is continuous, strictly increasing in $\boldsymbol{q}$, and strictly decreasing in $\boldsymbol{r}$.

For observations $\sigma,\tau \in \Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}; g)$ in the same gain-loss regime g, with $\boldsymbol{p}_\tau \cdot \boldsymbol{q}_\sigma \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_\tau$, loss costs cancel out and (13) reduces to the usual Afriat inequalities (with the Afriat utilities expressed not as single numbers but as sums of consumption plus gain-loss utilities). Thus by Afriat's Theorem, [A] prevents defections to affordable bundles in the same regime.

For gain-loss regimes g and $g' \neq g$, observation $\tau \in \Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}; g)$, and bundle $\boldsymbol{q} \in \Gamma(g';\boldsymbol{r}_\tau)$ with $\boldsymbol{p}_\tau \cdot \boldsymbol{q} \leq \boldsymbol{p}_\tau \cdot \boldsymbol{q}_t$,

$$u(\boldsymbol{q},\boldsymbol{r}_\tau) - U(\boldsymbol{r}_\tau) \equiv U(\boldsymbol{q}) + \sum_k [G_+^k(\boldsymbol{q},\boldsymbol{r}_\tau)\{v_+^k(q^k) - v_+^k(r_\tau^k)\} + G_-^k(\boldsymbol{q},\boldsymbol{r}_{\tau_\tau})\{v_-^k(q^k) - v_-^k(r_\tau^k)\}] - U(\boldsymbol{r}_\tau)$$

$$\equiv \left\{U(\boldsymbol{q}) + \sum_k [G_+^k(\boldsymbol{q},\boldsymbol{r}_\tau)v_+^k(q^k) + G_-^k(\boldsymbol{q},\boldsymbol{r}_\tau)v_-^k(q^k)]\right\} - \left\{\mathrm{U}(\mathbf{r}_\tau) + \sum_k [\mathrm{G}_+^k(\mathbf{q},\mathbf{r}_\tau)\{\mathrm{v}_+^k(\mathrm{r}_\tau^k) + \mathrm{G}_-^k(\mathbf{q},\mathbf{r}_\tau)\mathrm{v}_-^k(\mathrm{r}_\tau^k)]\right\}$$

$$\equiv min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g')}\{U_\rho + \textstyle\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{q} - \boldsymbol{q}_\rho)\}$$

$$-min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g')}\{U_\rho + \textstyle\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{r}_\tau - \boldsymbol{q}_\rho)\}$$

$$\leq min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g)}\{U_\rho + \textstyle\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{q}_\tau - \boldsymbol{q}_\rho)\} \tag{16}$$

$$-min_{\rho\in\Theta(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g)}\{U_\rho + \textstyle\sum_k[G_+^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho,\boldsymbol{r}_\tau)v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{r}_\tau - \boldsymbol{q}_\rho)\}$$

$$\equiv \left\{U(\boldsymbol{q}_\tau) + \sum_k [G_+^k(\boldsymbol{q}_\tau,\boldsymbol{r}_\tau)v_+^k(q_\tau^k) + G_-^k(\boldsymbol{q}_\tau,\boldsymbol{r}_\tau)v_-^k(q_\tau^k)]\right\} - \left\{U(\boldsymbol{r}_\tau) + \sum_k [G_+^k(\boldsymbol{q}_\tau,\boldsymbol{r}_\tau)\{v_+^k(r_\tau^k) + G_-^k(\boldsymbol{q}_\tau,\boldsymbol{r}_\tau)v_-^k(r_\tau^k)]\right\}$$

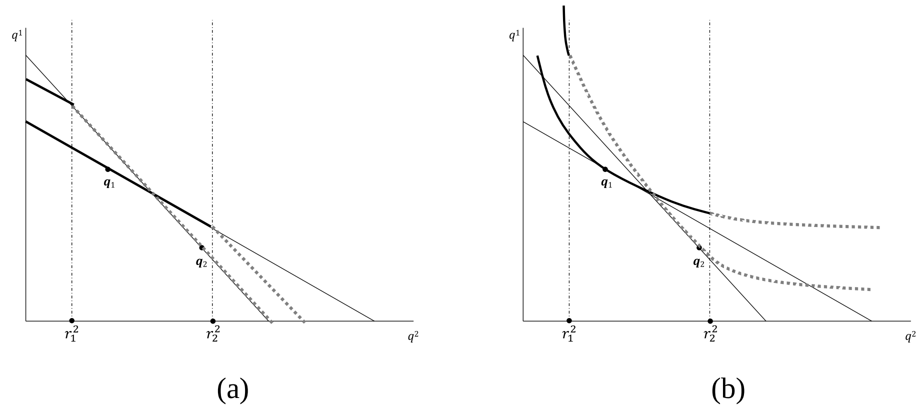
$$\equiv u(\boldsymbol{q}_\tau,\boldsymbol{r}_\tau)) - U(\boldsymbol{r}_\tau),$$

which prevents defections across regimes. ■

Although Proposition 3 shows that constant sensitivity and continuity require *gain-loss* utility to be additively separable across goods, because the consumption utility function is constant across gain-loss regimes, with nonparametric flexibility the sum of consumption and gain-loss utility that determines demand behavior need not be. Thus, neither Proposition 4 nor Proposition 5 requires KR's assumption of additive separability across goods.

Proposition 5 does rely on the choice of Afriat rationalizing regime utility functions.[15] As other choices might also suffice, its sufficient conditions are not necessary. For example, the Afriat regime preferences in Figure 5a do not yield a rationalization but the non-Afriat regime preferences in Figure 5b do.

