General Comments  I have completed reviewing the exams from my part of the class. Below I provide comments on grading, comments on the exams, and suggested answers.

I will put the exams of econ graduate students in your departmental mailbox. If you do not have a departmental mailbox (or I couldn’t find it), I gave your exam to David Miller, who will return it on Monday. On your paper is a score for the exam (out of 100) and final point count (out of 50), which is a weighted average of the homework and the exam (20% for homework and 80% for the exam). In addition, there is a letter grade. Letter grade is a monotonic function of the final point count, with cutoffs: 38.7 for A; 28.8 for A-; 24.4 for B+; 19 for B.

David and I agreed that the letter grade for the class will be an average of letter grades for the two halves. In case of indeterminacies (for example, averaging an A- with an A), we will refer to numerical scores to break ties.

The relationship between what you know and your point count is noisy at best. The relationship between points and letter grades is arbitrary. Here is my interpretation: grades of A- and A are in good position to pass (this portion) of the qual. These papers demonstrate, at least, a working knowledge of the basic concepts. B+ grades indicate answers that would lead to a pass on the qual if the graders were generous. Lower grades are certainly below what it takes to pass the qual. If you received less than 50% on the midterm you did not display adequate command of the material on the exam.

Please contact me if you have questions.

Comments about Exam and Grading

Range of grades: 29-88 with median 58.

General comment: The first and last questions basically require understanding what all of the terms mean, understanding of the definition of strategy (and the specific strategies described), and understanding of the relevant notion of equilibrium. If you understand these things, then answering the problem involves systematic checking. (Constructing an equilibrium in (c) and (d) of Question 4 requires a bit more, but writing down a candidate and checking to see whether it is an equilibrium does not.) I do not claim the requirements in the first sentence are easy, but they are basic and systematic. When faced with a representative game theory question ask: what is the game? have I fully specified strategies? what does it mean to be an equilibrium? have I checked all equilibrium conditions? Evidence that you understand and can carry out these steps is evidence that you “know what is going on.”

Comments on problems:

1. Part (d): explaining why Column does not gain from deviating requires an argument. Stating that Down is dominated in the stage game is not enough (you have seen plenty of examples in which players uses strictly dominated stage game strategies in a repeated game equilibrium. In fact, this is one such game). In part (e) many forgot that to check for SGPE. You must check that one-shot deviations are unattractive in all subgames. Several people concluded that the stated strategies constitute a SPNE but not a Nash equilibrium. But a SPNE must be a NE. This was an unfortunate conclusion.

2. In the first part Kristy noted four problems: failure to realize that the presence of multiple buyers implies that sellers will get all the surplus from trade; specifying that buyers will offer the expected value (15000−6000p) in all cases (this ignores adverse selection); incorrect notions of efficiency; claiming
that there were only two subgames (when in fact there is a subgame following any pair of possible offers. In the second part, some people seemed to misunderstand the formal game, so strategies were incorrectly specified and people wrote down optimization problems without specifying the strategy of the buyers.

3. Several students did not know how to specify full insurance, which made it difficult to get far with the problem.

4. (a) ok.

(b) Many did not guarantee zero-profit off the equilibrium path. The zero-profit condition must hold off the path too. Guaranteeing this requires some care because $e$ enters the profit function. Beyond that, there were problems constructing out-of-equilibrium wage schedule and beliefs. Finally, some people did not check the incentive conditions for the worker. The worker can deviate to any effort level (not just the equilibrium effort level of the other worker).

(c) The central problem was failure to check equilibrium conditions completely. The biggest error of commission was to keep the education level for $\theta = 3$ at 3 and reduce that for $\theta = 1$ to 0; then the wage schedule has to be changed to zero for other beliefs to keep $\theta = 1$ from deviating and satisfy zero profit. This violates the out-of-equilibrium zero profit condition.