Figure 5. A rationalization may require non-Afriat rationalizing regime preferences (solid lines for the loss map, dashed for the gain map)



Although Proposition 5's sufficient conditions are not necessary, Mas-Colell's (1978) and Forges and Minelli's (2009) results for the neoclassical case suggest a sense in which they should be asymptotically necessary. In the neoclassical case, they study the limit as the data become rich in the sense that as $T \to \infty$ the data come to include {reference point×budget set} combinations as close as desired to any possible combination, showing that the range of convexified rationalizing preferences then collapses on Definition 3's Afriat preferences.[16] With constant sensitivity this result cannot be immediately applied gain-loss regime by regime, because of Proposition 3's constraint that the component gain-loss utility functions must be the same in all regimes. But it is a plausible conjecture that in the limit, if the Afriat regime preferences do

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[15] Varian's (1982, Fact 4) bounds for the neoclassical case don't imply that all rationalizing preferences are convex, but examples show that requiring such convexity involves a loss of generality for some rationalizing regime preferences in Proposition 4. Proposition 5 avoids that difficulty by using the Afriat regime preferences, which are convex by construction.
[16] Requiring richness of consumption bundles as well would rule out non-convex preferences.

not yield a rationalization, neither can any other regime preferences. If so, Proposition 5's sufficient conditions are asymptotically necessary.

III. Empirical Illustration

Proposition 5's sufficient conditions for a reference-dependent rationalization with modelable reference points and constant sensitivity immediately suggest methods for recovering rationalizing preferences when they exist. Proposition 3's characterization of reference-dependent preferences in that case would be well suited to a structural econometric analysis of Farber's (2005, 2008) data, using sample proxies like CM's for KR's rational-expectations model of the targets. However, here we choose to illustrate those methods by using them to reconsider Farber's and CM's econometric analyses nonparametrically, again using sample proxies like CM's for the targets.

As in all previous work on labor supply with reference-dependent preferences, we consider preferences over levels of and changes in earnings and leisure. With two goods, GARP (Definition 2) reduces to the Weak Axiom of Revealed Preference ("WARP"). WARP is then necessary and sufficient for a neoclassical rationalization. In this section we use "WARP" for "GARP".

DEFINITION 6: [Weak Axiom of Revealed Preference ("WARP").] $\boldsymbol{q_s} R \boldsymbol{q_t}$ *and* $\boldsymbol{q}_s \neq \boldsymbol{q}_t$ *implies not* $\boldsymbol{q_t} R \boldsymbol{q_s}$*, where R indicates that there is some sequence of observations* $\boldsymbol{q}_h, \boldsymbol{q}_i, \boldsymbol{q}_j, \dots, \boldsymbol{q}_t$ *such that* $\boldsymbol{p}_h \cdot \boldsymbol{q}_h \geq \boldsymbol{p}_h \cdot \boldsymbol{q}_i, \boldsymbol{p}_i \cdot \boldsymbol{q}_i \geq \boldsymbol{p}_i \cdot \boldsymbol{q}_j, \dots, \boldsymbol{p}_s \cdot \boldsymbol{q}_s \geq \boldsymbol{p}_s \cdot \boldsymbol{q}_t$.

We relax Camerer et al.'s (1997), Farber's (2005, 2008), and CM's assumption that drivers have homogeneous preferences, as is usual in analyses of labor supply, instead allowing unrestricted heterogeneity of preferences, as is usual in nonparametric demand analyses. Section II's theory covers both cases, distinguished only by whether the data are pooled across drivers.

To guide future work, we compare several models of reference points, including expectations-based and recent experience-based alternatives to

CM's sample proxies for rational-expectations reference points.[17] Like CM, but unlike Camerer et al. and Farber, we allow different forms of reference-dependence: in earnings alone, in hours alone, or in both earnings and hours.

Section III.A reviews Farber's data. Section III.B outlines the models of reference-dependent preferences we compare. Section III.C discusses Selten and Krischker's (1983), Selten's (1991), and Beatty and Crawford's (2011) nonparametric notions of predictive success. Section III.D discusses estimation procedure. Section III.E reports the results.

A. *Data*

Like CM, we use Farber's (2005, 2008) data.[18] Farber collected 593 trip sheets for 13461 trips by 21 drivers between June 1999 and May 2001. Each sheet records the driver's name, hack number, date, each fare's start time and location, each fare's end time and location, and the fare paid. Nine sheets duplicate the day and driver, so there are only 584 shifts. Because our methods make some allowance for sample size, in addition to the 15 drivers Farber and CM studied we include the 6 with samples of 10 or fewer shifts they excluded.

Online Appendix B's Table B.1 reports descriptive statistics driver by driver. The values are the same as those in Farber's (2005) Table B1, except for the hourly wage variable and the Afriat efficiencies in the last two columns. Our earnings and wage variables differ from Farber's and CM's in two ways, which affect the Afriat efficiencies. First, we use the NY/NJ urban CPI to control for price level changes in the sample period. Second, Farber's and CM's wage variable is income per hour spent working, with working time defined as the sum of time driving with a passenger and time waiting for the next passenger. But as waiting time varies randomly from shift to shift with weather, the flow of customers, etc., and is not directly linked to earnings, it

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[17] We focus only on static models of reference points, unlike Farber (2015) and Thakral and Tô (2021). However, our theory allows reference points to be dynamic if they are modelable.
[18] The datasets are posted at https://www.aeaweb.org/aer/data/june08/20030605_data.zip, and https://www.aeaweb.org/aer/data/aug2011/20080780_data.zip. The CPI data are posted at https://data.bls.gov/timeseries/CUURS12ASA0, under the years 1999-2001.

appears to be largely exogenous. Accordingly, we treat waiting time as an exogenous fixed cost and define the wage as earnings per hour driving.

Our redefinition of the wage matters more in a nonparametric analysis than it does in Farber's and CM's structural analyses. Drivers' waiting times range from about 25-40% of their times on a shift. If we included waiting in working time, shift-to-shift wage variation would make a driver's observations' budget lines pivot around their common zero-hours end, they would never cross, he would trivially satisfy WARP, and a nonparametric analysis would give only a meaningless recapitulation of his data. By contrast, treating waiting times as a fixed cost allows a driver's budget lines to cross. Appendix B's Figure B.1 shows that with our wage definition drivers' budget lines cross frequently, making WARP a meaningful restriction and allowing a nonparametric analysis to provide a meaningful interpretation of the data.

Table B.1's last column reports each driver's Afriat efficiency index. The index is 1 for a driver whose data satisfy WARP; otherwise less than 1. Only seven of Farber's 21 drivers satisfy WARP. Except for drivers 12 (sample size 13), 14 (sample size 17), and 17 (sample size 10), the drivers with exact neoclassical fits (2, 3, 6, 9, 11, 13, and 15) are the ones with the smallest sample sizes among the 21 drivers. Except for drivers 2 (sample size 14), 9 (sample size 19), and 17 (sample size 10), those drivers are the same as the six (3, 6, 11, 13, 15, and 17) Farber and CM excluded due to small (≤ 10) sample sizes. Small samples make it easier to satisfy WARP by chance, and those drivers' data may simply be too under-powered to reject the neoclassical model. We return to the issue of correcting for power to reject in Section III.C.

B. *Alternative Models of Reference-dependent Preferences*

Our reference-dependent models vary along three dimensions. The first distinguishes reference-point models based on proxied rational expectations, as in CM, from those based on recent experience. Expectations-based reference points are leave-one-out means: sample averages of a driver's choices, excluding the current shift. Experience-based reference points are one-shift lags. For each kind of model we compare unconditional models and models that condition on Farber's and CM's variables that shift demand and influence waiting time: weather (rain, snow, or dry) and time of day (day or night). This yields 18 different *kinds* of reference-point model.

The second dimension distinguishes three *forms* of reference-dependence: with respect to hours, earnings, or both hours and earnings.

The third dimension distinguishes reference-dependent, and neoclassical, models that do or do not impose additive separability across goods.

C. *Nonparametric Notions of Predictive Success*

The simplest possible measure of a model's predictive success is the pass rate. A model's pass rate for driver *i*, denoted $r^i \in [0,1]$, is defined as the maximal proportion of the driver's observations that are consistent with the model. A closely related measure replaces r^i with a model's proximity π^i, defined as one minus the Euclidean distance, rescaled as a proportion of the maximum possible distance, between *i*'s set of observations and the set of sets of observations that fit the model exactly (Beatty and Crawford 2011, pp. 2786-87). Like r^i, $\pi^i \in [0,1]$, with higher values for more successful models.

However, neither measure is adequate for comparing models of varying flexibility. Reference-dependent models are more flexible than neoclassical models and must have pass rates and proximities at least as high. This accounts for much of the profession's skepticism about their parsimony. Even neoclassical models can be highly restrictive or without nonparametric content depending on the number of observations and whether budget lines cross.

To control for flexibility, Selten and Krischker (1983) and Selten (1991) penalize a model's pass rate for flexibility using what they call the model's "area", $a^i \in [0,1]$. The area is the size of the set of all model-consistent sets of observations for driver i, relative to the size of the set of all feasible sets of observations of the same size, or equivalently the probability that uniformly random data are consistent with the model. Noting that successful models have small values of a^i and large values of r^i, Selten and Krischker define a measure of predictive success, $m(r^i, a^i) \equiv r^i - a^i \in [-1,1]$.[19] As $m \to 1$ a model's restrictions grow tighter but behavior satisfies them: a highly successful model. As $m \to -1$ a model's restrictions become looser but behavior fails to satisfy them: a pathologically bad model. As $m \to 0$ a model's compliance approaches random: a harmless but useless model.

Selten and Krischker's all-or-nothing pass rate r^i is too undiscriminating for our application, in which drivers with more than a few trips have little chance of satisfying even a reference-dependent model exactly. Accordingly, we replace their pass rate r^i with Beatty and Crawford's proximity measure π^i, following them in continuing to penalize it via Selten and Krischker's area. Thus our proposed measure is $n(\pi^i, a^i) \equiv \pi^i - a^i \in [-1,1]$. Like Selten and Krischker's measure, $n(\pi^i, a^i)$ levels the playing field between more- and less-flexible models in a well-defined, objective way. Both measures are similar in spirit to the adjusted R^2 or the Akaike Information Criteria in structural econometrics, which penalize model fit and likelihood for a model's number of free parameters. From now on we use "Selten measure" loosely for Beatty and Crawford's proximity-based measure of predictive success.

D. *Estimation Procedure*

We estimate driver by driver and model by model.[20] For each model we fix whether preferences are additively separable across goods and the form and kind of reference-dependence.[21] The details are in our replication materials.

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[19]Selten (1991) shows that three axioms, *monotonicity* $m(1,0) > m(0,1)$; *equivalence of trivial theories* $m(1,1) = m(0,0)$; and *aggregability* $m(\lambda r_1 + (1-\lambda) r_2, \lambda a_1 + (1-\lambda) a_2) = \lambda m(r_1, a_1) + (1-\lambda) m(r_1, a_1)$ for $\lambda \in [0,1]$, characterize the measure $m(r^i, a^i) \equiv r^i - a^i$.