(d) Not every $e^*$ satisfies the incentive constraints if wages are set properly. For example, if you set $w(e) = 3e$ off-the-equilibrium path (this is the wage schedule that follows from using the “worst” beliefs and hence is the most likely to support an equilibrium), then you must check that $3e - e^3/3 \leq w(e^*) - (e^*)^3/3$ for all $e \neq e^*$.

(e) The same concerns from the first two parts arise.
1. The questions that follow refer to the infinitely repeated game with stage-game payoff matrix:

<table>
<thead>
<tr>
<th></th>
<th>Left</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up</td>
<td>9, 0</td>
<td>2, 2</td>
</tr>
<tr>
<td>Down</td>
<td>10, 0</td>
<td>5, 5</td>
</tr>
</tbody>
</table>

Assume players discount payoffs with the common discount factor $\delta \in (0, 1)$.

(a) Describe the set of feasible and strictly individually rational payoffs for this game.

Feasible: Convex hull of $\{(9, 0), (2, 2), (10, 0), (5, 5)\}$. Individually rational: Row has dominant strategy in the stage game (Down). The worst payoff available to Row if Row best responds is 5. Column has a dominant strategy (Right). The worst payoff available to Column in 2. Hence the minmax point is $(5, 2)$. The set of points that are strictly individually rational and feasible is the intersection of the feasible set with points strictly greater than $(5, 2)$. It is the interior of a triangle with vertices $(5, 2)$, $(5, 5)$, and $(8, 2)$.

Consider the strategy for the Row player: play Down initially and following any history in which Column player has played Left in all previous odd periods; otherwise, play Up. Consider the strategy for the Column player: play Left in an odd period provided that previously Row has always played Down and Column has played Left in all previous odd periods; otherwise, play Right.

(b) When the players use these strategies, what is the outcome?

Row obtains 10 in odd periods and 5 in even periods. Average payoff: $(10 + \delta 5)/(1 + \delta)$. Column obtains 0 in odd periods and 5 in even periods. Average payoff: $5\delta/(1 + \delta)$.

(c) What are the payoffs associated with this outcome? (above)

(d) For what values of $\delta$ do the strategies constitute a Nash equilibrium?

Row player has no incentive to deviate because there is no short-run gain and following a deviation Row receives its lowest payoff. Column may gain from deviating. Following a deviation, the best that Column can do is receive 2 in each period. Hence there are really only two things to check. If Column deviates in an odd period she receives $5\delta + 2(1 - \delta)$ so you must have

$$5(1 - \delta) + 2(\delta) \leq 5\delta/(1 + \delta)$$

and if Column deviates in an even period she receives $2\delta$ so you must also have

$$2\delta \leq 5/(1 + \delta).$$

The first inequality implies the second (the RHS of the second is larger than the RHS of the first and the LHS of the second is smaller than the LHS of the first).

A bit of algebra shows (I think) that you need $\delta > (\sqrt{69} - 3)/6$.

(e) For what values of $\delta$ do the strategies constitute a Subgame perfect equilibrium?

No values. After a history in which there has been a deviation (for example, if Column plays Right in the first period), the strategy profile specifies that Row play Up for all future histories and Column play Right for all future histories. Row can do better by playing Down (in all periods following the given history).
2. Suppose a used car is sold at an auction. The car may be either good quality or bad quality. The owner of the car knows its true value. The potential buyers have a prior distribution over quality; $p \in (0, 1)$ is the probability that the car is bad. Suppose that to the seller the value of a bad car is $6,000 and the value of a good car is $12,000. Buyers value a bad car at $9,000 and a good car at $15,000. These values are common knowledge. All players are risk neutral. Buyers seek to maximize the value of the car minus the price paid (a buyer receives zero if he does not buy). The seller seeks to maximize the price she receives (if she does not sell the car, her utility is her value of the car). Both buyers simultaneously offer prices and the seller accepts at most one of these. Describe the subgame perfect equilibrium of the game. For what values of $p$ is the equilibrium of the market efficient?