[20] Rather than nesting and estimating the form and kind of reference-dependence we condition on them and compare the resulting models. Nesting and estimating would be computationally complex, in part because the Afriat regime preferences are not invariant to merging regimes.

[21] With regard to additive separability across goods, Debreu's (1960) necessary and sufficient "double cancellation" condition shows that with two goods the Afriat rationalizing regime preferences preclude it in gain-loss regimes with more than one observation. We therefore use Varian's (1983, Theorem 6) linear program, specializing inequalities like those in condition [B] of Afriat's Theorem, and a version of condition (14) modified to require such separability. For proximities and Selten measures, we design and implement a computationally efficient search algorithm using the fact that that the proximity for a separable model cannot exceed that for its non-separable counterpart. Details and code are in our Replication files.

Proposition 5's conditions immediately suggest an estimation procedure:[22]

(i) Use the observations' modeled reference points to sort their consumption bundles into gain-loss regimes.

(ii) Pooling the data from all regimes, use linear programming to find Afriat numbers U_t, v_{t+}^k, v_{t-}^k, and $\lambda_t > 0$ for each $k = 1, \dots, K$ and $t = 1, \dots, T$ that satisfy [A]'s Afriat inequalities (13).

(iii) Use the fact that for each observation in a regime, (13) can hold with equality for another observation in the regime, to choose numbers so that for observation *t* in regime *g*, the rationalizing Afriat utilities are given as in (15) in the proof of Proposition 5:

$$U_t = u^g(\boldsymbol{q}_t, \boldsymbol{r}_t) \equiv min_{\rho \in \Theta(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g)} \left\{ U_\rho + \sum_k [G_+^k(\boldsymbol{q}_\rho, \boldsymbol{r}_t) v_{\rho+}^k + G_-^k(\boldsymbol{q}_\rho, \boldsymbol{r}_t) v_{\rho-}^k] + \lambda_\rho \boldsymbol{p}_\rho \cdot (\boldsymbol{q}_t - \boldsymbol{q}_\rho) \right\}.$$

(iv) Use (ii)'s Afriat numbers U_t, v_{t+}^k, and v_{t-}^k to check that [B]'s monotonicity restrictions are satisfied.

(v) Use (iii)'s rationalizing Afriat utilities to check, regime by regime and observation by observation, that [C]'s conditions (14) are satisfied by scanning along the budget surface.

Proposition 5's conditions (13) involve linear inequalities in a finite number of variables; and its conditions (14) involve nonlinear inequalities in a continuum of $\boldsymbol{q}$ values. Both sets of inequalities are finitely parameterized by the U_t, v_{t+}^k, v_{t-}^k, and λ_t that satisfy [A]'s (13). Thus our procedure satisfies most of footnote 8's desiderata and should inherit much of the tractability of Diewert's (1973) and Varian's (1982) methods for the neoclassical case.

We estimate Selten and Krischker's area a^i by checking the conditions for a rationalization repeatedly for random data, as in Beatty and Crawford.[23] For

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[22] This description ignores the choice of rationalizing regime preferences for drivers who are reference-dependent on less than *K* dimensions, or who are neoclassical. But Propositions 3-5 continue to hold, mutatis mutandis, for such preferences and our arguments extend to them.

[23] We calculate the area by numerical (Monte Carlo) integration over the budget sets. New sets of choices that satisfy the budget constraints are repeatedly drawn and the conditions of interest are tested for each draw. The area is the proportion of those draws that satisfy the conditions. The area estimate converges as the square root of the number of draws. We draw until the uncertainty of the estimate is confined to the fifth decimal place.

a neoclassical model we use WARP or, for models that impose additive separability across goods, Varian's (1983, Theorem 6) conditions.[24] For a reference-dependent model we use Proposition 5's conditions [A]-[C], with Varian's (footnote 21) modifications for additive separability across goods.

When a model of either type does not fit exactly for driver i, we define its proximity π^i as the Euclidean distance, rescaled as a proportion of the maximum possible distance, between driver i's set of observations and the set of sets of observations that fit exactly, with the latter estimated in the process of estimating the area a^i (Beatty and Crawford 2011, pp. 2786-87). However, the conditions for fitting a model exactly increase greatly in stringency with the number of observations, and for the drivers with the seven largest sample sizes of the 21 (1, 4, 10, 16, 18, 20, and 21; sample sizes 39 to 70), repeated sampling (up to 20,000 times) yielded no passes. For such drivers we set $\pi^i = 0$, as if we found passing observations only at the maximum possible distance.

Given Propositions 5's gap between the sufficient and necessary conditions for a reference-dependent rationalization, which precludes precise estimation of proximities and Selten measures for reference-dependent models, we bound them as follows.[25] Imposing Proposition 5's within-regime conditions [A] ((13)) and monotonicity conditions [B], but not its cross-regime conditions [C] ((14)), yields an approximate upper bound on the proximity—"approximate" because conditions [A] assume the Afriat regime utilities and so are sufficient but not necessary; so the true proximity could be higher than the upper bound. Imposing [A]-[C] yields an approximate lower bound on the proximity, which could also be higher than the lower bound. The approximate lower and upper bounds on a reference-dependent model's Selten measure follow similarly. In each case a one-sided approximate lower bound suffices for our purposes.

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[24] Because we need only WARP, not GARP, this is easily implemented for the non-additively separable model using R's `igraph` package. Details and code are in our Replication files.
[25] Bounds are unnecessary for a neoclassical model because GARP is necessary and sufficient for a rationalization without regard to Varian's (1982, Fact 4).

E. *Main Results*

We now summarize our estimation results. Due to the large number of models considered, we proceed sequentially. We first compare neoclassical and reference-dependent models that impose or relax additive separability across goods. This comparison so strongly favors relaxing separability that it can be seen by looking at aggregate summaries. Next, relaxing additive separability across goods, we compare reference-dependent models that differ in the kind and form of reference-dependence. Although reference-dependent models differ significantly from neoclassical models for many drivers, the kind and form of reference-dependence make little difference, as we show again via aggregate summaries. Finally, still relaxing additive separability across goods, we compare neoclassical and reference-dependent models more comprehensively, first via aggregate summaries and then driver by driver.

E.1 *Additive separability across goods*

Additive separability across goods has been assumed in all previous theoretical and empirical work on this topic, but it lacks theoretical or empirical justification, and Proposition 3's characterization of reference-dependent preferences with constant sensitivity shows that it is unnecessary.

Figures 6-9 give the empirical cumulative distribution functions ("CDFs") of proximities and Selten measures for neoclassical and reference-dependent models that impose or relax additive separability across goods.[26] Each CDF pools over all 21 drivers. For reference-dependent models each CDF also pools over all 18 kinds and three forms of reference-dependent model.

Figures 6-7 show that neoclassical models that relax additive separability across goods have significantly higher proximities and Selten measures than

[26] These comparisons also relax KR's constant-sensitivity constraints on how marginal rates of substitution vary across gain-loss regimes. A reference-dependent model must have at least as high a proximity as its neoclassical counterpart, but its Selten measure could be higher or lower. There is a minor exception for experience-based reference-point models, in which we lose one observation (two for models that condition on something) due to the construction of the lag. This can yield a slightly higher upper proximity bound for the neoclassical model.

models that impose it. These aggregate summaries don't show precisely for how many drivers relaxing additive separability improves a neoclassical model's fit enough to justify the added flexibility, but Figure 7's gap in Selten measures is large enough to confirm that relaxing separability is preferable for neoclassical models. Figures 8-9 show that reference-dependent models that relax additive separability across goods also have significantly higher proximities and Selten measures than models that impose it, with the gap in Selten measures again large enough to confirm that relaxing separability is also preferable for reference-dependent models.

We therefore set aside neoclassical and reference-dependent models that impose additive separability across goods and focus on models that relax it.

Figure 6: Empirical CDFs of Proximities for Neoclassical Models Imposing and Relaxing Additive Separability Across Goods

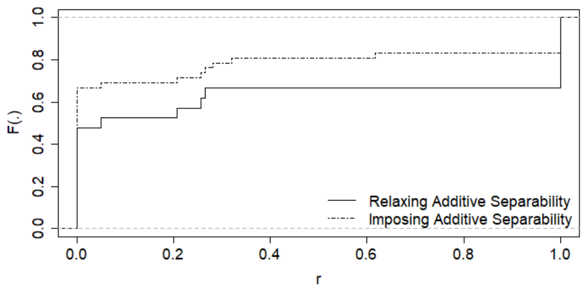


Figure 7: Empirical CDFs of Selten Measures for Neoclassical Models Imposing and Relaxing Additive Separability Across Goods

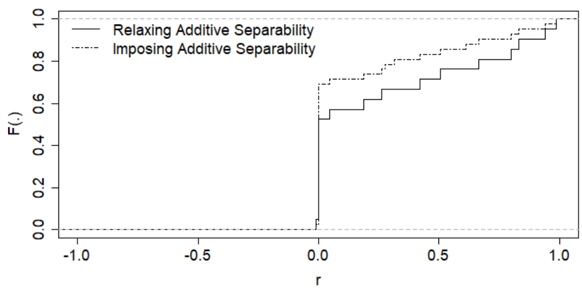


Figure 8: Empirical CDFs of Proximities for Reference-dependent Models Imposing and Relaxing Additive Separability Across Goods

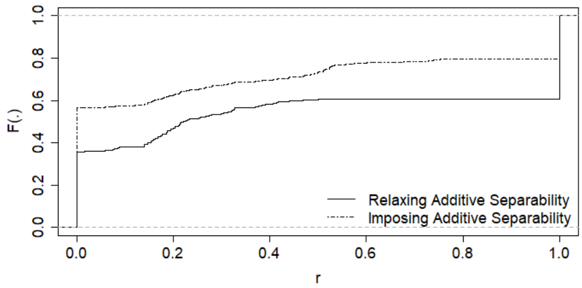
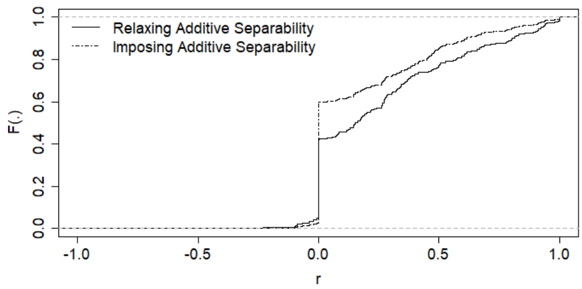


Figure 9: Empirical CDFs of Selten Measures for Reference-dependent Models Imposing and Relaxing Additive Separability Across Goods



E.2. *Reference-point models*

Online Appendix C's Figures C.1-4 give the empirical CDFs of proximities and Selten measures for the unconditional reference-dependent models we consider, again relaxing additive separability across goods. Figures C.1 and C.2 compare the CDFs for our 18 different kinds of reference-point model, again pooling over all 21 drivers. Figures C.3 and C.4 compare the CDFs for our three forms of reference-dependence, pooling over all 21 drivers.