At a price $> 12,000$ all seller sells both good and bad cars. The average quality from the standpoint of the buyer is $(1-p)$ $15,000 + p9,000 = 15,000 - 6,000p$. So as long as $p \leq 1/2$, there is an equilibrium in which the car is always sold at the price $15,000 - 6,000p$. At this price the buyers make zero profit and the seller always sells. More precisely, the strategies are: buyers offer $15000 - 6000p$, the seller accepts the highest price offered that is no more than her reservation value (and randomizes uniformly across buyers in case of indifference). When $p > 1/2$ the price is 9000 and only bad cars are sold. The outcome is efficient when $p \leq 1/2$ but when $p > 1/2$ it would be possible to make the seller and a buyer better off if high-valuation sellers could trade (at a price between the 9000 and 15000).

Suppose now that the seller is an intermediary. She can identify good and bad cars and buy as many as 100 cars for $6,000 per bad car and $12,000 per good car. (100 is the capacity of her car dealership.) A market equilibrium for this environment consists of $(n_G, n_B, P)$ where $n_i$ in the number of cars of quality $i$ that the seller sells ($i = G, B$), $P = (15,000 n_G + 9,000 n_B)/(n_G + n_B)$ is equal to the average value (to the buyer) of these cars, and $(n_G, n_B)$ maximizes the seller’s net payoff (subject to $n_G + n_B \leq 100, n_B, n_G \geq 0$). Characterize a used-car market equilibrium.

The market equilibrium involves a price $P$. Given the price, the seller decides what kind of car to buy. If the price is less than 6000, then the seller would never acquire cars for sale. Otherwise, the seller will acquire only bad cars. The buyers will make this inference in equilibrium, so the only market clearing prices are those associated with $p = 1$. Hence $P \leq 9000$. As there will be excess demand when $P < 9000$, market clearing requires $P = 9000$. Hence only bad cars are sold and they are sold at the valuation of the buyer.

3. Consider a market for insurance in which there are two types of agent. Each type agent has wealth 5 when he is not injured and wealth 1 when he is injured. High-risk types are injured with probability $3/4$; low-risk types are injured with probability $1/4$. Agents have a strictly concave utility function defined over wealth. Insurance firms cannot identify the risk type of agents. They can offer insurance contracts, which take the form of a price $P$ that the agent pays for the contract and an amount $I$ that the insurance company pays to the agent in the event of an injury. Hence an uninsured agent obtains expected utility $p_H U(5) + (1-p_H) U(1)$ for $p_H = 3/4$ and $p_L = 1/4$ and an agent who buys insurance $I$ for the price $P$ obtains expected utility $p_H U(5 - P) + (1-p_L) U(1 - P + I)$

(a) How much must a firm charge to earn non-negative profits on full insurance to the low-risk group?

Full insurance means $I = 4$. The expected payment is therefore 1 and the price must be at least one.

(b) How much must a firm charge to earn non-negative profits on full insurance to the high-risk group?

The price must be at least 3.
(c) How much must a firm charge to earn non-negative profits on full insurance when customers are equally likely to be high-risk or low-risk?

The price must be at least 2.

(d) Identify a pair of insurance contracts in which: one contract provides full insurance to high-risk customers and earns zero profits when purchased by only high-risk customers; the other contract earns zero profits when purchased only by low-risk customers; and high-risk customers obtain the same utility from either contract. (Your answer should take the form of a system of equations that describe the contracts.)

Full insurance for the high-risk group yields utility $U(2)$. The high-risk group would be indifferent between this contract and another one if

$$U(2) = 0.25U(5 - P) + 0.75U(1 - P + I)$$

for the contract to make zero profits on low-risk customers it must be that $P = I/4$. There is also a “participation” or individual rationality constraint for the low-risk type, but this will be satisfied automatically (by single-crossing) if the high-risk group is indifferent between the two contracts.