Figures C.2's and C.4's Selten measures show that in these data there are comparatively small differences among models' kinds and forms of reference-dependence. Expectations-based models usually have higher Selten measures than experience-based models, and unconditioned expectations-based models have measures almost as high as conditioned ones, though expectations-based models that are conditioned on day/night usually have even higher Selten measures. Expectations-based models with hours- and earnings-targeting have measures approximately as high as such models with only hours-targeting and somewhat higher measures than such models with only earnings-targeting. Figure C.4's demonstration that expectations-based models with only hours-targeting perform better than those with only earnings-targeting is surprising, given Camerer et al.'s (1997) and Farber's (2005, 2008) exclusive focus on earnings-targeting. We stress that this is not a "neoclassical" effect: Our analysis, like CM's, uses modelable targets to identify and distinguish the influence of hours via consumption utility from that via gain-loss utility (see (12) in Proposition 4's proof and (15) in Proposition 5's proof).

Accordingly, from now on we focus on unconditioned expectations-based models (still relaxing additive separability across goods), but we also report results for unconditioned experience-based reference-point models, in each case considering all three forms of reference-dependence.

E.3 *Comparing neoclassical and reference-dependent models*

Figures 10 and 11 give the empirical CDFs of proximities and Selten measures for neoclassical versus expectations-based reference-dependent models, pooling over drivers and kinds and forms of reference-dependent model. In these aggregate summaries, neoclassical models have higher Selten measures than reference-dependent models for measure values from 0 to 0.5, but slightly lower Selten measures for values from 0.5 to 1.0, so the comparison is inconclusive, we believe due mainly to driver heterogeneity.

Figure 10: Empirical CDFs of Proximities
for Neoclassical and Reference-dependent Models

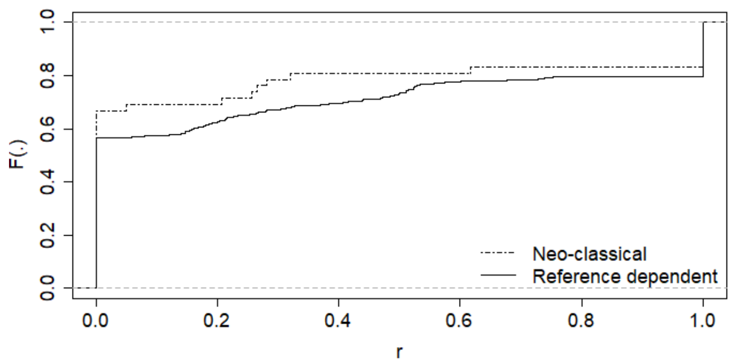
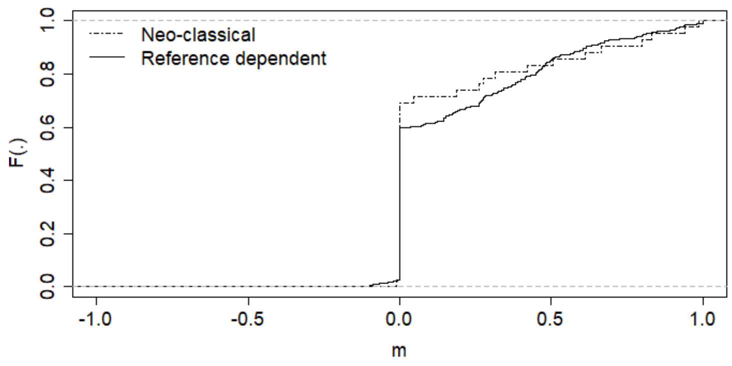


Figure 11: Empirical CDFs of Selten Measures
for Neoclassical and Reference-dependent Models



Figures 12-15 give driver-by-driver plots for neoclassical and expectations-based and experience-based reference-dependent models' proximities and Selten measures. (Online Appendix D's Tables D.1-D.4 give the precise numerical values behind the plots.) Each figure has separate plots for different forms of reference-dependence, with a separate "spoke" for each driver. Figures 12's and 14's proximity plots are centered at -0.25, for clarity a tick below the lowest possible value of 0; with outer rims at the highest possible value of 1. The solid lines trace proximities for the neoclassical model. The shaded areas depict Section III.C's approximate bounds on the proximities for the reference-dependent models. Figures 13's and 15's Selten measure plots are centered at the lowest possible value of -1, with outer rims at the highest possible value of 1. The solid lines trace measures for the neoclassical model.

Overall, the qualitative model comparisons differ only slightly across forms of reference-dependence, so we focus on models with reference-dependence in both hours and earnings, whose plots are in the left-most panels.

Neither model has uniformly higher Selten measures. In Figure 13, the expectations-based reference-dependent model has the same bounded Selten measure as the neoclassical model (thus possibly higher, Section III.C) for seven of 21 drivers (1, 4, 10, 16, 18, 20, and 21); an unambiguously higher measure for six (5, 7, 8, 12, 17, and 19); and an unambiguously lower measure

Figure 12. Proximities for neoclassical and unconditional expectations-based reference-dependent models

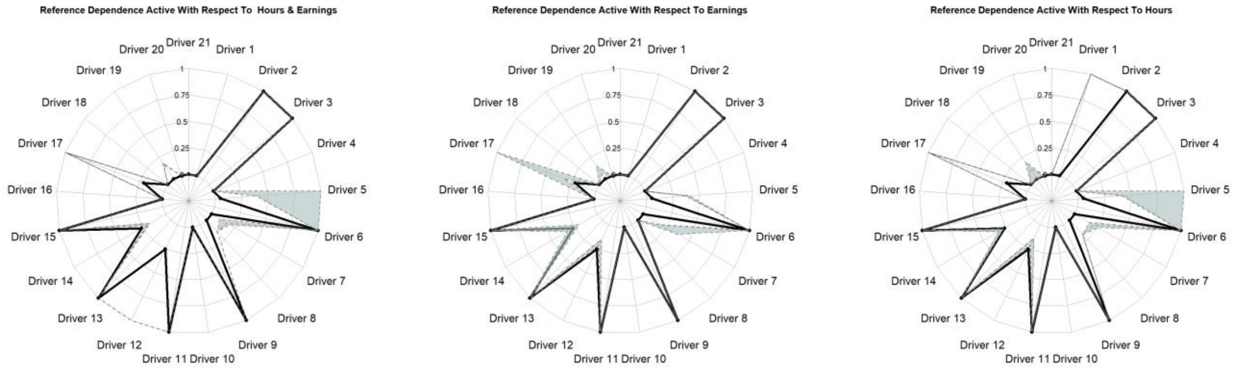


Figure 13. Selten measures for neoclassical and unconditional expectations-based reference-dependent models

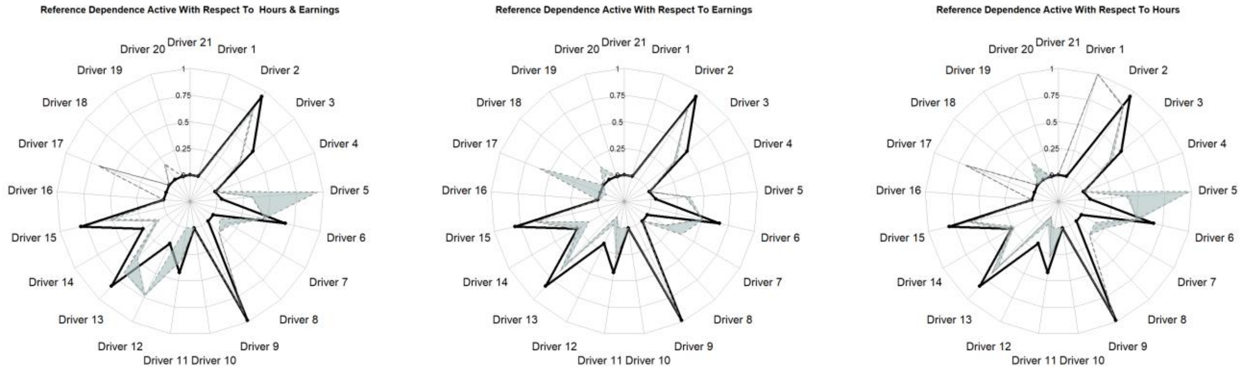


Figure 14. Proximities for neoclassical and unconditional experience-based reference-dependent models

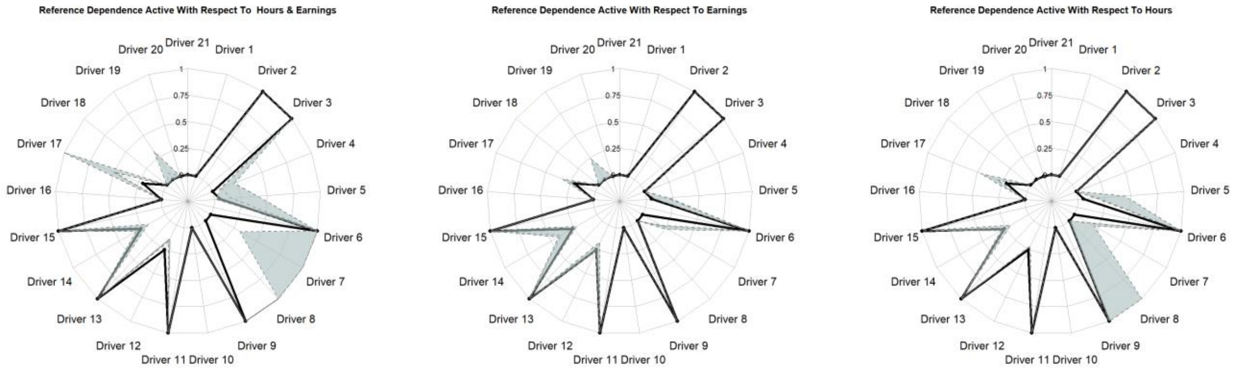
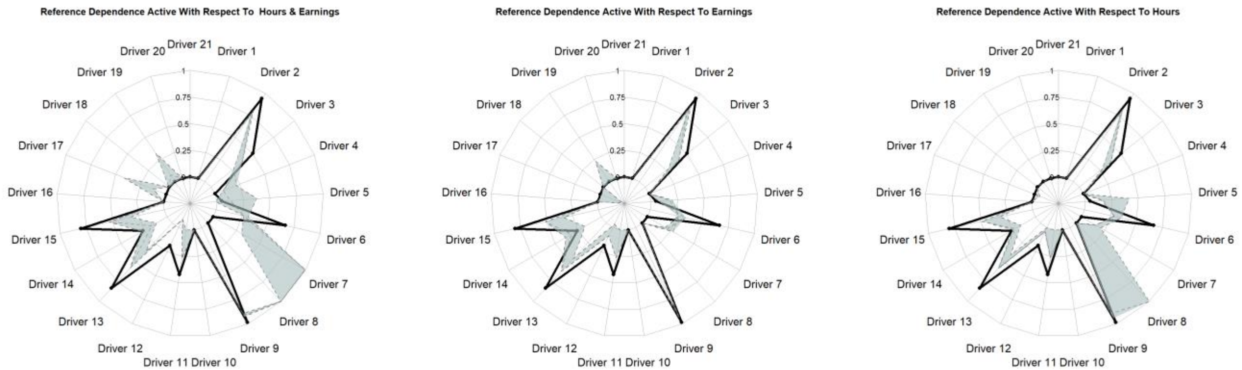