4. Suppose that, in the educational signaling model the payoff to a firm from hiring a worker of type $\theta$ with education $e$ at wage $w$ is

$$f(e, \theta) - w = 3e\theta - w.$$  

The utility of a worker of type $\theta$ with education $e$ receiving a wage $w$ is

$$w - c(e, \theta) = w - \frac{e^3}{\theta}.$$  

Suppose the support of the firms’ prior beliefs on $\theta$ is $\Theta = \{1, 3\}$. [In this model, first nature selects $\theta$ from a known distribution on $\Theta$. Next, the worker learns her type and chooses a level of education. After that, the firms (assume that there are two) simultaneously make wage offers. Finally, the worker accepts at most one of these offers.]

(a) The full-information education level $e^f(\theta)$ is defined as the education level that solves:

$$\max_{e \geq 0} f(e, \theta) - c(e, \theta).$$

Find $e^f(\theta)$ for $\theta \in \Theta$.

The problem asks you to maximize $3e\theta - \frac{e^3}{\theta}$ with respect to $e$. Hence the solution is;

$$e^f(\theta) = \theta$$

(b) Describe an equilibrium in which both types of worker choose their full information education level. (Your answer should fully specify the strategies of the worker and the firms. It should also confirm that the strategies satisfy equilibrium conditions.)

$e(\theta) = \theta; w(e) = 3e$ if $e < 3$ and $w(e) = 9e$ if $e \geq 3$. The firms believe that $\theta = 1$ for education less than 3 and $\theta = 3$ for $e \geq 3$. The wage schedule earns zero-profits given beliefs, so it is consistent with equilibrium behavior for the firms. The beliefs are consistent with the worker’s strategy (any
beliefs are consistent with education levels not equal to 1 and 3). Finally, the worker is responding optimally because the $\theta = 1$ worker prefers to get $3 - 1$ (by following the equilibrium) to $9e - e^3$ for picking $e \geq 3$ or $3e - e^3$ for picking $e < 3$ (and $e \neq 1$) and the $\theta = 3$ worker prefers the equilibrium utility of $27 - 9$ to $9e - e^3/3$ for any other $e$ and the type $\theta = 3$ worker cannot do better than $9e - e^3/3$ for any choice of $e$ (she does worse if $e < 3$).

(c) Are there other separating equilibria? If so, describe one. Do they depend on the prior distribution on $\Theta$?

Another separating equilibrium involves $w(e) = 3e$ if $e < 3.1$ and $w(e) = 9e$ if $e \geq 3.1$ and the $e(\theta) = 1$ if $\theta = 1$ and $3.1$ if $\theta = 3$. Here the higher type invests “too much” in education, but it is optimal to do so because if invests less she’ll be treated as a low type. (Concavity of the Sender’s full information utility function guarantees that higher education levels are not attractive.) The Sender is responding optimally because $9(3.1) - (3.1)^3/3 > 3e - e^3/3$ for $e < 3.1$. (If you insist, you can check the last inequality by noting that the right-hand side is maximized when $e = 3$ and attains the value 0, while the left-hand side is strictly positive.)

(d) Describe an equilibrium in which both types of worker select the same education level.

There are many equilibria of this kind. For concreteness, imagine that $w(e) = 3e$ for $e \neq \bar{\theta}$ and $w(\bar{\theta}) = 3\bar{\theta}^2$ ($\bar{\theta}$ is the average type). The firms believe that $e = \bar{\theta}$ signals average type and all other education levels signal low type. The worker always gets education $e = \bar{\theta}$. This is the best choice because $3e - e^3/\theta < 3\bar{\theta}^2 - \bar{\theta}^3/\theta$ for both $\theta$.

Now suppose that $\Theta = \{1, 2, 3\}$ (all three types arise with positive probability).

(e) Show that it is not an equilibrium for all three types of worker to choose their full information level.

When we try to construct the full-information equilibrium (in which type $\theta$ obtains education $\theta$), we need to check that the lowest type does not wish to deviate and set education 2 instead of 1: $2 \geq 12 - 8 = 4$, which of course is not true. (The right hand side is $w(2) - c(2, 1)$.)