Figure 15. Selten measures for neoclassical and unconditional experience-based reference-dependent models



for eight (2, 3, 6, 9, 11, 13, 14, and 15). Similarly, in Figure 15, the experience-based reference-dependent model has the same (possibly higher) bounded Selten measure as the neoclassical model for six drivers: 1, 10, 16, 18, 20, and 21; a higher measure for four: 4, 8, 17, and 19; a lower measure for nine: 2, 3, 6, 9, 11, 12, 13, 14, and 15; and ambiguous bounds for two: 5 and 7.

However, not all drivers' comparisons are equally informative. Consider first the expectations-based model with reference-dependence in both hours and earnings. With our CPI adjustment, all but one of the six drivers Farber and CM excluded due to small (≤ 10) sample sizes (3, 6, 11, 13, 15, and 17) has an exact neoclassical fit, and the neoclassical model has a higher Selten measure than its more flexible reference-dependent counterpart. This is good news for the neoclassical model, but might only reflect overfitting. For seven other drivers (1, 4, 10, 16, 18, 20, and 21) the sample sizes were too large for us to estimate the set of sets of observations that fit exactly. So for them the proximities are set to 0 for both models and the neoclassical model again has a higher Selten measure; but that does not truly favor the neoclassical over the reference-dependent model. For the eight remaining drivers (2, 5, 7, 8, 9, 12, 14, and 19), the expectations-based model with reference-dependence in hours and earnings has a higher Selten measure for five (5, 7, 8, 12, and 19) and the neoclassical model has a higher Selten measure for three (2, 9, and 14).

Similarly, the experience-based model with reference-dependence in hours and earnings has a higher Selten measure for four drivers (7, 8, 14, and 19) and the neoclassical model has a higher measure for four (2, 5, 9, and 12).

Thus, for many of Farber's drivers who violate rationality for a neoclassical model a reference-dependent model gives a coherent rationality-based account of their choices. Judging by Selten measures, for many of these drivers the reference-dependent model is more parsimonious despite its greater flexibility.

IV. Conclusion

This paper presents a nonparametric analysis of the theory of consumer demand and labor supply with reference-dependent preferences. Our nonparametric model of preferences closely follows KR structural analysis, maintaining their and others' assumption that preferences are additively separable across components of consumption and gain-loss utility, while relaxing some of KR's, Camerer et al.'s (1997), Farber's (2005, 2008), CM's, and other studies' assumptions on functional structure and form.

Our paper makes three main contributions. First, we show that unless reference points are precisely modelable or observable *and* sensitivity is constant, reference-dependent models of consumer demand are flexible enough to fit virtually any data. This suggests that analyses that treat reference points as latent variables may be as strongly influenced by the constraints they impose in estimating reference points as by reference-dependence per se.

Next, assuming modelable reference points and constant sensitivity, we show that reference-dependent models do imply meaningful restrictions in Samuelson's (1947) sense, and we characterize preferences that have constant sensitivity and are continuous. Our characterization derives, from continuity, KR's assumption that gain-loss utility is determined by the good-by-good differences between realized and reference consumption utilities, while relaxing KR's assumption that the sum of consumption and gain-loss utility that determines consumer demand is additively separable across goods, and KR's restrictions on how its marginal rates of substitution vary across regimes, both maintained in all previous theoretical or empirical work.

Our characterization suggests methods for recovering rationalizing preferences when they exist. Although our model would be well suited to a structural econometric re-analysis of Farber's (2005, 2008) data, using sample proxies like CM's for KR's rational-expectations targets, we illustrate it by reconsidering Farber's and CM's econometric analyses nonparametrically, again using sample proxies like CM's for the targets. We allow unrestricted driver heterogeneity and compare alternative models of reference points.

Although the GARP condition for a neoclassical rationalization is violated for most of Farber's drivers, our methods yield coherent reference-dependent rationalizations for almost all of most drivers' choices. We control for varying model flexibility using Beatty and Crawford's (2011) proximity-based variant of Selten and Krischker's (1983) and Selten's (1991) measure of predictive success. For most drivers, models that relax the assumption that preferences are additively separable across goods have significantly higher Selten measures than those that impose additive separability. Relaxing additive separability and KR's assumption on marginal rates of substitution, for many drivers an expectations-based reference-dependent model has at least as high or higher Selten measure than a neoclassical model and provides a parsimonious, rationality-based explanation of their labor supply.[27]

We hope our analysis shows that a generalized reference-dependent model of consumer demand is a useful supplement to neoclassical models of labor supply and that such models can be useful additions to the neoclassical toolkit for analyzing consumer demand, finance, and housing as well as labor supply.

[27] Our analysis also demonstrates the empirical importance of relaxing driver homogeneity. We do not advocate allowing full heterogeneity in all analyses, but the large differences across Farber's drivers suggest that a model with two or three latent classes may be more useful than a reference-dependent model with either homogeneous drivers or unrestricted heterogeneity.

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